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Exact Solution of Navier-Stokes Equations

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Abstract: *In Navier-Stokes equations (NASA's Navier-Stokes Equations, 3-dimensional-unsteady), we discover the exact solution by Newton potential function and time-function. We think the solution likely Newton potential function that be able to solve Laplace equation.*

Keywords: *Navier-Stokes Equations; Exact solutions; Newton potential function; Time function*

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1. Introduction

We discover the exact solution in Navier-Stokes equation by Newton potential function and time function.

According NASA's Navier-Stokes Equations (3-dimensional-unsteady),

Coordinate: (x,y,z), Time: t, Pressure: p , Heat Flux: q

Density: ρ , Stress: τ , Reynolds Number: R_e ,

Velocity Components: (u,v,w) Total Energy: E_t Plandtl Number: P_r

Continuity: $$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0 \tag{1}$$

X-Momentum: $$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u^2)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}+\frac{\partial(\rho uw)}{\partial z}$$

$$=-\frac{\partial p}{\partial x}+\frac{1}{R_e}[\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}] \tag{2}$$

Y-Momentum: $$\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho v^2)}{\partial y}+\frac{\partial(\rho vw)}{\partial z}$$

$$=-\frac{\partial p}{\partial y}+\frac{1}{R_e}[\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\tau_{yy}}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}] \tag{3}$$

Z-Momentum: $$\frac{\partial(\rho w)}{\partial t}+\frac{\partial(\rho uw)}{\partial x}+\frac{\partial(\rho wv)}{\partial y}+\frac{\partial(\rho w^2)}{\partial z}$$

$$=-\frac{\partial p}{\partial z}+\frac{1}{R_e}[\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\tau_{zz}}{\partial z}] \tag{4}$$

Energy: $$\frac{\partial E_t}{\partial t}+\frac{\partial (E_t u)}{\partial x}+\frac{\partial (E_t v)}{\partial y}+\frac{\partial (E_t w)}{\partial z}=-\frac{\partial (pu)}{\partial x}-\frac{\partial (pv)}{\partial y}-\frac{\partial (pw)}{\partial z}$$

$$-\frac{1}{R_e P_r}[\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}]$$

$$+\frac{1}{R_e}[\frac{\partial}{\partial x}(u\tau_{xx}+v\tau_{xy}+w\tau_{xz})+\frac{\partial}{\partial y}(u\tau_{xy}+v\tau_{yy}+w\tau_{yz})+\frac{\partial}{\partial z}(u\tau_{xz}+v\tau_{yz}+w\tau_{zz})] \quad (5)$$

2. Exact Solution in 3-Dimentional Navier-Stokes Equation (Include time)

For we solve equations, we use Newton potential function and time function. If we think the solution of Laplace equation,

$$u=\frac{C_1}{r^3}xf(t),\ v=\frac{C_1}{r^3}yf(t), w=\frac{C_1}{r^3}zf(t),\ r=\sqrt{x^2+y^2+z^2}$$

$$\rho=\rho_0\ ,\ \ r>0 \quad (6)$$

In this case, we solve Eq(1).

$$\frac{\partial \rho}{\partial t}+\frac{\partial (\rho u)}{\partial x}+\frac{\partial (\rho v)}{\partial y}+\frac{\partial (\rho w)}{\partial z}=f(t)\rho_0[-\frac{3C_1}{r^5}(x^2+y^2+z^2)+\frac{3C_1}{r^3}]=0 \quad (7)$$

