International Journal of Advanced Research in Physical Science (IJARPS)
Volume 7, Issue 1, 2020, PP 1-6
ISSN No. (Online) 2349-7882
www.arcjournals.org

The Impact of Gravitational Radiation Braking Torque on the Secular Speed Down of Spin of two - Components of Pulsar (The Theoretical Research)

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Abstract: *The secular speed down of the gravitational radiation breaking torque on the model of two-components of pulsar is studied by using the analytical method. The analytical solutions obtained are applied to the research for PSR0531+21 (Crab pulsar). The numerical results show that the spin of Crab pulsar speed down in Century* $-0.00021\left(rad/s\right)$*. The discussions and conclusion are held on the obtained results.*

Keywords: *pulsar - two - components - gravitational radiation torque - speed drown*

1. Introduction

Baym et al (1969) suggested a model of two - components of neutron stars in order to explain the glitches of pulsar. The two - component consists of a solid outer crust with charge and an internal neutron superfluid component which is coupled to the outer crust. The two - components rotates around their rotational axis under the coupled action of the external brailing torque. The rotation of the two - components are not identical around the rotational axis. The external torques consists of dipole, magnetic dipole and quadruple gravitational radiation torques. Baykal et al (1991) researched a shot noise model of two - component neutron star. Sedrakian (1997) researched the rotation of a two - component model neutron stars. Li (2016) researched the impact of magnetic radiation braking torque on the secular retardation braking torque on the secular retardation of spin of two - components of pulsar. In the present paper Li also research the impact of gravitational radiation braking torque on the secular speed drown of spin of two - component of pulsars.

2. The Coupled Equation System of two - Component of Pulsar under Braking of the External Torque

According to the theory of two - component model, the solid crust with moment inertia I_c and angular velocity Ω_c rotates around the rotational axis under the action of the external torque N ,and the neuronal super fluid component with moment inertia I_n and angular velocity Ω_n rotates around the rotational axis. The rotational angular velocities of two - components are not identical, i. e $\Omega_c \neq \Omega_n$.

The coupled equation system of two - component under braking of external torque is given by some authors (Baym et al, 1969, Shapiro et al 1983, D'Alessandro, 1997)

$$I_c \frac{d\Omega_c}{dt} = -N + \left(\frac{I_c}{\tau_c}\right)(\Omega_n - \Omega_c), \tag{1}$$

$$I_n \frac{d\Omega_n}{dt} = -\left(\frac{I_c}{\tau_c}\right)(\Omega_n - \Omega_c). \tag{2}$$

Where τ_c is the microscopic relaxation time, which describes the frictional coupling between the two - components. The total moment of inertia is given by

$$I = I_c + I_n. \tag{3}$$

The parameter Q $(0 \le Q \le 1)$ is the abundant level of the neutral super fluid component which measures the fraction of the frequency step that decay away. It is related to the moment of inertia of two - components given approximately by

$$Q = \frac{I_n}{I} \quad I_n = QI. \tag{4}$$

$$\therefore I_c = I - I_n = I(1-Q), \ \frac{I_c}{I_n} = \frac{1-Q}{Q}. \tag{5}$$

The macroscopic relaxation time τ is the time constant of exponential decay of the frequency step. It is related to microscopic relaxation time τ_c and moment of inertia given by Feibelman（1991）

$$\tau = \left(\frac{I_n}{I}\right)\tau_c = Q\tau_c. \tag{6}$$

3. The Coupling Equation System of two - Components under the Action of Braking of Gravitational Radiation Torque

According to the theory of the material radiation, the total radiation includes dipole, magnetic dipole and quadruple radiation. A body (its ellipticity $\in \neq 0$) proceeds quadruple gravitational radiation. The gravitational energy - loss can be transformed to the radiation power (Fang & Ruffini, 1983)

$$\frac{d\varepsilon}{dt} = -\frac{G}{45c^6}\dddot{Q}_{\alpha\beta}^{\ 2}. \tag{7}$$

Where $Q_{\alpha\beta}$ is the quadruple tensor of mass distribution body.

$$\dddot{Q}_{\alpha\beta} = 18I^2 \in^2 \Omega^6(1+15Sin^2\theta)Sin^2\theta.$$

When we take the special case $\theta = 90^o$.

$$\dddot{Q}_{\alpha\beta} = 288I^2 \in^2 \Omega^6. \tag{8}$$

Substituting (8) into equation (7)

$$\frac{d\varepsilon}{dt} = -\frac{32}{5c^5}GI^2 \in^2 \Omega^6. \tag{9}$$

This is consistent with the power of radiation $P = \dfrac{32GI \in^2 \Omega^6}{5c^5}$ given by Weber(1961).

