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Lorentz Force in Special Relativity theory

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Abstract: *In the special relativity theory, we know how Lorentz 4-force is invariant in special relativity theory.*

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1. Introduction

Our article's aim is that we tell how Lorentz 4-force is invariant by electro-magnetic field transformations in special relativity theory.

At first, the coordinate transformation is in Special relativity theory,

$$ct = \gamma(ct' + \frac{V}{c}x'),\ x = \gamma(x' + \frac{V}{c}ct'),\quad y = y', z = z' \tag{1}$$

Therefore, Minkowski 4-force is in Special Relativity theory[5]

$$f^0 = m_0 c\frac{d^2t}{d\tau^2} = m_0\gamma(c\frac{d^2t'}{d\tau^2} + \frac{V}{c}\frac{d^2x'}{d\tau^2}) = \gamma(f'^0 + \frac{V}{c}f'^1), \tag{2}$$

$$f^1 = m_0\frac{d^2x}{d\tau^2} = m_0\gamma(\frac{d^2x'}{d\tau^2} + \frac{V}{c}\frac{cd^2t'}{d\tau^2}) = \gamma(f'^1 + \frac{V}{c}f'^0) \tag{3}$$

$$f'^0 = m_0 c\frac{d^2t'}{d\tau^2},\ f'^1 = m\ \frac{d^2x'}{d\tau^2} \tag{4}$$

$$f^2 = m_0\frac{d^2y}{d\tau^2} = m_0\frac{d^2y'}{d\tau^2} = f'^2,\ f^3 = m_0\frac{d^2z}{d\tau^2} = m_0\frac{d^2z'}{d\tau^2} = f'^3 \tag{5}$$

Hence, in inertial frame, Lorentz 4-force is

$$F^0 = m_0\frac{d}{dt}(\frac{cdt}{d\tau}) = q\frac{\vec{u}}{c}\cdot\vec{E} \tag{6}$$

$$\vec{F} = m_0\frac{d}{dt}(\frac{d\vec{x}}{d\tau}) = q[\vec{E} + \frac{\vec{u}}{c}\times\vec{B}]\ ,\qquad \vec{u} = \frac{d\vec{x}}{dt} \tag{7}$$

$$F'^0 = m_0\frac{d}{dt'}(\frac{cdt'}{d\tau}) = q\frac{\vec{u}'}{c}\cdot\vec{E}' \tag{8}$$

$$\vec{F}' = m_0\frac{d}{dt'}(\frac{d\vec{x}'}{d\tau}) = q[\vec{E}' + \frac{\vec{u}'}{c}\times\vec{B}'],\quad \vec{u}' = \frac{d\vec{x}'}{dt'} \tag{9}$$

2. Invariant Lorentz 4-Force in Inertial Frame

In this time, Minkowski 4-force is in inertial frame.

$$f^0 = m_0 c \frac{d^2 t}{d\tau^2} = q\frac{\vec{u}}{c}\cdot\vec{E}\frac{dt}{d\tau} \qquad ,\vec{u} = \frac{d\vec{x}}{dt},\ \vec{u}' = \frac{d\vec{x}'}{dt'}$$

$$= m_0\gamma\left(c\frac{d^2 t'}{d\tau^2} + \frac{v}{c}\frac{d^2 x'}{d\tau^2}\right) = \gamma\left(f'^0 + \frac{v}{c}f'^1\right)$$

$$= \gamma q\frac{\vec{u}'}{c}\cdot\vec{E}'\frac{dt'}{d\tau} + \gamma\frac{v}{c}\left[qE_x' + q\frac{1}{c}(u_y'B_z' - u_z'B_y')\right]\frac{dt'}{d\tau} \tag{10}$$

$$f^1 = m_0\frac{d^2 x}{d\tau^2} = m_0\gamma\left(\frac{d^2 x'}{d\tau^2} + \frac{v}{c}\frac{cd^2 t'}{d\tau^2}\right) = \gamma\left(f'^1 + \frac{v}{c}f'^0\right)$$

$$= \gamma\left[qE_x' + q\frac{1}{c}(u_y'B_z' - u_z'B_y')\right]\frac{dt'}{d\tau} + \gamma\frac{v}{c}\left(q\frac{\vec{u}'}{c}\cdot\vec{E}'\right)\frac{dt'}{d\tau} \tag{11}$$

$$f'^0 = m_0 c\frac{d^2 t'}{d\tau^2} = q\frac{\vec{u}'}{c}\cdot\vec{E}'\frac{dt'}{d\tau},\ f'^1 = m_0\frac{d^2 x'}{d\tau^2} = q\left[E_x' + \frac{1}{c}(u_y'B_z' - u_z'B_y')\right]\frac{dt'}{d\tau} \tag{12}$$

In This Time, The Transformation Of Electromagnetic Field Is In Special Relativity Theory.

