International Journal of Advanced Research in Physical Science (IJARPS)
Volume 5, Issue 9, 2018, PP 46-56
ISSN No. (Online) 2349-7882
www.arcjournals.org

Electromagnetic Wave Function and Equation, Lorentz Force in Rindler Spacetime

Sangwha-Yi

Department of Math, Taejon University 300-716

Corresponding Author:** ***Sangwha-Yi, *Department of Math, Taejon University 300-716*

Abstract: *In the general relativity theory, we find the electro-magnetic wave function and equation in Rindler space-time. Specially, this article is that electromagnetic wave equation is corrected by the gauge fixing equation in Rindler space-time. We define the force in Rindler space-time We find Lorentz force (electromagnetic force) by electro-magnetic field transformations in Rindler space-time. In the inertial frame, Lorentz force is defined as 4-dimensional force. Hence, we had to obtain 4-dimensional force in Rindler space-time. We define energy-momentum in Rindler space-time.*

Keywords: *General relativity theory, Rindler spacetime; Electro-magnetic wave equation; Electromagnetic wave function; Lorentz force; Electro-magnetic field transformation, Energy-momentum*

PACS Number: *04, 04.90. +e,03.30, 41.20*

1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic wave equation and function and Lorentz force by electro-magnetic field transformations in Rindler space-time. This article correct the article "Electromagnetic Field Equation and Lorentz Gauge in Rindler space-time" about the existence proof of electromagnetic wave function and equation. We define energy-momentum in Rindler space-time.

The Rindler coordinate is

$$ct = \left(\frac{c^2}{a_0} + \xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right)$$

$$x = \left(\frac{c^2}{a_0} + \xi^1\right)\cosh\left(\frac{a_0\xi^0}{c}\right) - \frac{c^2}{a_0} \quad , y = \xi^2 , z = \xi^3 \tag{1}$$

Therefore,

$$cdt = c\cosh\left(\frac{a_0\xi^0}{c}\right)d\xi^0\left(1 + \frac{a_0}{c^2}\xi^1\right) + \sinh\left(\frac{a_0\xi^0}{c}\right)d\xi^1$$

$$dx = c\sinh\left(\frac{a_0\xi^0}{c}\right)d\xi^0\left(1 + \frac{a_0}{c^2}\xi^1\right) + \cosh\left(\frac{a_0\xi^0}{c}\right)d\xi^1, dy = d\xi^2, dz = d\xi^3 \tag{2}$$

Hence,

$$\frac{1}{c}\frac{\partial}{\partial t} = \frac{c\partial\xi^0}{c\partial t}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{c\partial t}\frac{\partial}{\partial\xi^1}$$

$$= \frac{\cosh\left(\frac{a_0\xi^0}{c}\right)}{\left(1 + \frac{a_0\xi^1}{c^2}\right)}\frac{\partial}{c\partial\xi^0} - \sinh\left(\frac{a_0\xi^0}{c}\right)\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial x} = \frac{c\partial \xi^0}{\partial x}\frac{1}{c}\frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial x}\frac{\partial}{\partial \xi^1}$$

$$= -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} + \cosh(\frac{a_0 \xi^0}{c})\frac{\partial}{\partial \xi^1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial \xi^3} \tag{3}$$

Hence,

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial \xi^0})^2 - \nabla_\xi{}^2$$

$$\vec{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}), \quad \vec{\nabla}_\xi = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3}) \tag{4}$$

2. CORRECTED ELECTROMAGNETIC WAVE EQUATION IN THE RINDLER SPACE-TIME

The electro-magnetic field $(\vec{E}, \vec{B})$ is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{c\partial t}, \vec{B} = \vec{\nabla} \times \vec{A} \tag{5}$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler space-time [1].

$$\vec{E}_\xi = -\frac{1}{(1+\frac{a_0 \xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0 \xi^1}{c^2})}\frac{\partial \vec{A}_\xi}{c\partial \xi^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi \times \vec{A}_\xi$$

In this time, $\vec{\nabla}_\xi = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3}), \vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$ (6)