If the gravitational energy - loss come from the rotational energy supplement $\varepsilon_{rot} = \dfrac{1}{5}I\Omega^2$ (for ellipsoid) the equation (9) can be written as

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon_{rot}}{dt} = -\frac{32}{5c^5}GI^2 \in^2 \Omega^6,$$

$$\therefore \frac{d\Omega}{dt} = -\frac{16}{c^5}GI \in^2 \Omega^5. \tag{10}$$

Next, we deduce the gravitational radiation braking torque. We can write (10) as the form:

$$\frac{d(I\Omega)}{dt} = -\frac{16}{c^5}GI^2 \in^2 \Omega^5, \ \ J = I\Omega,$$

$$\frac{dJ}{dt} = -\frac{16}{c^5}\frac{G \in^2}{I^3}J^5. \tag{11}$$

J is the angular momentum.

Integrating the above equation

$$\int_{J_0}^{J} \frac{dJ}{J^5} = -\frac{16G \in^2}{c^5 I^3} \int_{t_0}^{t} dt,$$

$$J = J_0 \left[1 + \frac{64G \in^2 J_0{}^4}{c^5 I^3}(t-t_0) \right]^{-1/4},$$

$$J_0 = I\Omega_{,0}$$

$$\therefore J = J_0[1+K(t-t_0)]^{-1/4}. \quad (12)$$

Where $K = \frac{64G \in^2 I\Omega_0{}^4}{c^5}$. (13)

The torque $N(t)$ can be written as

$$N(t) = -\frac{dJ}{dt} = \frac{1}{4} J_0 K [1+K(t-t_0)]^{-5/4}, \quad J_0 = I\Omega_0. \quad (14)$$

We add both sides of equation (1) and (2), one yelds

$$I_c \frac{d\Omega_c}{dt} + I_n \frac{d\Omega_n}{dt} = \frac{d}{dt}(I_c\Omega_c + I_n\Omega_n) = -N(t).$$

Substituting $N(t)$ for the expression (14) into the above equation and integrating it

$$J(t)_m = I_c\Omega_c + I_n\Omega_n = J(0)[1+K(t-t_0)]^{-1/4}, \quad (15)$$

$$\therefore \Omega_n = \frac{J(0)}{I_n}[1+K(t-t_0)]^{-1/4} - \frac{I_c}{I_n}\Omega_c. \quad (16)$$

$J(0)_m = I\Omega(0),$ and we use (4) – (5)

$$\frac{I}{I_n} = \frac{1}{Q}, \quad \frac{I_c}{I_n} = \frac{1-Q}{Q}.$$

$$\Omega_n(t) = \frac{\Omega\ (0)}{Q}\left|1+K(t-t_0)\ \right|^{-1,4} - \frac{1-Q}{Q}\Omega_c. \quad (17)$$

Substituting (14) and (17) into equation (1) and using $1/\tau_c = Q/\tau$ given by (6), we obtain the first order linear equation for the angular velocity Ω_c of the solid crust of pulsar

$$\frac{d\Omega_c}{dt} + \frac{1}{\tau}\Omega_c = \frac{\Omega(0)}{\tau}[1+K(t-t_0)]^{-1/4} - \frac{\Omega(0)K}{4(1-Q)}[1+K(t-t_0)]^{-5/4}. \quad (18)$$

We use (5) and (6)

$$I_c/I_n = 1-Q/Q, \quad \tau = \tau_c Q,$$

$$\therefore \frac{1}{\tau_c}\frac{I_c}{I_n} = \frac{1-Q}{\tau}.$$

Substituting the above expression into the equation (2), we obtain the equation for the angular velocity Ω_n of the neutral super fluid component:

$$\frac{d\Omega_n}{dt} + \frac{1-Q}{\tau}\Omega_n = \left(\frac{1-Q}{\tau}\right)\Omega_c. \quad (19)$$

4. The Analytical Solutions for the Equation System of two - Component under Braking of Gravitational Radiation Torque

At first we solve the equation (18) for the angular velocity Ω_c of the crust of pulsar, and the equation (19).