$$E_x = E'_x\ ,\ E_y = \gamma E'_y + \gamma\frac{v}{c}B'_z, E_z = \gamma E'_z - \gamma\frac{v}{c}B'_y$$

$$B_x = B'_x, B_y = \gamma B'_y - \gamma\frac{v}{c}E'_z, B_z = \gamma B'_z + \gamma\frac{v}{c}E'_y \tag{13}$$

Hence, If We Calculate For Proving Invariant Of Lorentz 4-Force By Eq(10),Eq(11),Eq(12),Eq(13), T-Component Is

$$f^0 = m_0 c\frac{d^2 t}{d\tau^2} = q\frac{\vec{u}}{c}\cdot\vec{E}\frac{dt}{d\tau}$$

$$= q\frac{1}{c}(u_x E_x + u_y E_y + u_z E_z)\frac{dt}{d\tau}$$

$$= q\frac{1}{c}\left[\left(\frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}\right)E'_x + \frac{u_y'}{\gamma(1 + \frac{u_x' v}{c^2})}\gamma\left(E'_y + \frac{v}{c}B'_z\right) + \frac{u_z'}{\gamma(1 + \frac{u_x' v}{c^2})}\gamma\left(E'_z - \frac{v}{c}B'_y\right)\right]$$

$$\times\gamma\frac{dt'}{d\tau}\left(1 + \frac{u_x' v}{c^2}\right)$$

$$= \gamma q\frac{1}{c}(u_x'E_x' + u_y'E_y' + u_z'E_z')\frac{dt'}{d\tau} + \gamma\frac{v}{c}\left[qE_x' + q\frac{1}{c}(u_y'B_z' - u_z'B_y')\right]\frac{dt'}{d\tau}$$

$$= \gamma\left(f'^0 + \frac{v}{c}f'^1\right) \tag{14}$$

X-component is

$$f^1 = m_0\frac{d^2 x}{d\tau^2} = \left[qE_x + q\frac{1}{c}(u_y B_z - u_z B_y)\right]\frac{dt}{d\tau}$$

$$= \left[qE'_x + q\left(\frac{1}{c}\left(\frac{u_y'}{\gamma(1 + \frac{u_x' v}{c^2})}\right)\gamma\left(B'_z + \frac{v}{c}E'_y\right) - \frac{u_z'}{\gamma(1 + \frac{u_x' v}{c^2})}\gamma\left(B'_y - \frac{v}{c}E'_z\right)\right)\right]$$

$$\times\gamma\frac{dt'}{d\tau}\left(1 + \frac{u_x' v}{c^2}\right)$$

$$= \gamma[qE_x' + q\frac{1}{c}(u_y'B_z' - u_z'B_y')]\frac{dt'}{d\tau} + \gamma\frac{V}{c}[q\frac{1}{c}(u_x'E_x' + u_y'E_y' + u_z'E_z')]\frac{dt'}{d\tau}$$

$$= \gamma(f'^1 + \frac{V}{c}f'^0) \quad (15)$$

Y-component is

$$f^2 = m_0\frac{d^2y}{d\tau^2} = [qE_y + q\frac{1}{c}(u_zB_x - u_xB_z)]\frac{dt}{d\tau}$$

$$= [q\gamma(E'_y + \frac{V}{c}B'_z) + q(\frac{1}{c}(\frac{u_z'}{\gamma(1+\frac{u_x'V}{c^2})})B'_x - \frac{u_x'+V}{1+\frac{u_x'V}{c^2}}\gamma(B'_z + \frac{V}{c}E'_y))]$$

$$\times\gamma\frac{dt'}{d\tau}(1+\frac{u_x'V}{c^2})$$

$$= [qE_y' + q\frac{1}{c}(u_z'B_x' - u_x'B_z')]\frac{dt'}{d\tau} = f'^2 \quad (16)$$

Z-component is

$$f^3 = m_0\frac{d^2z}{d\tau^2} = [qE_z + q\frac{1}{c}(u_xB_y - u_yB_x)]\frac{dt}{d\tau}$$

$$= [q\gamma(E'_z - \frac{V}{c}B'_y) + q(\frac{1}{c}(\frac{u_x'+V}{1+\frac{u_x'V}{c^2}})\gamma(B'_y - \frac{V}{c}E'_z) - \frac{u'_y}{\gamma(1+\frac{u_x'V}{c^2})}B'_x)]$$

$$\times\gamma\frac{dt'}{d\tau}(1+\frac{u_x'V}{c^2})$$

$$= [qE_z' + q\frac{1}{c}(u_x'B_y' - u_y'B_x')]\frac{dt'}{d\tau} = f'^3 \quad (17)$$

3. Conclusion

We know Lorentz 4-force is invariant by the Lorentz transformation in special relativity theory .

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