Hence, Lorentz gauge condition is in Rindler space-time [1],

$$\phi_\xi \to \phi_\xi - \frac{1}{c}\frac{\partial \Lambda}{\partial \xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, \ \vec{A}_\xi \to \vec{A}_\xi + \vec{\nabla}_\xi \Lambda, \ \Lambda \text{ is a scalar function.} \tag{7}$$

$$A^\mu{}_{;\mu} = \frac{\partial A^\mu}{\partial \xi^\mu} + \Gamma^\mu{}_{\mu\rho}A^\rho \to \partial_\mu(A^\mu + g^{\mu\upsilon}\partial_\upsilon \Lambda) + \Gamma^0{}_{01}(A^1 + \frac{\partial \Lambda}{\partial \xi^1}) \tag{8}$$

Lorentz gauge fix condition is in Rindler space-time [1],

$$0 = \frac{1}{c}\frac{\partial \phi_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{A_{\xi^1}}{c^2}\frac{a_0}{(1+\frac{a_0\xi^1}{c^2})}$$

$$\to \frac{1}{c}\frac{\partial \phi_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{A_{\xi^1}}{c^2}\frac{a_0}{(1+\frac{a_0\xi^1}{c^2})} - [\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial \xi^0})^2 - \nabla_\xi{}^2]\Lambda + \frac{\partial \Lambda}{\partial \xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}$$

$$= 0 \tag{9}$$

Hence, the gauge equation is

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\Lambda-\frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0 \quad (10)$$

We can use Eq(10) as an electromagnetic wave equation because we can apply electromagnetic wave function instead of the gauge function Λ to Eq(10) in Rindler space-time. Hence,

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]E_{\xi^1}-\frac{\partial E_{\xi^1}}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]B_{\xi^1}-\frac{\partial B_{\xi^1}}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0 \quad (11)$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]E_y-\frac{\partial E_y}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]B_y-\frac{\partial B_y}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0 \quad (12)$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]E_z-\frac{\partial E_z}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]B_z-\frac{\partial B_z}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0 \quad (13)$$

The electro-magnetic wave function is

$$E_x=E_{x0}\sin\Phi, E_y=E_{y0}\sin\Phi, E_z=E_{z0}\sin\Phi$$

$$B_x=B_{x0}\sin\Phi, B_y=B_{y0}\sin\Phi, B_z=B_{z0}\sin\Phi \quad (14)$$

$$E_{\xi^1}=E_X=E_{X0}\sin\Phi, B_{\xi^1}=B_X=B_{X0}\sin\Phi$$

$$E_{\xi^2}=E_y\cosh(\frac{a_0\xi^0}{c})-B_z\sinh(\frac{a_0\xi^0}{c}),$$

$$=(E_{y0}\sin\Phi)\cosh(\frac{a_0\xi^0}{c})-(B_{z0}\sin\Phi)\sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^2}=B_y\cosh(\frac{a_0\xi^0}{c})+E_z\sinh(\frac{a_0\xi^0}{c})$$

$$=(B_{y0}\sin\Phi)\cosh(\frac{a_0\xi^0}{c})+(E_{z0}\sin\Phi)\sinh(\frac{a_0\xi^0}{c})$$

$$E_{\xi^3} = E_z \cosh(\frac{a_0\xi^0}{c}) + B_y \sinh(\frac{a_0\xi^0}{c}) = (E_{z0}\sin\Phi)\cosh(\frac{a_0\xi^0}{c}) + (B_{y0}\sin\Phi)\sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^3} = B_z \cosh(\frac{a_0\xi^0}{c}) - E_y \sinh(\frac{a_0\xi^0}{c})$$

$$= (B_{z0}\sin\Phi)\cosh(\frac{a_0\xi^0}{c}) - (E_{y0}\sin\Phi)\sinh(\frac{a_0\xi^0}{c}) \quad (15)$$

$$\Phi = \omega(t - l\frac{x}{c} - m\frac{y}{c} - n\frac{z}{c}),$$

$$= \omega\{(\frac{c}{a_0} + \frac{\xi^1}{c})(\sinh(\frac{a_0\xi^0}{c}) - l\cosh(\frac{a_0\xi^0}{c})) + \frac{c}{a_0} - m\frac{\xi^2}{c} - n\frac{\xi^3}{c}\},$$

