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A Dynamic Model for Single-Walled Carbon Nanotubes with Small Initial Curvature based on Nonlocal-Strain Gradient Theory

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Abstract: *Many experimental observations have shown that Carbon nanotubes (CNTS) have the characteristics of waviness along their axial dimension. Both Non-local stress effect and strain gradient effect, which have great influence on the mechanical properties of CNTS. In order to simultaneously investigate the effects of initial curvature and different types of scale effects on the mechanical properties of carbon nanotubes, a dynamic Bernoulli-Euler beam model was proposed based on the nonlocal strain gradient theory and the forced vibration of single-walled CNTS was studied. The results indicate that the strain gradient effect strengthened the stiffness of CNTS, while the non-local stress effect softened the stiffness of CNTS. In addition, small initial curvature complicates the relationship between frequency difference and vibration amplitude in forced vibration of the CNTS.*

Keywords: *Non-local stress; Strain gradient; Carbon nanotubes; Euler-Bernoulli beam; Galerkin method; Multi-scale method; Vibration characteristics*

1. Introduction

Since Carbon nanotubes (CNTS) was found in 1991, it is always the hot spot of the research [1]. CNTS has many excellent mechanics, electricity and chemical performance with high strength and toughness [2-3]. As CNTS have high length-diameter ratio, it can be used as an ideal composites of strengthening phase [4]. In order to excavate the huge potential application of CNTS, the mechanical properties of it have been studied extensively [5-6].

At present, many achievements have been got on CNTS both on experimental and theoretical aspect. For experimental study, Treacy et al. [7] measured the amplitude of thermally induced vibration of the free end of CNTS through TEM in 1996. Wong et al. [8] applied an AFM probe to press CNTS, and then measured the force and deflection values during the deformation of CNTS. However, it is extremely difficult to obtain the mechanical properties of CNTS experimentally at the nanoscale. Therefore, the method of studying CNTS is mainly in theoretical analysis. The main theoretical analysis methods are molecular dynamics, continuum mechanics and so on. Among these methods, molecular dynamics simulation is a reliable and highly recognized method. Yakobson et al. [9] used molecular dynamics to study the buckling position and energy curves of CNTS in axial compression bending. Kowaki et al. [10] studied the relationship between the melting point and the geometry size of SWCNTS. But, in the molecular dynamics method, this process requires high ability of computer operation, which is quite time comsuming[11-12].therefore, in order to overcome the deficiency of molecular dynamics simulation, the classical continuum model has been widely applied. YAN et al. [13-14] studied the stability and dynamics of CNTS based on the classical continuum model.

However, it should be pointed out that the classical continuum model can't simulate CNTS under the environment of micro nano small scale effect. As the scale effect of material mechanical properties is very remarkable, it restricts the application of continuous medium model [15]. In order to solve this problem, reseracher introduced the theory of nonlocal elasticity and combined this theory with the continuum mechanics model, which can not only simulate the influence of microscopic scale effect on nanostructures, but also have the characteristic of simple calculation. Huang Kun et al. [16] combined the theory of nonlocal elasticity with Euler-Bernoulli beam to study the static and dynamic characteristics of small initial curvature CNTS. Sudak [17], based on the theory of nonlocal elasticity, used the beam model to study the buckling problem of CNTS under the consideration of small-scale effects and van der Waals forces. Although this method is widely used, it also has its shortcomings. For example, the accuracy and rationality of the model is not satisfactory in the study of high-frequency waves [18]. Moreover, since the scale effect affects the stress and strain of CNTS, the non-local theory is not enough to analyze the scale effect only from the perspective of stress. Aifantis [19-20] proposed to analyze the impact of scale effect on material strain by strain gradient elasticity theory. Based on strain gradient theory, Chen et al. [21] calculated the intermolecular forces which is identical with the mimetic results of molecular dynamics. Challamel [22] used the strain gradient elastic model and Eringen's non-local elastic model to discuss the main properties of small-scale effects in vibration analysis. Combined the high-order non-local elasticity theory with the strain gradient theory, Lim et al. [23] given the beam model governing equation and boundary conditions of analyzing the wave characteristics of CNTS, and then discussed the law of the influence of scale effect on the wave parameters of the beam model.

In this paper, to investigate the effects of different scale effects and initial curvature of materials on the dynamic characteristics of single-walled carbon nanotubes (SWCNTS), a Bernoulli–Euler beam model with non-local stress and strain gradient coupling constitutive relation is proposed. Based on the model, the mechanical response of SWCNTS is analyzed.

