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Approximation of Scattering Phases for Reid93 Potential

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Abstract: *For a single-channel nucleon-nucleon scattering, a well-known and convenient variable phase approach is considered, which is widely used for practical problems of atomic and nuclear physics. Approximation of the pp- and np- scattering phases obtained for the modern realistic phenomenological nucleon-nucleon potential Reid93 was carried out. The approximation function is used as a well-known formula for a parabolic-type quadratic function.*

Keywords: *variable phase approach, nucleon-nucleon scattering, nucleon-nucleon state, phase shifts, potential Reid93.*

1. Introduction

From the experimentally observed values, first of all, information is obtained about the phase and amplitude of scattering, rather than the wave functions. The latter are the main object of research in the standard approach. That is, in the experiment there are not the wave functions themselves, but their changes caused by the interaction [1]. Therefore, it is interesting to obtain and use such an equation that directly bundles the phase and scattering amplitudes with potential and does not find the wave functions at the same time.

The exact solution of the scattering problem for the purpose of calculating the scattering phases is possible only for individual phenomenological potentials. For realistic potentials, the scattering phase is approximated. This is due to the use of physical approximations or numerical calculations. The influence of the choice of a numerical algorithm on the solution of the scattering problem is given in [2].

The main methods for solving the Schrödinger equation for the purpose of obtaining scattering phases are: the method of successive approximations, the Born approximation, the variable phase approach (or phase-function method) and others.

In previous papers [3-6] we obtain scattering phases for a set of nucleon-nucleon potentials. The variable phase approach (VPA) was used to find phase shifts. On the received scattering phases were calculated scalar scattering amplitude, the full cross-section and the partial scattering amplitude

This paper deals with the analysis of calculated phase shifts of nucleon-nucleon scattering in various spin states for the modern realistic phenomenological nucleon-nucleon potential Reid93 by using the variable phase approach (results according to paper [7]) and phase shifts from the original paper [8] for the same potential.

2. The Variable Phase Approach

In the scattering of a spin-free particle with energy E and an orbital moment l on the spherical-symmetric potential $V(r)$, the Schrödinger equation for the radial wave function $u_l(r)$ has the form [1]:

$$u''_l(r)+\left(k^2-\frac{l(l+1)}{r^2}-U(r)\right)u_l(r)=0, \tag{1}$$

where $U(r)=2mV(r)/\hbar^2$ - the renormalized interaction potential, m - the reduced mass, $k^2=2mE/\hbar^2$ - the wave number.

VPA is a extra, specific and special method for solving the Schrödinger radial equation (1). This method is convenient for obtaining scattering phases. This is due to the fact that this method does not need to first calculate radial wave functions in a wide area and then, by their asymptotic behavior, find these phases.

Two linearly independent solutions of the free Schrödinger equation (1) are the known Riccati-Bessel functions $j_l(kr)$ and $n_l(kr)$. The free motion is only responsible for the solution $j_l(kr)$ at the point r=0. In this case, the solution becomes asymptotically large for r values

$$u_l(r) \approx const \cdot \sin(kr - l\pi/2).$$

The presence of the potential leads to the fact that now, in the field of potential disappearance $U(r)$, the wave function includes an additive $n_l(kr)$ for an irregular solution of the free equation. And in particular, the measure of this additive is the scattering phase δ_l:

$$u_l(r) \approx const \cdot \left[j_l(kr) - tg\delta_l \cdot n_l(kr) \right],$$

$$u_l(r) \to const \cdot \sin(kr - l\pi/2 + \delta_l),\ r\to\infty.$$

A standard and generally accepted method for calculating scattering phases is the direct solution of the Schrödinger equation (1) with an asymptotic boundary condition. VPA is the transition from the Schrödinger equation (1) to the equation for the phase function. To do this, make the following simple replacement [1, 9]:

$$u_l(r) = A_l(r)\left[\cos\delta_l(r) \cdot j_l(kr) - \sin\delta_l(r) \cdot n_l(kr)\right]. \quad (2)$$