$$l^2 + m^2 + n^2 = 1 \quad (16)$$

3. ELECTRO-MAGNETIC FORCE IN RINDLER SPACE-TIME

In inertial frame, Lorentz 4-force is

$$F^0 = m_0 \frac{d}{dt}(\frac{cdt}{d\tau}) = q\frac{\vec{u}}{c}\cdot\vec{E} \quad (17)$$

$$\vec{F} = m_0 \frac{d}{dt}(\frac{d\vec{x}}{d\tau}) = q[\vec{E} + \frac{\vec{u}}{c}\times\vec{B}], \quad \vec{u} = \frac{d\vec{x}}{dt} \quad (18)$$

We want to obtain the Lorentz 4-force in Rindler space-time. Hence, we define the force in Rindler space-time.

$$F_{\xi}^{\ 0} = m_0 \frac{d}{d\xi^0}(\frac{cd\xi^0}{d\tau})$$

$$\vec{F}_{\xi} = m_0 \frac{d}{d\xi^0}(\frac{d\vec{\xi}}{d\tau}) \quad (19)$$

Or

$$F_{\xi}^{\ \mu} = m_0 \frac{d}{d\xi^0}(\frac{d\xi^\mu}{d\tau}) \quad (20)$$

Hence, 4-force is in inertial frame

$$F^{\alpha} = m_0 \frac{d}{dt}(\frac{dx^\alpha}{d\tau}) \quad (21)$$

In this time, Minkowski force is in inertial frame or in Rindler space-time [13].

$$f^{\alpha} = m_0 \frac{d}{d\tau}(\frac{dx^\alpha}{d\tau}) = m_0 \frac{d^2x^\alpha}{d\tau^2} \quad (22)$$

$$f_{\xi}^{\ \mu} = m_0 \frac{d}{d\tau}(\frac{d\xi^\mu}{d\tau}) = m_0 \frac{d^2\xi^\mu}{d\tau^2} \quad (23)$$

Minkowski force is

$$f^{\alpha}=m_0\frac{d^2x^{\alpha}}{d\tau^2}=m_0\frac{d}{d\xi^0}(\frac{\partial x^{\alpha}}{\partial\xi^{\mu}}\frac{d\xi^{\mu}}{d\tau})\frac{d\xi^0}{d\tau}$$

$$=m_0\frac{d}{d\xi^0}(\frac{\partial x^{\alpha}}{\partial\xi^{\mu}})\frac{d\xi^{\mu}}{d\tau}\frac{d\xi^0}{d\tau}+m_0\frac{\partial x^{\alpha}}{\partial\xi^{\mu}}\frac{d}{d\xi^0}(\frac{d\xi^{\mu}}{d\tau})\frac{d\xi^0}{d\tau}$$

$$=m_0\frac{d}{d\xi^0}(\frac{\partial x^{\alpha}}{\partial\xi^{\mu}})\frac{d\xi^{\mu}}{d\tau}\frac{d\xi^0}{d\tau}+F_{\xi}^{\ \mu}\frac{\partial x^{\alpha}}{\partial\xi^{\mu}}\frac{d\xi^0}{d\tau}$$

$$F_{\xi}^{\ \mu}=m_0\frac{d}{d\xi^0}(\frac{d\xi^{\mu}}{d\tau}) \tag{24}$$

Hence,

$$f^0=m_0\frac{cd^2t}{d\tau^2}=m_0\frac{d}{d\xi^0}(\frac{\partial t}{\partial\xi^0})\frac{cd\xi^0}{d\tau}\frac{d\xi^0}{d\tau}+F_{\xi}^{\ 0}\frac{\partial t}{\partial\xi^0}\frac{d\xi^0}{d\tau}$$

$$+m_0\frac{d}{d\xi^0}(\frac{c\partial t}{\partial\xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}+F_{\xi}^{\ 1}\frac{c\partial t}{\partial\xi^1}\frac{d\xi^0}{d\tau} \tag{25}$$

$$f^1=m_0\frac{d^2x}{d\tau^2}=m_0\frac{d}{d\xi^0}(\frac{\partial x}{c\partial\xi^0})\frac{cd\xi^0}{d\tau}\frac{d\xi^0}{d\tau}+F_{\xi}^{\ 0}\frac{\partial x}{c\partial\xi^0}\frac{d\xi^0}{d\tau}$$