Second point, in Eq(2)

$$\tau_{xx}=\frac{C_2}{r^6}x^2[f(t)]^2,\ \tau_{xy}=\frac{C_2}{r^6}xy[f(t)]^2,\quad \tau_{xz}=\frac{C_2}{r^6}xz[f(t)]^2\ ,$$

$$,\dots,\tau_{ij}=\frac{C_2}{r^6}x^i x^j[f(t)]^2,\ r>0 \quad (8)$$

$$\frac{\partial (\rho u)}{\partial t}+\frac{\partial (\rho u^2)}{\partial x}+\frac{\partial (\rho uv)}{\partial y}+\frac{\partial (\rho uw)}{\partial z}$$

$$=\rho_0\dot{f}(t)\frac{C_1}{r^3}x+[f(t)]^2\rho_0[-\frac{6C_1^{\ 2}}{r^8}(x^3+xy^2+xz^2)+\frac{C_1^{\ 2}}{r^6}(2x+x+x)]$$

$$=\rho_0\dot{f}(t)\frac{C_1}{r^3}x-[f(t)]^2\rho_0\frac{2C_1^{\ 2}}{r^6}x \quad (9)$$

In this time, if $f(t)$ is,

$$\frac{1}{[f(t)]^2}\frac{d}{dt}[f(t)]=1\rightarrow f(t)=\frac{1}{C'-t},\ t\geq 0,\ C'<0 \text{ is constant} \quad (10)$$

Therefore,

$$=\rho_0\frac{1}{(C'-t)^2}\frac{C_1}{r^3}x-\frac{1}{(C'-t)^2}\rho_0\frac{2C_1^{\ 2}}{r^6}x$$

$$=-\frac{\partial p}{\partial x}+\frac{1}{R_e}[\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}]$$

$$= -\frac{\partial p}{\partial x} + \frac{1}{R_e}[-\frac{6C_2}{r^8}(x^3 + xy^2 + xz^2) + \frac{C_2}{r^6}(2x + x + x)]\frac{1}{(C'-t)^2}$$

$$= -\frac{\partial p}{\partial x} - \frac{1}{R_e}\frac{2C_2}{r^6}x\frac{1}{(C'-t)^2} \tag{11}$$

Hence, in Eq(2),in Eq(3) and in Eq(4),

$$\frac{\partial p}{\partial x} = [\frac{2x}{r^6}(\rho_0 C_1^{\ 2} - \frac{C_2}{R_e}) - \rho_0 \frac{C_1}{r^3}x]\frac{1}{(C'-t)^2} = [\frac{2C_3}{r^6}x - \rho_0\frac{C_1}{r^3}x]\frac{1}{(C'-t)^2} ,$$

$$\frac{\partial p}{\partial y} = [\frac{2y}{r^6}(\rho_0 C_1^{\ 2} - \frac{C_2}{R_e}) - \rho_0 \frac{C_1}{r^3}y]\frac{1}{(C'-t)^2} = [\frac{2C_3}{r^6}y - \rho_0\frac{C_1}{r^3}y]\frac{1}{(C'-t)^2} ,$$

$$\frac{\partial p}{\partial z} = [\frac{2z}{r^6}(\rho_0 C_1^{\ 2} - \frac{C_2}{R_e}) - \rho_0 \frac{C_1}{r^3}z]\frac{1}{(C'-t)^2} = [\frac{2C_3}{r^6}z - \rho_0\frac{C_1}{r^3}z]\frac{1}{(C'-t)^2} \quad C_3 = \rho_0 C_1^{\ 2} - \frac{C_2}{R_e} \tag{12}$$

Therefore,

$$p = [-\frac{1}{2}\frac{C_3}{r^4} + \rho_0\frac{C_1}{r}]\frac{1}{(C'-t)^2} \ , \ C_3 = \rho_0 C_1^{\ 2} - \frac{C_2}{R_e}, \ r > 0 \tag{13}$$

In Eq(5), if E_t, q_i is,

$$E_t = E_0, \ q_x = \frac{C_4}{r^4}\frac{x}{(C'-t)^3}, q_y = \frac{C_4}{r^4}y\frac{1}{(C'-t)^3}, q_z = \frac{C_4}{r^4}z\frac{1}{(C'-t)^3}, \ r > 0 \tag{14}$$

$$\frac{\partial E_t}{\partial t} + \frac{\partial(E_t u)}{\partial x} + \frac{\partial(E_t v)}{\partial y} + \frac{\partial(E_t w)}{\partial z} = 0 \tag{15}$$