2. Non-Local Strain Gradient Bernoulli-Euler Beam Model with Small Initial Curvature

As shown in Fig.1, the single-walled carbon nanotubes (SWCNTS) were expressed in a three-dimensional Cartesian coordinate system, where L is the pipe length and w_0 is the initial deformation, x and y are axial and Horizontal coordinates, respectively.

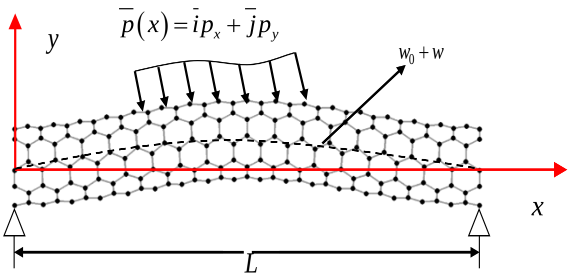

Fig1. *Schematic configuration of a nanobeam*

According to Lim's research [23], the non-local elastic stress constitutive and strain gradient constitutive are coupled, and the new carbon nanotubes stress-strain constitutive equation can be obtained as follows:

$$\left[1-(e_1a)^2\nabla^2\right]\left[1-(e_0a)^2\nabla^2\right]\sigma_{xx}=E\left[1-(e_1a)^2\nabla^2\right]\varepsilon_{xx}-El^2\left[1-(e_0a)^2\nabla^2\right]\nabla^2\varepsilon_{xx} \tag{1}$$

Where e_0a and e_1a is the non-local scale effect parameter, l is the strain gradient effect parameter, which represent the size of the influence range of the small-scale effect and the strength of the scale effect. σ_{xx} is the stress tensor, ε_{xx} is the strain tensor, ∇^2 is Laplace operator. E is the modulus of elasticity. For the Euler-Bernoulli beam model with initial curvature shown in Fig.1, it can be assumed that $e_0=e_1=e$, then the Eq. (1) can be simplified as:

$$\left[1-(ea)^2\nabla^2\right]\sigma_{xx}=E\left(1-l^2\nabla^2\right)\varepsilon_{xx} \tag{2}$$

For the one-dimensional Euler-Bernoulli beam model, if only the normal stress and normal strain in the axial direction are considered, Eq. (2) can be further simplified as:

$$\left[1-\mu^2\frac{\partial^2}{\partial x^2}\right]\sigma(x)=E\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\varepsilon(x) \tag{3}$$

where $\sigma(x),\varepsilon(x)$ are stress and strain functions, and $\mu=ea$.

we obtain bending moment and axial force as following

$$M=\int y\sigma(x)dA \tag{4}$$

$$N=\int \sigma(x)dA \tag{5}$$

Where A is the cross-sectional area of a SWCNTS beam.

the displacement strain equation of Von Karman is used to represent the X-axis strain tensor [24]:

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}-y\frac{\partial^2 w}{\partial x^2}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x} \tag{6}$$

From Eq. (3), (4), (5) and (6), the relationship between bending moment, axial force and displacement is obtained as follows

$$M-\mu^2\frac{\partial^2 M}{\partial x^2}=-EI\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 w}{\partial x^2} \tag{7}$$

$$N-\mu^2\frac{\partial^2 N}{\partial x^2}=EA\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right) \tag{8}$$

Where $I=\int y^2 dA$ is the moment of inertia of the cross section.

For the classical long beam (Bernoulli-Euler beam), compared to the axial stress, the influence of shear stress is small. Therefore, the axial stress and strain can be deduced without considering the influence of shear force. The control equation of Euler-Bernoulli beam theory with Newton's second law is as follows [25-26]:

$$\frac{\partial^2 M}{\partial x^2}+\frac{\partial}{\partial x}\left[N\left(\frac{\partial w}{\partial x}+\frac{\partial w_0}{\partial x}\right)\right]+p_y=m\frac{\partial^2 w}{\partial t^2} \tag{9}$$

$$\frac{\partial N}{\partial x} + p_x = m\frac{\partial^2 u}{\partial t^2} \tag{10}$$

where p_x and p_y are the components of the external load in the x and y directions, respectively.