$$+m_0\frac{d}{d\xi^0}(\frac{\partial x}{\partial\xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}+F_{\xi}^{\ 1}\frac{\partial x}{\partial\xi^1}\frac{d\xi^0}{d\tau} \tag{26}$$

$$F_{\xi}^{\ 0}=m_0\frac{d}{d\xi^0}(\frac{cd\xi^0}{d\tau}),\ F_{\xi}^{\ 1}=m_0\frac{d}{d\xi^0}(\frac{d\xi^1}{d\tau})$$

Therefore,

$$F_{\xi}^{\ 0}\frac{\partial t}{\partial\xi^0}\frac{d\xi^0}{d\tau}+F_{\xi}^{\ 1}\frac{c\partial t}{\partial\xi^1}\frac{d\xi^0}{d\tau}$$

$$=f^0-m_0\frac{d}{d\xi^0}(\frac{\partial t}{\partial\xi^0})\frac{cd\xi^0}{d\tau}\frac{d\xi^0}{d\tau}-m_0\frac{d}{d\xi^0}(\frac{c\partial t}{\partial\xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}=A \tag{27}$$

$$F_{\xi}^{\ 0}\frac{\partial x}{c\partial\xi^0}\frac{d\xi^0}{d\tau}+F_{\xi}^{\ 1}\frac{\partial x}{\partial\xi^1}\frac{d\xi^0}{d\tau}$$

$$=f^1-m_0\frac{d}{d\xi^0}(\frac{\partial x}{c\partial\xi^0})\frac{cd\xi^0}{d\tau}\frac{d\xi^0}{d\tau}-m_0\frac{d}{d\xi^0}(\frac{\partial x}{\partial\xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}=B \tag{28}$$

If we represent by the matrix,

$$\begin{pmatrix}A\\B\end{pmatrix}=\begin{pmatrix}\frac{\partial t}{\partial\xi^0}\frac{d\xi^0}{d\tau} & \frac{c\partial t}{\partial\xi^1}\frac{d\xi^0}{d\tau}\\ \frac{\partial x}{c\partial\xi^0}\frac{d\xi^0}{d\tau} & \frac{\partial x}{\partial\xi^1}\frac{d\xi^0}{d\tau}\end{pmatrix}\begin{pmatrix}F_{\xi}^{\ 0}\\F_{\xi}^{\ 1}\end{pmatrix} \tag{29}$$

In this time, Rindler coordinate is

$$ct=(\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0\xi^0}{c}),x=(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})-\frac{c^2}{a_0}\quad ,y=\xi^2,z=\xi^3 \tag{30}$$

So,

$$\frac{\partial t}{\partial \xi^0}=(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2},\ \frac{\partial x}{c\partial \xi^0}=(\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}$$

$$\frac{c\partial t}{\partial \xi^1}=\sinh(\frac{a_0\xi^0}{c}),\ \frac{\partial x}{\partial \xi^1}=\cosh(\frac{a_0\xi^0}{c}) \tag{31}$$

Hence,

$$Det=(\frac{\partial t}{\partial \xi^0}\frac{\partial x}{\partial \xi^1}-\frac{c\partial t}{\partial \xi^1}\frac{\partial x}{c\partial \xi^0})(\frac{d\xi^0}{d\tau})^2=(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})^2 \tag{32}$$

Therefore,

$$\begin{pmatrix} F_\xi^{\ 0} \\ F_\xi^{\ 1} \end{pmatrix}=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})^2}\begin{pmatrix} \frac{\partial x}{\partial \xi^1}\frac{d\xi^0}{d\tau} & -\frac{c\partial t}{\partial \xi^1}\frac{d\xi^0}{d\tau} \\ -\frac{\partial x}{c\partial \xi^0}\frac{d\xi^0}{d\tau} & \frac{\partial t}{\partial \xi^0}\frac{d\xi^0}{d\tau} \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} \tag{33}$$