As,

$$(\rho u, \rho v, \rho w) = [-\frac{1}{2}\frac{C_1 C_3}{r^7} + \rho_0\frac{C_1^{\ 2}}{r^4}](x, y, z)\frac{1}{(C'-t)^3}, \ r > 0 \tag{16}$$

Hence,

$$-\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho w)}{\partial z}$$

$$= [\{-7\frac{C_3 C_1}{2r^9}(x^2 + y^2 + z^2) + 3\frac{C_3 C_1}{2r^7}\} + \{4\rho_0\frac{C_1^{\ 2}}{r^6}(x^2 + y^2 + z^2) - 3\rho_0\frac{C_1^{\ 2}}{r^4}\}]\frac{1}{(C'-t)^3}$$

$$= [-\frac{2C_1 C_3}{r^7} + \rho_0\frac{C_1^{\ 2}}{r^4}]\frac{1}{(C'-t)^3} \tag{17}$$

So,

$$-\frac{1}{R_e P_r}[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}]$$

$$= -\frac{1}{R_e P_r}[\{-\frac{4C_4}{r^6}(x^2 + y^2 + z^2) + \frac{3C_4}{r^4}\}\frac{1}{(C'-t)^3}] = \frac{1}{R_e P_r}\frac{C_4}{r^4}\frac{1}{(C'-t)^3} \tag{18}$$

In this time,

$$\tau_{ij} = \frac{C_2}{r^6} x^i x^j [f(t)]^2 = \frac{C_2}{r^6} x^i x^j \frac{1}{(C'-t)^2}, r > 0$$

$$(\tau_{xj}u, \tau_{yj}v, \tau_{zj}w) = \frac{C_1 C_2}{r^9}(x^2, y^2, z^2)x^j \frac{1}{(C'-t)^3}, r > 0, t \geq 0, C' < 0 \text{ is constant}$$

$$\frac{1}{R_e}[\frac{\partial}{\partial x}(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y}(u\tau_{xy} + v\tau_{yy} + w\tau_{yz}) + \frac{\partial}{\partial z}(u\tau_{xz} + v\tau_{yz} + w\tau_{zz})]$$

$$= \frac{1}{R_e}[-9\frac{C_1C_2}{r^{11}} x^2(x^2 + y^2 + z^2) + \frac{C_1C_2}{r^9}(3x^2 + y^2 + z^2)$$

$$-9\frac{C_1C_2}{r^{11}} y^2(x^2 + y^2 + z^2) + \frac{C_1C_2}{r^9}(x^2 + 3y^2 + z^2)$$

$$-9\frac{C_1C_2}{r^{11}} z^2(x^2 + y^2 + z^2) + \frac{C_1C_2}{r^9}(x^2 + y^2 + 3z^2)]\frac{1}{(C'-t)^3}$$

$$= \frac{1}{R_e}[-9\frac{C_1C_2}{r^7} + 5\frac{C_1C_2}{r^7}]\frac{1}{(C'-t)^3} = -\frac{1}{R_e}\frac{4C_1C_2}{r^7}\frac{1}{(C'-t)^3} \quad (19)$$

Hence, Eq(5) is

$$0 = [(-2\frac{C_1C_3}{r^7} + \rho_0\frac{C_1^2}{r^4}) + \frac{1}{R_eP_r}\frac{C_4}{r^4} - \frac{1}{R_e}\frac{4C_1C_2}{r^7}]\frac{1}{(C'-t)^3},$$

$$C_3 = -\frac{2C_2}{R_e} \quad C_4 = -\rho_0 C_1^2 R_e P_r$$

$$C_3 = \rho_0 C_1^2 - \frac{C_2}{R_e} = -\frac{2C_2}{R_e}, \quad C_2 = -\rho_0 R_e C_1^2 \quad (20)$$

