International Journal of Advanced Research in Physical Science (IJARPS)
Volume 6, Issue 7, 2019, PP 18-22
ISSN No. (Online) 2349-7882
www.arcjournals.org

Wave Function and Radial Moments for the Deuteron

V. I. Zhaba*

Uzhgorod National University, Department of Theoretical Physics,54, Voloshyna St., Uzhgorod, UA-88000, Ukraine

***Corresponding Author:** ***V. I. Zhaba,** Uzhgorod National University, Department of Theoretical Physics,54, Voloshyna St., Uzhgorod, UA-88000, Ukraine*

Abstract: *The radial moments of the deuteron and inverse moments of the radius are calculated. These results are compared with the data for other potential models. The values of the radius and quadrupole moment of the deuteron are the same as in previous works. For numerical calculations of these physical quantities, the deuteron wave function was used in the coordinate representation for the four potentials of Nijmegen group (NijmI, NijmII, Nijm93 and Reid93) and for one potential of Argonne group (Argonne v18).*

Keywords: *deuteron, wave function, radial moments, inverse moments, radius, quadrupole moment.*

1. Introduction

Wave function describes the quantum mechanical system and is the main characteristic of microobjects. Knowledge of the deuteron wave function (DWF) in coordinate and momentum representations allows obtaining maximum information about the connected neutron-proton system and theoretically calculating and predicting those characteristics that are measured in the experiment.

In chronological order (from 1940 to 2015 years) in a detailed review [1], the static parameters of deuteron were systematized, which in the cited literature are obtained by DWF for various potential models. The presence or absence of knots at the origin of coordinate for the radial DWF is described. According to the notations mentioned in the cited literature, an overview of analytical forms for DWF in the coordinate representation was conducted.

It should be noted [2] that the choice of analytical forms of DWF depends on the satisfaction of the parameters (characteristics) of the deuteron calculated for these forms. These include radial moments.

For calculations of radial moments and other parameters of deuteron in this paper, DWF was used in the coordinate representation in the form [3-5]

$$\begin{cases} u(r) = r^{3/2} \sum_{i=1}^{N} A_i \exp(-a_i r^3), \\ w(r) = r \sum_{i=1}^{N} B_i \exp(-b_i r^3). \end{cases} \tag{1}$$

This form of DWF was applied to potentials of the Nijmegen group (NijmI, NijmII, Nijm93 and Reid93) and for the Argonne v18 potential. All coefficients in (1) for these potentials are given in papers [4, 5]. DWFs for these potentials do not contain an unnecessary knots at the origin of coordinate. Therefore, it can be used for numerical calculations.

2. The Radial Moments for the Deuteron

If DWF are known in the coordinate representation, then you can calculate the parameters of the deuteron [6-8]:

- radius («matter radius») of deuteron:

$$r_d = \frac{1}{2} \left\{ \int_0^\infty r^2 \left[u^2(r) + w^2(r) \right] dr \right\}^{1/2}; \tag{2}$$

- electric quadrupole moment:

$$Q_d = \frac{1}{20}\int_0^\infty r^2 w(r)\left[\sqrt{8}u(r) - w(r)\right]dr = \frac{1}{\sqrt{50}}\int_0^\infty r^2\left[u(r)w(r) - \frac{w^2(r)}{\sqrt{8}}\right]dr\,; \tag{3}$$

- magnetic moment:

$$\mu_d = \mu_s - \frac{3}{2}(\mu_s - \frac{1}{2})P_D\,; \tag{4}$$

- contribution of D- state:

$$P_D = \int_0^\infty w^2(r)dr\,; \tag{5}$$

- the asymptotics of D/S- state:

$$\eta = A_D / A_S\,, \tag{6}$$

where A_S i A_D – the asymptotics of the normalization for S- and D- states.

In formula (4) the value $\mu_s = \mu_n + \mu_p$ is the sum of the magnetic moments of the neutron and the proton. The value of the calculated magnetic moment of the deuteron is given in the nuclear magnetons μ_N.