Hence,

$$F_\xi^{\ 0}=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})^2}[A\frac{\partial x}{\partial \xi^1}\frac{d\xi^0}{d\tau}-B\frac{c\partial t}{\partial \xi^1}\frac{d\xi^0}{d\tau}]$$

$$=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\frac{\partial x}{\partial \xi^1}[f^0-m_0\frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2-m_0\frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\frac{c\partial t}{\partial \xi^1}[f^1-m_0\frac{d}{d\xi^0}(\frac{\partial x}{c\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2-m_0\frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \tag{34}$$

As,

$$F_\xi^{\ 1}=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})^2}[-A\frac{\partial x}{c\partial \xi^0}\frac{d\xi^0}{d\tau}+B\frac{\partial t}{\partial \xi^0}\frac{d\xi^0}{d\tau}]$$

$$=-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\frac{\partial x}{c\partial \xi^0}[f^0-m_0\frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2-m_0\frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\frac{\partial t}{\partial \xi^0}[f^1-m_0\frac{d}{d\xi^0}(\frac{\partial x}{c\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2-m_0\frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \tag{35}$$

In this time,

$$\frac{d}{d\xi^0}\left(\frac{\partial t}{\partial \xi^0}\right)=\frac{d\xi^1}{d\xi^0}\cosh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c^2}+\left(\frac{c^2}{a_0}+\xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0{}^2}{c^3},$$

$$\frac{d}{d\xi^0}\left(\frac{\partial x}{c\partial \xi^0}\right)=\frac{d\xi^1}{d\xi^0}\sinh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c^2}+\left(\frac{c^2}{a_0}+\xi^1\right)\cosh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0{}^2}{c^3}$$

$$\frac{d}{d\xi^0}\left(\frac{c\partial t}{\partial \xi^1}\right)=\cosh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c},\quad \frac{d}{d\xi^0}\left(\frac{\partial x}{\partial \xi^1}\right)=\sinh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c} \tag{36}$$

Therefore, Eq (34) is by Eq (31), Eq (36). Lorentz force $F_\xi{}^0$ is in Rindler space-time.

$$F_\xi{}^0$$

$$=\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\cosh\left(\frac{a_0\xi^0}{c}\right)$$

$$\times[f^0-m_0\{\frac{d\xi^1}{d\xi^0}\cosh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c^2}+\left(\frac{c^2}{a_0}+\xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0{}^2}{c^3}\}c\left(\frac{d\xi^0}{d\tau}\right)^2$$

$$-m_0\cosh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\sinh\left(\frac{a_0\xi^0}{c}\right)$$

$$\times[f^1-m_0\{\frac{d\xi^1}{d\xi^0}\sinh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c^2}+\left(\frac{c^2}{a_0}+\xi^1\right)\cosh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0{}^2}{c^3}\}c\left(\frac{d\xi^0}{d\tau}\right)^2$$

$$-m_0\sinh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \tag{37}$$

Therefore, Eq (35) is by Eq (31), Eq (36).Lorentz force $F_\xi{}^1$ is in Rindler space-time.

$$F_\xi{}^1$$

$$=-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\left(\frac{c^2}{a_0}+\xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c^2}$$

$$\times[f^0-m_0\{\frac{d\xi^1}{d\xi^0}\cosh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c^2}+\left(\frac{c^2}{a_0}+\xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0{}^2}{c^3}\}c\left(\frac{d\xi^0}{d\tau}\right)^2$$

$$-m_0\cosh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}$$

$$\times[f^1-m_0\{\frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}+(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0^{\ 2}}{c^3}\}c(\frac{d\xi^0}{d\tau})^2$$

$$-m_0\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \tag{38}$$

In this time, the transformation of electromagnetic field is [1]

$$E_x=E_{\xi^1}\ ,$$

$$E_y=E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z=E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$B_x=B_{\xi^1},$$

$$B_y=B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z=B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}) \tag{39}$$