The equation (18) is the first order linear equation, its integrating form is

$$\Omega_c(t)=\Omega(0)e^{-\int\frac{1}{\tau}dt}\int\frac{1}{\tau}[1+K(t-t_0)]^{-1/4}e^{\int\frac{1}{\tau}dt}dt-$$

$$-\Omega(0)e^{-\int\frac{1}{\tau}dt}\frac{K}{4(1-Q)}\int[1+K(t-t_0)]^{-5/4}e^{\int\frac{1}{\tau}dt}dt+C.$$

Letting the form of the solution:

$$\Omega_c(t)=e^{-1/\tau}(I_1+I_2)+Ce^{-1/2}. \qquad (20)$$

C is a constant

$$I_1=\Omega(0)\frac{1}{\tau}\int[1+K(t-t_0)]^{-1/4}e^{t/\tau}dt.$$

We may prove the second term $K(t-t_0)\le 1,$ so we can expand series by using the binomial theories and neglect the third term.

$$\therefore I_1=\Omega(0)\frac{1}{\tau}\int\left[1-\frac{1}{4}K(t-t_0)\right]e^{t/\tau}dt,$$

$$=\Omega(0)\frac{1}{\tau}\int\left[\left(1+\frac{1}{4}Kt_0\right)e^{t/\tau}\frac{1}{4}Kte^{t/\tau}\right]dt,$$

$$=\Omega(0)e^{t/\tau}\left[\left(1+\frac{1}{4}Kt_0\right)-\frac{1}{4}K\tau^2\left(t/\tau-1\right)\right],$$

$$=\Omega(0)e^{t/\tau}\left[\left(1+\frac{1}{4}K\tau\right)-\frac{1}{4}K\left(t-t_0\right)\right]. \qquad (21)$$

As the same in (21)

$$I_2=-\Omega(0)\frac{K}{4(1-Q)}\int[1+K(t-t_0)]^{-5/4}e^{t/\tau}dt,$$

$$=-\Omega(0)\frac{K}{4(1-Q)}\int\left[1-\frac{5}{4}K(t-t_0)\right]e^{t/\tau}dt,$$

$$=-\Omega(0)e^{t/\tau}\frac{K\tau}{4(1-Q)}\int\left[\left(1+\frac{5}{4}K\tau\right)-\frac{5}{4}K(t-t_0)\right]. \qquad (22)$$

Substituting (21) and (22) into (20), we obtain

$$\Omega_c(t)=\Omega_c(0)\left\{1+\frac{1}{4}K\tau\left[1-\frac{1}{1-Q}\left(1+\frac{5}{4}K\tau\right)\right]-\frac{1}{4}K\left[1-\frac{5K\tau}{4(1-Q)}\right](t-t_0)\right\}+Ce^{-t/\tau}. \qquad (23)$$

Letting $t=t_0$,

$$C=-\frac{1}{4}K\tau\left[1-\frac{1}{1-Q}\left(1+\frac{5}{4}K\tau\right)\right]e^{t_0/\tau}. \qquad (24)$$

Substituting (24) into (23), we obtain the solution for the angular velocity Ω_c of the crust component

$$\Omega_c(t)=\Omega_c(0)\left\{1+\left[\frac{1}{4}K\tau-\frac{K\tau}{4(1-Q)}\left(1+\frac{5}{4}K\tau\right)\right]\left(1-e^{-(t-t_0)/\tau}\right)\right.$$

$$\left.-\frac{1}{4}K\left[1-\frac{5K\tau}{4(1-Q)}\right](t-t_0)\right\}. \qquad (25)$$

$$\delta\Omega_c=\Omega(t)-\Omega(0).$$

Substituting $\Omega_c(t)$ for (25) into equation (19) and solving it, we obtain the angular velocity Ω_n of the superfluied component. Because this component in the crust interior can not be observed, we neglect the calculation for the angular velocity Ω_n of the superfluied component.