In addition to these parameters, the asymptotics of S- and D- states (A_S and A_D respectively), the effective radius ρ_d in [fm], the inverse radius in [1/fm], and the radii of the deuteron $R_{r.}$ and $R_{n.r.}$ in relativistic or nonrelativistic kinematics respectively are also determined. And the radius is obtained as $R = {}^1\!/_\beta$. The relationship between the binding energy B_d and the parameter β, that determines the radius of the deuteron [8] for relativistic kinematics:

$$B_d = M_p + M_n - \sqrt{M_p^2 - \beta^2} - \sqrt{M_n^2 - \beta^2}\,, \tag{7}$$

where M_p, M_n – the masses of the proton and the neutron.

Also important parameters are the values of normalization for wave function [9]

$$N^2 = A_S^2(1+\eta^2) \tag{8}$$

and radial moments of the deuteron [10, 11]

$$< r^n >_u = \int_0^\infty r^n u^2(r)dr\,; \tag{9}$$

$$< r^n >_w = \int_0^\infty r^n w^2(r)dr\,; \tag{10}$$

$$< r^n >_{uw} = \int_0^\infty r^n u(r)w(r)dr\,; \tag{11}$$

$$< r^{2k} > = 2^{-2k}\int_0^\infty r^{2k}\left[u^2(r) + w^2(r)\right]dr\,. \tag{12}$$

The radial moments are constituent parts in the radius r_d (2) and the quadrupole moment Q_d (3) for the deuteron

$$r_d = \frac{1}{2}\left\{\int_0^\infty r^2\left[u^2(r) + w^2(r)\right]dr\right\}^{1/2} = \frac{1}{2}\left\{\int_0^\infty r^2u^2(r)dr + \int_0^\infty r^2w^2(r)dr\right\}^{1/2} = \frac{1}{2}\left\{< r^2 >_u + < r^2 >_w\right\}^{1/2} \tag{13}$$

$$r_d = \sqrt{< r^{2k} >},\ k=1; \tag{14}$$

$$Q_d = \frac{1}{20}\int_0^\infty r^2 w(r)\left[\sqrt{8}u(r) - w(r)\right]dr = \frac{\sqrt{8}}{20}\int_0^\infty r^2 u(r)w(r)dr - \frac{1}{20}\int_0^\infty r^2 w^2(r)dr = \frac{\sqrt{8}}{20} < r^2 >_{uw} - \frac{1}{20} < r^2 >_w \quad (15)$$

In [12], another characteristic is given, which is determined by DWF in the coordinate representation – the inverse moments of the radius

$$< r^{-n} >= \int_0^\infty r^{-n}\left[u^2(r) + w^2(r)\right]dr \,. \quad (16)$$

The inverse moment $<r^{-1}>$ is written as [13]

$$< r^{-1} >= \int_0^\infty r^{-1}\left[u^2(r) + w^2(r)\right]dr \,, \quad (17)$$

which appears in the multiple expansion of the π-deuteron scattering length and appears in low energy pion-deuteron scattering [14].

3. Calculations and Conclusion

The radial moments (9)-(11) for n=0;1;2 were published in [15]. They were calculated on DWF in the coordinate representation [16, 17]

$$R_l(r) = r^l \sum_{k=1}^{N} C_k \exp(-\alpha_k r^2) \,. \quad (18)$$

Calculations in [15] were performed for potentials NijmI, NijmII, Nijm93, Reid93 and Argonne v18.

In Table 1 shows the results of calculations of radial moments (9)-(11) for orders n=-3;-2;-1;0;1;2;3. DWF [4, 5] in the coordinate representation in the form (1) was used for the four potentials of Nijmegen group (NijmI, NijmII, Nijm93 and Reid93) and for one potential of Argonne group (Argonne v18). The obtained numerical results (Table 1) for the indicated potentials in this paper coincide within the same order with the data [10] for chiral potentials.