Hence,

$$f^0=F^0\frac{dt}{d\tau}=q\frac{\vec{u}}{c}\cdot\vec{E}\frac{dt}{d\tau}=q\frac{\vec{u}\,'}{c}\cdot\vec{E}\ ,\vec{u}=\frac{d\vec{x}}{dt},\vec{u}\,'=\frac{d\vec{x}}{d\tau}$$

$$=q\frac{1}{c}[\frac{dx}{d\tau}E_x+\frac{dy}{d\tau}E_y+\frac{dz}{d\tau}E_z],\ \frac{dx}{d\tau}=\frac{\partial x}{\partial\xi^0}\frac{d\xi^0}{d\tau}+\frac{\partial x}{\partial\xi^1}\frac{d\xi^1}{d\tau}$$

$$+\frac{d\xi^2}{d\tau}\{E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})\}\ +\frac{d\xi^3}{d\tau}\{E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})\}] \tag{40}$$

$$f^1=F^1\frac{dt}{d\tau},$$

$$F^1=q[E_x+\frac{1}{c}(u_yB_z-u_zB_y)]\qquad\qquad ,\vec{u}=\frac{d\vec{x}}{dt}$$

$$f^1=F^1\frac{dt}{d\tau}=q[E_x\frac{dt}{d\tau}+\frac{1}{c}(\frac{dy}{d\tau}B_z-\frac{dz}{d\tau}B_y)],\ \frac{dt}{d\tau}=\frac{\partial t}{\partial\xi^0}\frac{d\xi^0}{d\tau}+\frac{\partial t}{\partial\xi^1}\frac{d\xi^1}{d\tau}$$

$$=q[\{(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}\frac{d\xi^0}{d\tau}+\sinh(\frac{a_0\xi^0}{c})\frac{1}{c}\frac{d\xi^1}{d\tau}\}E_{\xi^1}$$

$$+\frac{1}{c}\{\frac{d\xi^2}{d\tau}(B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}))$$

$$-\frac{d\xi^3}{d\tau}(B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c}))\}] \quad (41)$$

In Eq(24), f^2 is

$$f^2=m_0\frac{d^2y}{d\tau^2}=F_\xi^{\ 2}\frac{d\xi^0}{d\tau},\ F_\xi^{\ 2}=m_0\frac{d}{dt}(\frac{dy}{d\tau})\frac{dt}{d\xi^0}=F^2\frac{dt}{d\xi^0} \quad (42)$$

Therefore, Lorentz force $F_\xi^{\ 2}$ is in Rindler space-time.

$$F_\xi^{\ 2}=m_0\frac{d}{d\xi^0}(\frac{d\xi^2}{d\tau})=F^2\frac{dt}{d\xi^0}$$

$$=q[E_y+\frac{1}{c}(u_zB_x-u_xB_z)]\frac{dt}{d\xi^0}$$

$$=q[\{E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})\}$$

$$\times\{(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}+\frac{1}{c}\frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\}+\frac{1}{c}\{\frac{d\xi^3}{d\xi^0}B_{\xi^1}$$

$$-\{(\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}+\cosh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\xi^0}\}$$

$$\times\{B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})\}\}] \quad (43)$$

In Eq (24), f^3 is

$$f^3=m_0\frac{d^2z}{d\tau^2}=F_\xi^{\ 3}\frac{d\xi^0}{d\tau},\ F_\xi^{\ 3}=m_0\frac{d}{dt}(\frac{dz}{d\tau})\frac{dt}{d\xi^0}=F^3\frac{dt}{d\xi^0} \quad (44)$$

Therefore, Lorentz force $F_\xi^{\ 3}$ is in Rindler space-time.

$$F_\xi^{\ 3}=m_0\frac{d}{d\xi^0}(\frac{d\xi^3}{d\tau})=F^3\frac{dt}{d\xi^0}$$

$$=q[E_z+\frac{1}{c}(u_xB_y-u_yB_x)]\frac{dt}{d\xi^0}$$

$$=q[\{E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})\}$$

$$\times\{(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}+\frac{1}{c}\frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\}$$

$$+\frac{1}{c}\{\{(\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}+\cosh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\xi^0}\}$$

$$\times\{B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})\}-\frac{d\xi^2}{d\xi^0}B_{\xi^1}\}] \quad (45)$$

