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Pulsar Spin- Down Under Braking of Ohmic Decay

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Abstract: *spin down of pulsar under the braking of Ohmic magnetic decay in the magnetic dipole model is studied. The theoretical formulas for the influence of Ohmic magnetic decay on the variation of the pulse period in the magnetic dipole model are given. The numerical solutions for slow down of period due to the braking of Ohmic magnetic decay are calculated by using the given formulas for PSR0531+21(Crab). The numerical results are given. Discussions and Conclusion are drawn.*

Keywords: *Pulsars-- Ohmic decay —slow down of period*

1. Introduction

At first, the gravitational radiation plays a leading role after pulsar was born in a shot time (almost 18 year), ,and then, the magnetic radiation plays a second role in the pulsar's life. On the other hand, when the pulsar was born the magnetic field is very strong, （10^{14} G） The magnetic field decay with time from beginning to pulsar's life. Therefore, the magnetic decay follows pulsar's life, the magnetic decay can brake spin down. It is very important for research of the magnetic decay.

In the past year some authors studied evolution of the magnetic decay of pulsars, such as Ostriker and Gunn **[1],** Wang et al **[2]**, Mitra et al **[3]**, Urpin and Gil **[4]**. However, these authors studied the surface magnetic decay and can not studied the internal magnetic decay, Hensel, Urpin and Yakovlev [5] studied the internal magnetic decay. They term as Ohmic decay. Howerer, they did not study pulsar spin down under the braking of Ohmic decay. In this paper the present author studied spin down of pulsar under the braking of the magnetic Ohmic decay.

2. Spin Down of Pulsar under Braking of Ohmic Decay

The pulsar radiating power is transformed from the rotational energy at a rate $\frac{dE}{dt}$. According to the theory of magnetic dipole model (Shapiro & Teukolsky **[6]**,

$$\frac{dE}{dt} = -\frac{2}{3c^3}(M\sin\alpha)^2\Omega^4 = -\frac{32}{3c^3}\frac{\pi^4\mu^2}{P^4},, \quad \Omega = \frac{2\pi}{P} \ , \tag{1}$$

Where P is the period of pulsar, and μ is the projection of the magnetic dipole moment M on the direction perpendicular to the rotational axis. α denotes magnetic inclination.

$$\mu^2(t) = M^2(t)Sin^2\alpha \ ,. \ \ M^2(t) = R^6B^2(t). \tag{2}$$

The total rotational energy of pulsar can be written as

$$E = \frac{1}{2}I\Omega^2 = {2\pi^2 I}/{P^2} \ ,. \tag{3}$$

Here I is moment of inertia. If we consider the moment of inertia as a constant

$$\frac{dE}{dt} = -\frac{4\pi^2 I}{P^3}\frac{dP}{dt} .. \qquad (4)$$

We obtain from the equations (1) and (4)

$$\frac{dP}{dt} = \frac{8\pi^2 \mu^2(t)}{3c^3 I} P^{-1} \quad \text{i}$$

Or $2P\frac{dP}{dt} = \frac{16\pi^2\mu^2}{3c^3 I}..,$

i.e $\frac{dP^2}{dt} = \frac{16\pi^2\pi^2}{3c^3 I}..$

According to the expression (2)

$$\mu^2 - M^2{}_0 Sin^2\alpha = R^6 B^2 Sin\alpha,,$$

$$\frac{dP^2}{dt} = \frac{16\pi^2 R^6 Sin^2\alpha}{3c^3 I} B^2(t). . \qquad (5)$$

Haensel et al (1990) researched Ohmic decay of internal magnetic fields in neutron stars. They obtained the theoretical formulas for decay of internal magnetic field B(t) with time [5]

$$B(t) = B(0)\left(1 + \frac{B_0{}^2}{B_{\max}(t)^2}\right)^{-1/2} , \qquad (6)$$

With $B_{\max}(t) = [\left(\frac{2}{D}\right)\int^t T_9{}^{-2}(t')dt']^{-1/2}.$

$$B_{\max}(t) = 1.85\times10^{18}(\frac{t}{1Yr})^{-2/3} \text{ G}. \qquad (7)$$

This paper cited the result of the expression (6) (7) given by Haensel et al. Substitution of the expression (7) into the formula (6)

$$B(t) = B(0)\left\langle 1 + 2.9218\times10^{-79} B^2(0)(t/yr)^{4/3}\right\rangle^{-1/2},.,$$

$$B^2(t) = B^2(0)\left\langle 1 + KB^2(0)(t/yr)^{4/3}\right\rangle^{-1}.., \qquad (8)$$

Where $K = 2.9218\times10^{-37}$.