Table1. *The radial moments of the deuteron (9)-(11)*

$<r^n>$	NijmI	NijmII	Nijm93	Reid93	Av18
$<r^3>_u$, fm^3	100.652	100.733	100.585	100.899	96.9108
$<r^3>_w$, fm^3	1.35305	1.3375	1.34506	1.33651	1.32063
$<r^3>_{uw}$, fm^3	10.2779	10.2534	10.3934	10.3772	10.1369
$<r^2>_u$, fm^2	15.1123	15.1344	15.1067	15.1511	14.9302
$<r^2>_w$, fm^2	0.34807	0.343574	0.344999	0.342744	0.342347
$<r^2>_{uw}$, fm^2	2.03849	2.02919	2.03521	2.03208	2.01689
$<r^1>_u$, fm^1	3.1319	3.13634	3.129001	3.13795	3.1241
$<r^1>_w$, fm^1	0.121744	0.120332	0.121438	0.120511	0.121300
$<r^1>_{uw}$, fm^1	0.561439	0.558499	0.559754	0.55849	0.558301
$<r^0>_u$	0.943355	0.941465	0.942006	0.941553	0.941230
$<r^0>_w$	0.056627	0.056300	0.057496	0.056900	0.057594
$<r^0>_{uw}$	0.217086	0.215815	0.217893	0.216596	0.217551
$<r^{-1}>_u$, fm^{-1}	0.424457	0.417427	0.422954	0.417709	0.419602
$<r^{-1}>_w$, fm^{-1}	0.033960	0.034139	0.035582	0.035112	0.0355295
$<r^{-1}>_{uw}$, fm^{-1}	0.115841	0.114809	0.118019	0.116439	0.117265
$<r^{-2}>_u$, fm^{-2}	0.291567	0.276583	0.287889	0.277412	0.281360
$<r^{-2}>_w$, fm^{-2}	0.025628	0.026081	0.028052	0.027771	0.027758
$<r^{-2}>_{uw}$, fm^{-2}	0.083772	0.082074	0.087215	0.085502	0.085797
$<r^{-3}>_u$, fm^{-3}	0.348542	0.310063	0.336333	0.312298	0.321691
$<r^{-3}>_w$, fm^{-3}	0.073364	0.040228	0.087602	0.144299	0.116521
$<r^{-3}>_{uw}$, fm^{-3}	0.088064	0.081218	0.093849	0.094461	0.092648

In Table 2 shows the results of calculations of radial moments (12) for orders k=0;1;2;3;4;5,. These results coincide in the same order with the data [11] for a group of thirteen nucleon-nucleon potentials. The values $<r^2>$ and $<r^4>$ are similar to the results in the pionless and pionful theories for various values of range R of the short-distance potential V_{short} [18]. The radius (14) and the quadrupole moment (15) of the deuteron are the same as their values in papers [4, 5].

Table2. *The radial moments (12), radius and quadrupole moment of the deuteron*

$<r^{2k}>$, r_d, Q_d	NijmI	NijmII	Nijm93	Reid93	Av18
$<r^{10}>$, fm^{10}	8.94E+6	8.02E+6	7.51E+6	8.40E+6	2.96E+6
$<r^{8}>$, fm^{8}	107946.0	102767.0	99664.2	105154.0	56551.6
$<r^{6}>$, fm^{6}	1850.72	1824.42	1805.97	1840.06	1376.55
$<r^{4}>$, fm^{4}	54.9438	54.8824	54.7597	55.0268	50.2706
$<r^{2}>$, fm^{2}	3.8651	3.86951	3.86293	3.87347	3.81814
$r_d=(<r^2>)^{1/2}$, fm	1.96599	1.96711	1.96543	1.96811	1.95401
Q_d, fm^2	0.270883	0.269793	0.270572	0.270243	0.268113

In Table 3 gives a set of inverse moments of the radius, calculated by the formula (16). The inverse moment $<r^{-1}>$ is commensurate with the results of [13] for OBE, NijmII, Reid93 potentials and the results of [14] for OPE and chiral potentials. The inverse moments $<r^{-1}>$ and $<r^{-2}>$ are the same as for the renormalization of the NN interaction with Lorentz-invariant chiral two-pion exchange [19] and for chiral potentials LO, NLO-Δ, N2LO-Δ [12].

In [20] it was noted, that the deuteron matrix elements of two-nucleon operators in chiral effective theories of the two-nucleon system will be proportional to $1/r^n$ at short distances and converge for $n\leq 2$ and diverge for $n\geq 3$. The matrix elements are evaluated using the leading-order DWF.