5. The Theoretical Numerical Results for the PSR0531+21 (Crab)

This paper researches the secular spin down of the Crust of PSR0531+21 (Crab) under gravitational radiation braking torque. The physical parameters of this pulsar are listed in Table 1. For data Ω_0, Q, τ are cited from Tang (1975), $\in$ is cited from Rees et at (1974) and I is cited from Shapiro (et al 1983)

Table1. *Data for PSR0531+21*

Pulsar	$\Omega_0(rad/s)$	$Q=I_n/I$	$\tau(d)$	$\in$	$I(cm^2\cdot g)$
RSR0531+21	190	0.96	7.7	2×10^{-4}	1.4×10^{45}

$$G=6.67\times10^{-8}(d,\ cm^2,\ g^{-2}),\ c=3\times10^{10}(cm/s).$$

Substituting the above data into (13), we obtain $K=1.2864\times10^{-15}$, $\tau=7.7d=6.6528\times10^5(s)$, $K\tau=8.5581\times10^{-10}$, $5(K\tau)^2/16(1-Q)\sim10^{-20}$ may be neglected. In this paper we use $t-t_0=100year$, $e^{-(t-t_0)/\tau}=e^{-4743}\to0$. The formula (25) can be written as

$$\Omega(t)=\Omega_0\left[1-\frac{1}{4}K\tau\left(\frac{Q}{1-Q}\right)-\frac{1}{4}K(t-t_0)\right]. \qquad (26)$$

Substituting K, τ, Q and $t-t_0=\Delta t=100yr$ into (26), we obtain the numerical results in Table2

Table2. *The numerical results for PSR0531+21 ($\Delta t=100yr$)*

Pulsar	$\Omega(t)/\Omega(0)$	$\Omega(t)(rad/s)$	$\delta\Omega(rad/s)$	K
PSR0531+21	0.9999989	189.99979	-0.00021	1.2864×10^{-15}

6. Discussion and Conclusion

(1) It can be seen from Table2 that the angular velocity of the outer crust decreases with time under the action of gravitational radiation braking torque: $\delta\Omega=-0.00021(rad/s)$ in century. The is a very small value due to the weak gravitational radiation.

(2) Comparison with the results of the magnetic dipole radiation.

In the previous work Li (2016) the speed down of spin of the magnetic dipole radiation is given by $\delta\Omega=-0.2450(rad/s)$.

Hence, the speed down of spin of the magnetic radiation is largest than that the quadropole gravitational radiation. These is due to that for the magnetic radiation $K \sim 10^{-10},\ c^{-3}$; for the gravitational radiation $K \sim 10^{-15},\ c^{-5},\ \in^{-4}$. So, the former is largest than the later.

(3) In the formula (25) $t - t_0 = \Delta t.$ The starting time t_0 may be selected as any time. It may be selected as the present time or the time after stellar quake (glitches), but it can not be seleced as the born time of pulsar.

(4) In the integration of the expressions (20) and (21) we used the binomial theorem to expand the expressions (20) and (21). Because

$K(t - t_0) = 1.2864 \times 10^{-15} \times 3.1556926 \times 10^{9}(s) \approx 3 \times 10^{-6} \prec 1.$ Hence we can use binomial theorem to expand the expressions (20) and (21).

(5) The pulsar PSR0531+21 Speeds up suddenly due to stellar quake (glitches) in three years. However it is temporary happening and it is not secular happening. It does not influence the secular variation of spin of two - components due to braking of gravitational radiation torque.

(6) We conclude that the action of gravitational radiation braking torque on the spin down of two - component is very small, but it is existing in the model of two - components indeed.

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Citation: *Lin-sen Li, (2020). "The Impact of Gravitational Radiation Braking Torque on the Secular Speed Down of Spin of two – Components of Pulsar (The Theoretical Research)". International Journal of Advanced Research in Physical Science (IJARPS) 7(1), pp.1-6, 2020.*

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