4. ENERGY-MOMENTUM IN RINDLER SPACETIME

In initial frame, energy-momentum is

$$\vec{p}=m_0\frac{d\vec{x}}{d\tau}\ ,\ E=m_0c^2\frac{dt}{d\tau} \quad (46$$

$$p_x=m_0\frac{dx}{d\tau}=m_0\frac{\partial x}{\partial\xi^\alpha}\frac{d\xi^\alpha}{d\tau}$$

$$=m_0\frac{\partial x}{c\partial\xi^0}\frac{cd\xi^0}{d\tau}+m_0\frac{\partial x}{\partial\xi^1}\frac{d\xi^1}{d\tau}$$

$$=m_0(1+\frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{cd\xi^0}{d\tau}+m_0\cosh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\tau} \quad (47)$$

$$E=m_0c^2\frac{dt}{d\tau}=m_0c\frac{cdt}{\partial\xi^\alpha}\frac{d\xi^\alpha}{d\tau}$$

$$=m_0c\frac{\partial t}{\partial\xi^0}\frac{cd\xi^0}{d\tau}+m_0c\frac{c\partial t}{\partial\xi^1}\frac{d\xi^1}{d\tau}$$

$$=m_0c(1+\frac{a_0}{c^2}\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{cd\xi^0}{d\tau}+m_0c\sinh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\tau} \quad (48)$$

Hence, we can define energy-momentum in Rindler spacetimee.

$$\vec{p}_\xi=m_0\frac{d\vec{\xi}}{d\tau}\ ,E_\xi=m_0c^2(1+\frac{a_0}{c^2}\xi^1)\frac{d\xi^0}{d\tau} \quad (49)$$

$$p_x=m_0\frac{dx}{d\tau}=\sinh(\frac{a_0\xi^0}{c})\frac{E_\xi}{c}+\cosh(\frac{a_0\xi^0}{c})p_{\xi^1}$$

$$E=m_0c^2\frac{dt}{d\tau}=\cosh(\frac{a_0\xi^0}{c})E_\xi+\sinh(\frac{a_0\xi^0}{c})p_{\xi^1}c$$

$$p_y=m_0\frac{dy}{d\tau}=m_0\frac{d\xi^2}{d\tau}=p_{\xi^2}\ ,p_z=m_0\frac{dz}{d\tau}=m_0\frac{d\xi^3}{d\tau}=p_{\xi^3} \quad (50)$$

Therefore, general case is

$$E_\xi{}^2-p_\xi{}^2c^2=m_0{}^2c^4[(1+\frac{a_0\xi^1}{c^2})^2\frac{(d\xi^0)^2}{d\tau^2}-\frac{1}{c^2}\frac{d\vec{\xi}\cdot d\vec{\xi}}{d\tau^2}]=m_0{}^2c^4 \quad (51)$$

In special case, light is

$$E_\xi=p_\xi c \quad (52)$$

5. CONCLUSION

We find the electro-magnetic wave equation and function and the electro-magnetic force in uniformly accelerated frame. We define energy-momentum in Rindler space-time.

References

[1] S.Yi, “Electromagnetic Field Equation and Lorentz Gauge in Rindler space-time”, The African review of physics,**11,**33(2016)-INSPIRE-HEP

[2] S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[3] W.Rindler, Am.J.Phys.**34**.1174(1966)

[4] P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[5] C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[6] S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[7] R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)

[8] A.Miller, Albert Einstein’s Special Theory of Relativity(Addison-Wesley Publishing Co., Inc., 1981)

[9] W.Rindler, Special Relativity(2nd ed., Oliver and Boyd, Edinburg,1966)

[10] Massimo Pauri, Michele Vallisner, "Marzke-Wheeler coordinates for accelerated observers in special relativity":Arxiv:gr-qc/0006095(2000)

[11] A. Einstein, “ Zur Elektrodynamik bewegter K¨orper”, Annalen der Physik. 17:891(1905) [12]J.W.Maluf and F.F.Faria,”The electromagnetic field in accelerated frames”:Arxiv:gr-qc/1110.5367v1(2011)

[12] D.J. Griffith,” Introduction To Electrodynamics”,(2nd ed.,Prentice Hall,Inc.1981)

Citation: *Sangwha-Yi, " Electromagnetic Wave Function and Equation, Lorentz Force in Rindler Spacetime", International Journal of Advanced Research in Physical Science (IJARPS), vol. 5, no. 9, pp. 46-56, 2018.*

Copyright:© *2018 Authors, This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.*