By substituting Eq. (9) and (10) into Eq. (7) and (8), we can get:

$$M - \mu^2\left\{m\frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial x}\left[N\left(\frac{\partial w}{\partial x} + \frac{\partial w_0}{\partial x}\right)\right] - p_y\right\} = -EI\left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 w}{\partial x^2} \tag{11}$$

$$N - \mu^2\frac{\partial}{\partial x}\left(m\frac{\partial^2 u}{\partial t^2} - p_x\right) = EA\left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right) \tag{12}$$

Integrating Eq. (11) twice with respect to x, we have:

$$\frac{\partial^2 M}{\partial x^2} = \mu^2\frac{\partial^2}{\partial x^2}\left\{m\frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial x}\left[N\left(\frac{\partial w}{\partial x} + \frac{\partial w_0}{\partial x}\right)\right] - p_y\right\} - \frac{\partial^2}{\partial x^2}\left[EI\left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 w}{\partial x^2}\right] \tag{13}$$

Substitute Eq. (13) into Eq. (9), we get:

$$\begin{aligned}&\mu^2\frac{\partial^2}{\partial x^2}\left\{m\frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial x}\left[N\left(\frac{\partial w}{\partial x} + \frac{\partial w_0}{\partial x}\right)\right] - p_y\right\} - \frac{\partial^2}{\partial x^2}\left[EI\left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 w}{\partial x^2}\right]\\&+\frac{\partial}{\partial x}\left[N\left(\frac{\partial w}{\partial x} + \frac{\partial w_0}{\partial x}\right)\right] + p_y = m\frac{\partial^2 w}{\partial t^2}\end{aligned} \tag{14}$$

Substituting Eq. (12) into Eq. (14) and neglecting the terms of μ^4 , the transverse motion equation can be obtained:

$$\begin{aligned}&\mu^2\frac{\partial^2}{\partial x^2}\left\{m\frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial x}\left[EA\left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right)\left(\frac{\partial w}{\partial x} + \frac{\partial w_0}{\partial x}\right)\right]\right\}\\&+\frac{\partial}{\partial x}\left\{\left\{EA\left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right) + \mu^2\frac{\partial}{\partial x}\left(m\frac{\partial^2 u}{\partial t^2} - p_x\right)\right\}\left(\frac{\partial w}{\partial x} + \frac{\partial w_0}{\partial x}\right)\right\}\\&= m\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left[EI\left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 w}{\partial x^2}\right] + \mu^2\frac{\partial^2 p_y}{\partial x^2} - p_y\end{aligned} \tag{15}$$

Similarly, the first partial derivative of Eq. (12) with respect to x is obtained as follows:

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left[\mu^2\frac{\partial}{\partial x}\left(m\frac{\partial^2 u}{\partial t^2} - p_x\right) + EA\left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right)\right] \tag{16}$$

Substituting Eq. (16) into Eq. (10), and leting $p_x = 0$, we get:

$$\frac{\partial}{\partial x}\left[\mu^2\frac{\partial}{\partial x}\left(m\frac{\partial^2 u}{\partial t^2}\right) + EA\left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right)\right] = m\frac{\partial^2 u}{\partial t^2} \tag{17}$$

For slender beams, longitudinal inertia terms can be ignored. Therefore, longitudinal displacement u is mainly caused by transverse deformation. Then, it can be got from Eq. (17):

$$\frac{\partial}{\partial x}\left[EA\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right)\right]=0 \tag{18}$$

From Eq. (18), it can be obtained:

$$\frac{\partial^2 u}{\partial x^2}=-\frac{\partial}{\partial x}\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]+\cdots \tag{19}$$

The equation (18) is integrated twice with respect to x:

$$\frac{\partial u}{\partial x}=-\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]+c_1 \tag{20}$$

$$u=-\int\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]dx+c_1(x)+c_2 \tag{21}$$

Where $c_1(t), c_2(t)$ are functions of time, which are determined by the boundary conditions of u .

