

Reissner-Nodstrom Solution and Energy-Momentum Density's Conservation Law

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Abstract: We find Reissner-Nodstrom solution hold by the energy-momentum density's conservation law (Noether's theorem) of electromagnetic field in general relativity theory.

Keywords: General relativity theory, Reissner-Nodstrom solution, Energy-momentum density's conservation law, Noether's theorem

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1. INTRODUCTION

Our article's aim is that we find Reissner-Nodstrom solution hold by the energy-momentum density's conservation law (Noether's theorem) of electromagnetic field.

In general relativity theory, the energy-momentum tensor $T^{\mu\nu}$ of the electromagnetic field is

$$T^{\mu\nu} = \frac{1}{4\pi C} (F^{\mu\rho} F^{\nu\sigma} - \frac{1}{4} g^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}) \quad (1)$$

In this time, spherical coordinates is in general relativity theory

$$ds^2 = -A(t,r)dt^2 + B(t,r)dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (2)$$

Hence, Faraday tensors $F^{\mu\nu}, F_{\mu\nu}$ are in general relativity theory.

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E(t,r) & 0 & 0 \\ E(t,r) & 0 & 0 & 0 \\ 0 & 0 & 0 & -B(t,r) \\ 0 & 0 & B(t,r) & 0 \end{pmatrix} \quad (3-i)$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E(t,r) & 0 & 0 \\ -E(t,r) & 0 & 0 & 0 \\ 0 & 0 & 0 & -B(t,r) \\ 0 & 0 & B(t,r) & 0 \end{pmatrix} \quad (3-ii)$$

Hence, Einstein-Maxwell equations is

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{C^4} T_{\mu\nu} = -\frac{2G}{C^5} (F_{\mu\rho} F^{\rho\nu} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}) \quad (4)$$

$$F^{\mu\nu}_{;\nu} = 0 \quad (5)$$

$$\frac{\partial F_{\mu\nu}}{\partial x^\rho} + \frac{\partial F_{\nu\rho}}{\partial x^\mu} + \frac{\partial F_{\rho\mu}}{\partial x^\nu} = 0 \quad (6)$$

2. REISSNER-NODSTROM SOLUTION AND NOETHER'S THEOREM

In this time, energy-momentum tensor $T^{\mu\nu}$ is

$$\begin{aligned} T^{\mu\nu} &= \frac{1}{4\pi C} (F^\mu{}_\rho F^{\nu\rho} - \frac{1}{4} g^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}) \\ &= \frac{1}{4\pi C} (g^{\mu\nu} F_{\nu\rho} F^{\nu\rho} - \frac{1}{4} g^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}) \end{aligned} \tag{7}$$

Hence, $T^{\mu\nu}$ is

$$\begin{aligned} T^{\mu\nu} &= \frac{E^2 + B^2}{8\pi C} \begin{pmatrix} -g^{00} & 0 & 0 & 0 \\ 0 & -g^{11} & 0 & 0 \\ 0 & 0 & g^{22} & 0 \\ 0 & 0 & 0 & g^{33} \end{pmatrix} \\ &= \frac{E^2 + B^2}{8\pi C} \begin{pmatrix} 1/A & 0 & 0 & 0 \\ 0 & -1/B & 0 & 0 \\ 0 & 0 & 1/r^2 & 0 \\ 0 & 0 & 0 & 1/r^2 \sin^2 \theta \end{pmatrix} \end{aligned} \tag{8}$$

Therefore, energy-momentum tensor $T_{\mu\nu}$ is

$$\begin{aligned} T_{\mu\nu} &= \frac{1}{4\pi C} (F_{\mu\rho} F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}) \\ &= \frac{1}{4\pi C} (g_{\mu\nu} F_{\nu\rho} F^{\nu\rho} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}) \end{aligned} \tag{9}$$

Hence, if we calculate $T_{\mu\nu}$,

$$T_{\mu\nu} = \frac{E^2 + B^2}{8\pi C} \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & -B & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} \tag{10}$$

Eq(5) is

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} + \Gamma^\mu{}_{\rho\nu} F^{\rho\nu} + \Gamma^\nu{}_{\rho\nu} F^{\mu\rho} = \frac{\partial F^{\mu\nu}}{\partial x^\nu} + \Gamma^\nu{}_{\rho\nu} F^{\mu\rho} \tag{11}$$

In this time,

$$\Gamma^\nu{}_{\rho\nu} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\rho} \sqrt{-g} \tag{12}$$

Eq(5) is

$$F^{\mu\nu}{}_{;\nu} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} (\sqrt{-g} F^{\mu\nu}) = 0 \tag{13}$$

Hence, Eq(13) is

$$\frac{\partial}{\partial r} (r^2 E) = 0, \quad E = k \frac{Q}{r^2} \tag{14}$$

Eq(6) is

$$\frac{\partial F_{23}}{\partial x} + \frac{\partial F_{31}}{\partial x^2} + \frac{\partial F_{12}}{\partial x^3} = -\frac{\partial B}{\partial r} = 0, B = 0 \quad (15)$$

Hence, we know the following formula.

$$E = k \frac{Q}{r^2}, \quad B = 0 \quad (16)$$

The solution of Eq(4),Eq(5),Eq(6) is Reissner-Nodstrom solution. Hence,

$$A = 1 - \frac{C_1}{r} + \frac{C_2}{r^2}, \quad B = 1 / (1 - \frac{C_1}{r} + \frac{C_2}{r^2}) \quad (17)$$

In this time, if energy-momentum tensor $T^{\mu\nu}$ satisfy the energy-momentum conservation law (Noether's theorem)

$$\begin{aligned} T^{\mu\nu}_{;\nu} &= T^{00}_{;\nu} + T^{\dot{\mu}\dot{\nu}}_{;\dot{\mu}} = T^{00}_{;\nu} + \frac{\partial T^{\dot{\mu}}}{\partial x^i} + 2\Gamma^i_{\dot{\mu}} T^{\dot{\mu}} \\ &= \frac{E^2 + B^2}{8\pi c} \frac{\partial (-g^{11})}{\partial r} + 2 \cdot \frac{1}{2} g^{11} \left(\frac{\partial g_{11}}{\partial r} \right) \cdot \frac{E^2 + B^2}{8\pi c} \cdot -g_{11} \\ &= \frac{E^2 + B^2}{8\pi c} \left(-\frac{C_1}{r^2} + \frac{2C_2}{r^3} \right) + \frac{E^2 + B^2}{8\pi c} \left(\frac{C_1}{r^2} - \frac{2C_2}{r^3} \right) = 0 \end{aligned} \quad (18)$$

3. CONCLUSION

Reissner-Nodstrom solution hold by the energy-momentum density's conservation law of electromagnetic field in general relativity theory.

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