Expanding the expression (.6)

$$B^2(t) = B^2(0)\langle 1 - KB^2(0)(t/yr)^{4/3} + K^2B^4(t/yr)^{8/3} ..,,,,,...\rangle .. \quad (9)$$

Substituting (7) into the equation (5) and integrating

$$P^2(t) - P^2(t) = \frac{16\pi^2 R_0^6 Sin^2\alpha}{3c^3 I}\int B^2(t)\, dt$$

$$= \frac{16\pi^2 R_0^6 Sin^2\alpha}{3c^3 I}\int (1 - KB_0^2(t//yr)^{4/3} + k^2 B_0^4(t/yr)^{8/3}.....)dt \ .$$

$$P^2(t) = P^2(0) + \frac{16\pi^2 R_0^6 Sin^2\alpha}{3c^3 I}\int_{t_0}^{t} \langle (1yr) - (1yr)KB^2(t/yr)^{4/3} + (1yr)K^2B^4{}_0(t/yr)^{8/3}\rangle d(\frac{t}{1yr})$$

Integrating this expression, we obtain

$$P^2(t) = P^2(0) + \frac{16\pi^2 R_0^6 Sin^2\alpha}{3c^3 I}[(t - t_0) - \frac{3}{7}(1yr)KB_0^2\{(\frac{t}{1yr})^{3/7} - (\frac{t_0}{1yr})^{7/3}] +$$

$$+\frac{3}{11}(1yr)K^2B_0^4\{(\frac{t}{1yr})^{11/3} - (\frac{t_0}{1yr})^{11/3}\}]. \quad (10)$$

As an example we calculate the period of PSR0531+21 is prolonged by the braking of Ohmic decay . For this pulsar P(0)=0.033085(s) [7], $R = 1.2\times10^6 cm,, I = 1.4\times10^{45}(gcm^2),$ [6] $\alpha = 59.2^0$ [8] · In the internal magnetic field B_0 **=10 ^{14} ~10^{13}** G and on the surface magnetic field B_0 =10^{12} G '[5] . In this paper we use the internal magnetic field B_0 = 10^{13} G and take $t - t_0 = 100 yr$. 1yr =3.1556926 (s),

$K = 2.9218\times10^{37}$.

Substituting the above data into the expression (10), yields

$$P^2(t) = P^2(0) + 3.0677\times10^{-13}(3.1556926\times10^9 - 18.3415+,,,,,,)$$

$$= P^2(0) + 9.68\times10^{-4} - 5.6266\times10^{-12} + ... \quad (11)$$

The second term is very small , which may be neglected. The numerical results are listed in Table 1..

Table 1 Numerical results for slow down of the period of PSR0531+21 due to Ohmic decay (s/cy)

Pulsar	$P(s)$	α(deg)	$B(0)(G0$	$P(t)(s/cy)$	$\delta P(s/cy)$
PSR0531+21	0.033085	59.2	10^{13}	0.04516	0.01233

3. Discussion and Conclusion

1. Pulsar sin-down and braking of the surface magnetic with time According to three curves of log (B / G) = 14, 13, 12 in Fig.3 of Ref [5] , log(B / G) : 14, 13 vary with time and 12 does not vary with time. Hence the theory of Ohmic decay does not suite to study the surface magnetic field B = 10^{12} G. . Although Ref [4] gives the formula for the construct of surface magnetic field, it can not transform to the surface magnetic decay with time.. So in this paper we use the theory of the

exponential magnetic decay with time..

$$\mu^2 = M^2 Sin^2\alpha = M_0{}^2 Sin^2\alpha e^{-\xi t} = R^6 Sin^2\alpha B^2(0) e^{-\xi t}.. \quad (12)$$

Substituting （12）into the formula (5) and integrating the equation

$$P^2(t) = P^2(9) + \frac{16\pi^2 R_0{}^6 Sin^2\alpha_0 B^2(0)}{3c^3 I}\int_{t_0}^{t} e^{-\xi t} dt = p^2(0) + \frac{16\pi R_0{}^6 Sin\ \alpha_0 B(0)}{3c^3 I(-\xi)}(e^{-\xi t} - e^{-\xi t_0}) \quad (13)$$

In this paper we use $t - t_0 = 100y = 3.1556926\times10^9(s)$ and setting $t_0 = 0$

$$-(e^{\xi t} - e^{-\xi t_0})/\xi = -(e^{-\xi t} - 1)/\xi \quad (14)$$

According to Ref [2] $\tau_D = 1.8\times10^6$ yr

$\xi = 2/\tau_D = 2/1.8\times10^6 = 1.1111\times10^{-6}/yr = 3.5\times10^{14}/s$. [2]

For PSR0351+21, $B_0 = 3.78\times10^{12}$ G [7]

Substituting the above data into (12) and (13), we obtain the numerical results are listed in Table.2

Table2. *The numerical results for slow down of the period of PSR0531+21 due to the surface magnetic decay (s/cy)*

Pulsar	$P(0)$	α(deg)	$B(0)(G)$	$P(t)(s/cy)$	$\delta P(s/cy)$
PSR0531+21	0.033085	59.2	3.78×10^{12}	0.03705	0.00396

2. Comparison and conclusion

It can be seen from Table.1 and Table.2 that the influence of Ohmic magnetic decay in the pulsar interior on the period is larger than that of the surface magnetic decay. Hence we conclude that it is important for study Ohmic decay in pulsar interior than that of the surface magnetic decay.

3. In this paper PSR0531+21 has spin down with time due to braking of Ohmis decay in the secular variation. However PSR 0531+21 also spin up suddenly in order $\Delta\omega/\omega \sim 10^{-8}$ undergone star quake (glitches) per almost three years [9] . When after glitches it has spin down gradually and resumes to the original velocity $\omega = 190(rad/s)$ in the time of relaxation 7.7 days. Hence the sudden spin up due to the glitches does not influence spin down in secular variation.

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