For a beam with immovable ends (hinged or clamped end), imposing the boundary conditions $u(0)=u(L)=0$, yields:

$$c_2=0 \quad c_1=\frac{1}{L}\int_0^L\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]dx \tag{22}$$

Substitute Eq. (22) into Eq. (20) and we get:

$$\frac{\partial u}{\partial x}=-\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]+\frac{1}{L}\int_0^L\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]dx \tag{23}$$

Substitute Eq. (23) into Eq. (15) and we get:

$$\begin{aligned}&\mu^2\frac{\partial^2}{\partial x^2}\left\{m\frac{\partial^2 w}{\partial t^2}-\frac{\partial}{\partial x}\left[EA\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left\{\frac{1}{L}\int_0^L\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]dx\right\}\left(\frac{\partial w}{\partial x}+\frac{\partial w_0}{\partial x}\right)\right]\right\}\\&+\frac{\partial}{\partial x}\left\{\left\{EA\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left\{\frac{1}{L}\int_0^L\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]dx\right\}+\mu^2\left(m\frac{\partial^2 u}{\partial t^2}-p_x\right)\right\}\left(\frac{\partial w}{\partial x}+\frac{\partial w_0}{\partial x}\right)\right\}\\&=m\frac{\partial^2 w}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 w}{\partial x^2}\right]-\mu^2\frac{\partial^2 p_y}{\partial x^2}-p_y\end{aligned} \tag{24}$$

Eq. (24) omits the nonlinear inertia term, and can be simplified as:

$$\begin{aligned}&\mu^2\frac{\partial^2}{\partial x^2}\left\{m\frac{\partial^2 w}{\partial t^2}-\frac{\partial}{\partial x}\left[EA\left\{\frac{1}{L}\int_0^L\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]dx\right\}\left(\frac{\partial w}{\partial x}+\frac{\partial w_0}{\partial x}\right)\right]\right\}\\&+\frac{\partial}{\partial x}\left\{\left\{EA\left\{\frac{1}{L}\int_0^L\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]dx\right\}\right\}\left(\frac{\partial w}{\partial x}+\frac{\partial w_0}{\partial x}\right)\right\}\\&=m\frac{\partial^2 w}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 w}{\partial x^2}\right]-\mu^2\frac{\partial^2 p_y}{\partial x^2}-p_y\end{aligned} \tag{25}$$

Here we have introduced dimensionless variables $\bar{x} = x/L,\quad \bar{w} = w/L,\quad \bar{t} = \omega_0 t,\quad \overline{w_0} = w_0/L$, and drop the caps of $\bar{x},\quad \bar{w},\quad \bar{t},\quad \overline{w_0}$ in Eq.(25) for the simplifications, $\omega_0 = \sqrt{\pi^4 EI / L^4 m}$ is the nature frequency for a hinged–hinged beam without the nonlocal effect and the initial curvature.

$$\frac{EA}{m\omega_0^2 L^2}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w_0}{\partial x^2}\right)\left\{\int_0^L\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]dx\right\}-\frac{\mu^2 EA}{m\omega_0^2 L^4}\left(\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w_0}{\partial x^4}\right)\left\{\int_0^L\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}\right]dx\right\}$$
$$=\frac{\partial^2 w}{\partial t^2}-\frac{\mu^2}{L^2}\frac{\partial^4 w}{\partial x^2\partial t^2}+\frac{1}{\pi^4}\frac{\partial^4 w}{\partial x^4}-\frac{l^2}{L^2\pi^4}\frac{\partial^6 w}{\partial x^6}+\frac{\mu^2}{m\omega_0^2 L^3}\frac{\partial^2 p_y}{\partial x^2}-\frac{p_y}{m\omega_0^2 L} \tag{26}$$

If the beam-ends are not subjected to external moments, for example a hinged–hinged beam, the associated boundary conditions are $w(0,t) = w(1,t) = \frac{\partial^2 w(0,t)}{\partial x^2} = \frac{\partial^2 w(1,t)}{\partial x^2} = 0$.

As Eq. (26) is a nonlinear partial differential equation with integral term, it is difficult to get an analytic solution. The Galerkin method is one of the most widely used approximate methods, and it is employed to solve the partial differential equations. Under the normalized boundary conditions, the approximate solution of differential Eq. (26) can be set as:

$$w = \sum_{n=1}^{\infty}\eta_n \sin n\pi x \tag{27}$$

Here only the first term is used. For the sake of simplification, we assume $w_0 = d\sin\pi x$. Substituting the solution into Eq. (26), and multiplying it by $\sin\pi x$ and integrating over the interval [0,1] leads to:

$$\frac{1}{2}\left(1+\frac{\mu^2}{L^2}\pi^2\right)\ddot{\eta}+\lambda_1\eta+\lambda_2\eta^2+\lambda_3\eta^3 = p \tag{28}$$