Table3. *The inverse moments of the radius (16)*

$<r^{-n}>$	NijmI	NijmII	Nijm93	Reid93	Av18
$<r^{-1}>$, fm^{-1}	0.458417	0.451566	0.458536	0.452822	0.455131
$<r^{-2}>$, fm^{-2}	0.317194	0.302665	0.315941	0.305184	0.309118
$<r^{-3}>$, fm^{-3}	7.09294	2.46443	8.52901	16.2051	12.5376

REFERENCES

[1] V.I. Zhaba, Deuteron: properties and analytical forms of wave function in coordinate space // e-print arXiv:nucl-th/1706.08306 (2017).

[2] V.I. Zhaba, Analytical forms of the wave function in coordinate space And tensor polarization of the deuteron for potentials Nijmegen group, J. Phys. Stud. 20(3), 3101 (2016).

[3] V.I. Zhaba, New analytical forms of the deuteron wave function and polarization characteristics for potential Reid93, Cherkasy Univ. Bull., Phys. and Mathem. Scienc. 349(16), 50 (2015).

[4] V.I. Zhaba, New analytical forms of a deuteron wave function for potentials nijmegen groups, Nucl. Phys. Atom. Energy 17, 22 (2016).

[5] V.I. Zhaba, New analytical forms of wave function in coordinate space and tensor polarization of deuteron, Mod. Phys. Lett. A 31, 1650139 (2016).

[6] J.M. Blatt,V.F. Weisskopf, Theoretical nuclear physics (New York, Wiley, 1958), 864 p.

[7] S. de Benedetti, Nuclear interactions (Wiley, London, 1964), 636 p.

[8] R. Machleidt, High-precision, charge-dependent Bonn nucleon-nucleon potential, Phys. Rev. C 63, 024001 (2001).

[9] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen et al., Construction of high-quality NN potential models, Phys. Rev. C 49, 2950 (1994).

[10] E.R. Arriola, M.P. Valderrama, Deuteron radial moments for renormalized chiral potentials, Eur. Phys. J. A 31, 549 (2007).

[11] M.M. Mustafa, Radius and radial moments of the deuteron, Phys. Rev. C 48, 929 (1993).

[12] M.P. Valderrama, E.R. Arriola, Renormalization of chiral two-pion exchange NN interactions with Δ excitations: Central phases and the deuteron, Phys. Rev. C 79, 044001 (2009).

[13] A.C. Cordon, E.R. Arriola, Renormalization versus strong form factors for one-boson-exchange potentials, Phys. Rev. C 81, 044002 (2010)

[14] M.P. Valderrama, E.R. Arriola, Renormalization of the NN interaction with a chiral two-pion-exchange potential: Central phases and the deuteron, Phys. Rev. C 74, 054001 (2006).

[15] V.I. Zhaba, Approximation of the wave function in the coordinate representation and the radial moments of the deuteron, Abstracts of the XXIII annual scientific conference of the Institute of Nuclear Research of the National Academy of Sciences of Ukraine, Kyiv, 2016, February 01-05, 28-29 p.

[16] S.B. Dubovichenko, Deuteron Form Factors for the Nijmegen Potentials, Phys. Atom. Nucl. 63, 734 (2000).

[17] S.B. Dubovichenko, Properties of light atomic nuclei in the potential cluster model (Almaty, Daneker, 2004), 247 p.; e-print arXiv:nucl-th/1006.4944 (2010).

[18] D.R. Phillips, T.D. Cohen, Deuteron electromagnetic properties and the viability of effective field theory methods in the two-nucleon system, Nucl. Phys. A 668, 45 (2000).

[19] R. Higa, M.P. Valderrama, E.R. Arriola, Renormalization of the NN interaction with Lorentz-invariant chiral two-pion exchange, Phys. Rev. C 77, 034003 (2008).

[20] L. Platter, D.R. Phillips, Deuteron matrix elements in chiral effective theory at leading order, Phys.Lett.B 641, 164(2006).

Citation: *V. I. Zhaba, (2019). Wave Function and Radial Moments for the Deuteron. International Journal of Advanced Research in Physical Science (IJARPS) 6(7), pp.18-22, 2019.*