Therefore, P is

$$p = [-\frac{1}{2}\frac{C_3}{r^4} + \rho_0\frac{C_1}{r}]\frac{1}{(C'-t)^2} = [\frac{C_2}{R_e}\frac{1}{r^4} + \rho_0\frac{C_1}{r}]\frac{1}{(C'-t)^2} = [-\rho_0\frac{C_1^2}{r^4} + \rho_0\frac{C_1}{r}]\frac{1}{(C'-t)^2} \quad (21)$$

$$C_4 = -\rho_0 C_1^2 R_e P_r, \; q_x = \frac{C_4}{r^4} x \frac{1}{(C'-t)^3} = -\rho_0\frac{C_1^2 R_e P_r}{r^4}\frac{x}{(C'-t)^3}$$

$$q_y = \frac{C_4}{r^4} y \frac{1}{(C'-t)^3} = -\rho_0\frac{C_1^2 R_e P_r}{r^4} y \frac{1}{(C'-t)^3} \; q_z = \frac{C_4}{r^4} z \frac{1}{(C'-t)^3} = -\rho_0\frac{C_1^2 R_e P_r}{r^4} z \frac{1}{(C'-t)^3}$$

$$r > 0, C' < 0, t \geq 0 \quad (22)$$

3. CONCLUSION

Therefore, the exact solution of Navier-Stokes equations (3-dimensional-unsteady) is

Pressure: $p = [-\rho_0\frac{C_1^2}{r^4} + \rho_0\frac{C_1}{r}]\frac{1}{(C'-t)^2}, \; r > 0, C' < 0, t \geq 0$

Heat Flux:

$$q_x = -\rho_0 \frac{C_1^{\,2} R_e P_r}{r^4} \frac{x}{(C'-t)^3}, q_y = -\rho_0 \frac{C_1^{\,2} R_e P_r}{r^4} \frac{y}{(C'-t)^3}, q_z = -\rho_0 \frac{C_1^{\,2} R_e P_r}{r^4} \frac{z}{(C'-t)^3}$$

$r > 0, C' < 0,\ t \geq 0$

Density: $\rho = \rho_0$,

Stress: $\tau_{ij} = \dfrac{C_2}{r^6} x^i x^j \dfrac{1}{(C'-t)^2} = -\rho_0 \dfrac{R_e}{r^6} C_1^{\,2} x^i x^j \dfrac{1}{(C'-t)^2}, r > 0, C' < 0, t \geq 0$

Reynolds Number: R_e, Plandtl Number: P_r

Velocity Components: $(u, v, w) = \dfrac{C_1}{r^3}(x, y, z)\dfrac{1}{(C'-t)},\ r > 0, C' < 0,\ t \geq 0$

Total Energy: $E_t = E_0$

REFERENCES

[1] Three-dimensional unsteady form of the Navier-Stokes equations: Glenn Research Center, NASA

[2] D.Yi, Ya. Sinaj, Blow ups of complex solutions of the 3-d-Navier-Stokes system and renormalization group method, J. Eur. Math. Soc **10** (2008), no. 2. 267-313

[3] I.Gallagher, M. Paicu, Remarks on the blow-up of solutions to a toy model for the Navier-Stokes equations, Proc. Amer. Math. Soc**. 17** (2009), no. 6, 2075-3083

[4] N. H.Katz, A. Tapai, A note on the slightly supercritical Navier-Stokes equations in the plane, preprint

[5] S. Albeverio, A.B. Cruzeiro, Global flow and invariant (Gibbs) measures for Euler and Navier-Stokes two dimensional fluid. Commun. Math. Phys.**129**, 431-444(1990)

[6] A. Bensoussan, R. Temman, Equations stochastiques du type Navier-Stokes. J. Funct. Anal**. 13**, 195-222(1973)

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