where: $p = \frac{1}{m\omega_0^2 L}\int_0^1\left(1-\frac{\mu^2}{L^2}\frac{\partial^2}{\partial x^2}\right)p_y \sin x dx,\quad \lambda_1 = \frac{1}{2}\left[1+\frac{l^2\pi^2}{L^2}+\frac{d^2\pi^4 EA}{2\omega_0^2 L^4 m}\left(1+\frac{\mu^2}{L^2}\pi^2\right)\right]$

$$\lambda_2 = \frac{3d\pi^4 EA}{8\omega_0^2 L^3 m}\left(1+\frac{\mu^2}{L^2}\pi^2\right),\quad \lambda_3 = \frac{\pi^4 EA}{8\omega_0^2 L^2 m}\left(1+\frac{\mu^2}{L^2}\pi^2\right),\quad \omega_0^2 = \frac{\pi^4 EI}{l^4 m}$$

The transfer term of Eq. (28) is simplified

$$\ddot{\eta}+k_1^2\eta+k_2\eta^2+k_3\eta^3 = p_0 \tag{29}$$

where：

$$k_1^2 = \left(\frac{L^2}{L^2+\mu^2\pi^2}+\frac{l^2\pi^2}{L^2+\mu^2\pi^2}+\frac{d^2\pi^4 EA}{2\omega_0^2 L^4 m}\right),\quad k_2 = \frac{3d\pi^4 EA}{4\omega_0^2 L^3 m},\quad k_3 = \frac{\pi^4 EA}{4\omega_0^2 L^2 m}$$

$$p_0 = \frac{2L}{m\omega_0^2\left(L^2+\mu^2\pi^2\right)}\int_0^1\left(1-\frac{\mu^2}{L^2}\frac{\partial^2}{\partial x^2}\right)p_y \sin x dx$$

3. FIRST-ORDER FORCED VIBRATION EQUATION

By adding damping term in Eq. (29), the damped forced vibration equation of CNTS in first-order mode can be obtained

$$\ddot{\eta}+C\dot{\eta}+k_1^2\eta+k_2\eta^2+k_3\eta^3 = p_0 \tag{30}$$

In order to make the quadratic and cubic strongly nonlinear terms playing a role in the process of the same order perturbation, the above equation can be written as:

$$\frac{\partial^2\eta}{\partial t^2}+2\varepsilon^2 c\frac{\partial\eta}{\partial t}+k_1^2\eta+\varepsilon k_2\eta^2+\varepsilon^2 k_3\eta^3=\varepsilon^2 f\cos\omega t \tag{31}$$

where

$$k_1^2=\left(\frac{L^2}{L^2+\mu^2\pi^2}+\frac{l^2\pi^2}{L^2+\mu^2\pi^2}+\frac{d^2\pi^4 EA}{2\omega_0^2L^4m}\right),\quad k_2=\frac{3d\pi^4 EA}{4\omega_0^2L^3m},\quad k_3=\frac{\pi^4 EA}{4\omega_0^2L^2m}$$

$$\varepsilon^2 f=\frac{2L}{m\omega_0^2\left(L^2+\mu^2\pi^2\right)}\int_0^1\left(1-\frac{\mu^2}{L^2}\frac{\partial^2}{\partial x^2}\right)p_{y1}\sin x dx$$

We seek an approximate solution to this equation by letting:

$$\eta=\eta_0\left(T_0,T_1,T_2\right)+\varepsilon\eta_1\left(T_0,T_1,T_2\right)+\varepsilon^2\eta_2\left(T_0,T_1,T_2\right)+\cdots \tag{32}$$

Since the excitation is $O\left(\varepsilon^2\right),\omega-k_1$ is assumed to be $O\left(\varepsilon^2\right)$ for consistency. Hence we put:

$$\varepsilon^2\sigma=\omega-k_1 \tag{33}$$

Substituting Eq. (32) and Eq. (33) into Eq. (31) and equating the coefficients of ε^0, ε and ε^2 on both sides, we obtain

$$D_0^2\eta_0+k_1^2\eta_0=0 \tag{34}$$

$$D_0^2\eta_1+k_1^2\eta_1=-2D_0D_1\eta_0-k_2\eta_0^2 \tag{35}$$

$$D_0^2\eta_2+k_1^2\eta_2=-2D_0D_1\eta_1--2D_0D_2\eta_0-D_1^2\eta_0-2cD_0\eta_0-2k_2\eta_0\eta_1-k_3\eta_0^3+f\cos\left(k_1T_0+\sigma T_2\right) \tag{36}$$

The general solution of Eq. (34) can be written in the following form:

$$\eta_0=A\left(T_1,T_2\right)\exp\left(ik_1T_0\right)+\bar{A}\left(T_1,T_2\right)\exp\left(-ik_1T_0\right) \tag{37}$$

Substituting η_0 into Eq. (35) yields

$$D_0^2\eta_1+k_1^2\eta_1=-2ik_1D_1A\exp\left(ik_1T_0\right)-k_2\left[A^2\exp\left(2ik_1T_0\right)+A\bar{A}\right]+cc \tag{38}$$

Eliminating the terms in Eq.(38) that produce secular terms in η_1 yields $D_0A=0$ or $A=A\left(T_2\right)$.Hence the solution of Eq.(38) becomes:

$$\eta_1=\frac{k_2}{k_1^2}\left[-2A\bar{A}+\frac{1}{3}A^2\exp\left(2ik_1T_0\right)+\frac{1}{3}\overline{A^2}\exp\left(-2ik_1T_0\right)\right] \tag{39}$$

Substituting η_0 and η_1 into Eq. (36) gives

$$D_0^2\eta_2+k_1^2\eta_2=-\left[2ik_1\left(A'+cA\right)+\left(3k_3-\frac{10k_2^2}{3k_1^2}\right)A^2\bar{A}--\frac{1}{2}f\exp\left(i\sigma_1T_2\right)\right]\exp\left(ik_1T_0\right)+cc+NST \tag{40}$$

Where the prime denotes the derivatives with respect to T_2 and NST stands for terms proportional to $\exp\left(\pm 3ik_1T_0\right)$. Secular terms will be eliminated from η_2 if:

$$2ik_1\left(A'+cA\right)+\left(3k_3-\frac{10k_2^2}{3k_1^2}\right)A^2\bar{A}-\frac{1}{2}f\exp\left(i\sigma T_2\right)=0 \tag{41}$$

Letting $A=\frac{1}{2}\alpha\exp(i\beta)$ in (41) and separating real and imaginary parts, we have

$$D_2\alpha=-c\alpha+\frac{f}{2k_1}\sin\gamma \tag{42}$$

$$\alpha D_2\beta=\frac{9k_3k_1^2-10k_2^2}{24k_1^3}\alpha^3-\frac{f}{2k_1}\cos\gamma \tag{43}$$

Where

$$\gamma=\sigma T_2-\beta \tag{44}$$

Eliminating β from (43) and (44) yields:

$$\alpha D_2\gamma=\alpha\sigma-\frac{9k_3k_1^2-10k_2^2}{24k_1^3}\alpha^3+\frac{f}{2k_1}\cos\gamma \tag{45}$$

Therefore to the second approximation:

$$\eta=\alpha\cos\left(\omega t-\gamma\right)+\frac{1}{2}\varepsilon k_2k_1^{-2}\alpha^2\left[-1+\frac{1}{3}\cos\left(2\omega t-2\gamma\right)\right]+O(\varepsilon^2) \tag{46}$$

Where α and γ are defined by (42) and (45)

Letting Eq. (42) and (45) into $D_2\alpha=D_2\gamma=0$,and the steady-state solution can be deduced:

$$c\alpha=\frac{f}{2k_1}\sin\gamma \tag{47}$$

$$\alpha\sigma-\frac{9k_3k_1^2-10k_2^2}{24k_1^3}\alpha^3=-\frac{f}{2k_1}\cos\gamma \tag{48}$$

The sum of the square of the above two equations is:

$$\left[c^2+\left(\sigma-\frac{9k_3k_1^2-10k_2^2}{24k_1^3}\alpha^2\right)^2\right]\alpha^2=\frac{f^2}{4k_1^2} \tag{49}$$

where $\alpha\neq 0$, Eq. (49) can be rewritten as:

$$\sigma=\frac{9k_3k_1^2-10k_2^2}{24k_1^3}\alpha^2\pm\left(\frac{f^2}{4k_1^2\alpha^2}-c^2\right)^{\frac{1}{2}} \tag{50}$$

4. RESULTS AND DISCUSSION

In the present study, a (15,15) SWCNTs is used as an example. The diameter is R =2.034 nm, the thickness of the tube wall is h =0.0066nm. The other physical and geometrical parameters are:

$L = 8\text{nm}, m = 4.866\times10^{-15}\text{kg/m}, I = 0.218\text{nm}^4, A = 0.422\text{nm}^2, \omega_0^2 = 5.848\times10^{24}, E = 5.5TPa$

Using these data to Eq. (50), a graph of the amplitude and frequency difference in the forced vibration can be obtained (in Figs.2,3 and 4).

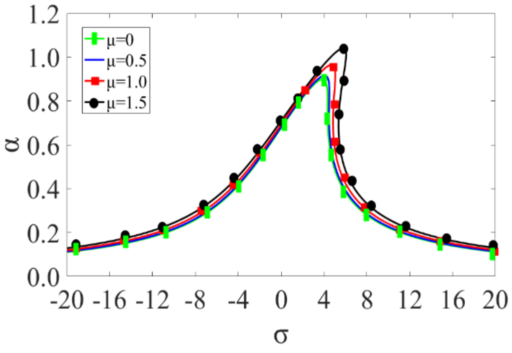

Fig2. *Influence of non-local scale effect on dynamic characteristic curve (Strain gradient parameters and initial deformation amplitude are* $l = 5, d = 0.3$*)*

It can be seen from Fig.2 that no matter what the value μ is taken, the amplitude changes in the same trend, that is, the amplitude of α increases with the increase of frequency difference σ. However, when the frequency difference σ reaches a certain critical value, the amplitude drops sharply, and the magnitude of this critical amplitude varies with the difference of μ. When the strain gradient effect is present, the vibration effect of CNTS is enhanced with the increase of non-local scale parameter μ, therefore the non-local stress effect softens the stiffness of CNTS.

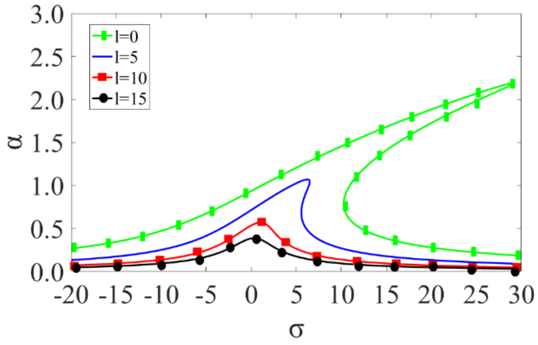

Fig3. *Influence of strain gradient effect on dynamic characteristic curve (Non-local stress parameters and initial deformation amplitude are* $\mu = 1.5, d = 0.3$*)*

As shown in Fig.3, with the increase of strain gradient parameters, the amplitude of carbon nanotubes becomes smaller at the frequency difference of the same value, which plays a strong damping role and obviously strengthens the stiffness of carbon nanotubes.

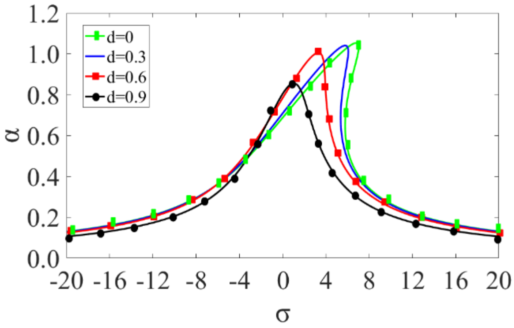

Fig4. *Influence of initial curvature on dynamic characteristic curve (Nonlocal stress parameters and strain gradient parameters are* $\mu = 1.5, l = 5$

As can be seen from Fig. 4, when the frequency difference decreases slowly, the stiffness of CNTS changes from hardening to softening and then to hardening with the increase of d. Therefore, the initial curvature complicates the relationship between frequency difference and vibration amplitude in the forced vibration.

5. CONCLUSION

In this paper, the non-local elastic stress constitutive and strain gradient constitutive are coupled, and the non-local elastic theory and strain gradient theory are combined to analyze the microscopic scale effect, therefore the scale effect subjected to CNTS can be more accurately simulated. Based on the new constitutive relation, a new Bernoulli–Euler beam model with a small initial curvature is established and applied to the forced vibration of carbon nanotubes. The results indicate that the initial curvature of the nanobeam have a significant influence on the mechanics of nanobeam. First, the strain gradient effect strengthened the stiffness of CNTS and the non-local stress effect softened the stiffness of CNTS. Second, initial curvature complicates the relationship between frequency difference and vibration amplitude in forced vibration of CNTS.

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