The two new functions $\delta_l(r)$ and $A_l(r)$ introduced have the physical content of the corresponding scattering phases and the rationing constants (or amplitudes) of wave functions for scattering on a definite sequence of truncated potentials. They are called according to their physical content by phase and amplitude function. The term "phase function" was first used in the paper of Morse and Allis [10]. The equation for phase and amplitude functions with initial conditions is written in this form [1, 9]:

$$\delta'_l = -\frac{1}{k} U \left[\cos\delta_l \cdot j_l - \sin\delta_l \cdot n_l\right]^2,\ \delta_l(0) = 0; \quad (3)$$

$$A'_l = -\frac{1}{k} A_l U \left[\cos\delta_l \cdot j_l - \sin\delta_l \cdot n_l\right]\left[\sin\delta_l \cdot j_l + \cos\delta_l \cdot n_l\right],\ A_l(0) = 1. \quad (4)$$

The phase equation (3) was first obtained by Drukarev [11], and then independently in the papers of Bergmann, Kynch [12], Olson, Calogero [13] and Dashen [14]. VPA proved to be convenient in solving many practical problems of atomic and nuclear physics.

3. APPROXIMATION OF SCATTERING PHASES

In the original paper [8], scattering phases were obtained for the potentials of Nijmegen group (Nijm I, Nijm II and Reid93) and for Nijmegen multienergy partial-wave analysis. In [7], phase shifts *nn*-, *pp*-, *np*- scattering for the nucleon-nucleon potential Reid93 were obtained using the variable phase approach.

According to a detailed analysis in [7], we can draw the following conclusions. Comparison of phase shifts for *pp*- and *np*- scattering calculated for the same Reid93 potential by different methods indicates that the difference between the results is not more than two percent. Comparison of the results of phase shift calculations for the Reid93 potential obtained with VPA and phase shifts for other potential models (NijmI, NijmII [8], Argonne v18 [15] and CD-Bonn [16]) and for partial wave analysis [8] indicates that the deviation between these data is up to five percent. The results of calculations of single-channel scattering phases for the Reid93 potential are in good agreement with the data obtained in the framepaper of the chiral perturbation theory [17] and for the partial wave analysis below the pion formation threshold [18].

The structure of the Paris potential in paper [19] is analyzed from the point of view of the independence of the coefficients of various components. In addition, in [19] the approximation of the scattering phases obtained for the Paris potential is carried out.

In this paper, the approximation of the phase's of *pp*- and *np*- scattering from papers [7] and [8] for the Reid93 potential is carried out. A parabolic-type quadratic function was used, which Dolgopolov, Minin and Rabotkin used for scattering phases for the Paris potential [19]

$$y_i(x) = a + bx + cx^2. \tag{5}$$

The obtained coefficients *a*, *b*, *c* for the approximation of phases *pp*- and *np*- scattering are given in Tables 1 and 2, respectively.

The following values are calculated for the estimation of the quality of approximation of the phase scattering δ_i:

1) standard deviation of the fit:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(\delta_i - y_i(x))^2}{N-P}};$$

2) χ^2 per degree of freedom of function;

3) correlation coefficient *R*.

According to the data of Tables 1 and 2 within a single spin state, it is difficult to find the difference between the coefficients for the approximation form (5) and the approximation parameters. Only one can estimate for which of the states the approximation with the quadratic function (5) will be "better" or "worse". In the end, the minimum value of the correlation coefficient will be for the $^3P_0$- state (for both *pp*- scattering and *np*- scattering), and the maximum value - for the $^1D_2$- state.

Table1. *Parameters of approximation for pp- scattering phases for the Reid93 potential of papers [7, 8]*

State	σ	χ^2	*R*	*a*	*b*	*c*
$^1S_0$ [8]	7.23422	52.33388	0.96423	50.21339	-26.42685	2.51043
$^1S_0$ [7]	7.10054	50.41768	0.96600	50.54306	-27.22381	2.71849
$^3P_0$ [8]	3.49740	12.23184	0.92675	4.04607	6.15318	-3.52370
$^3P_0$ [7]	3.51237	12.33673	0.92790	4.10873	6.08925	-3.53163
$^3P_1$ [8]	0.75864	0.57554	0.99819	-0.87440	-13.22648	1.38537
$^3P_1$ [7]	0.76060	0.57851	0.99822	-0.91270	-13.43629	1.42428
$^1D_2$ [8]	0.12204	0.01489	0.99963	-0.24939	4.41924	-0.39851
$^1D_2$ [7]	0.12482	0.01558	0.99962	-0.25045	4.49823	-0.41495

Table2. *Parameters of approximation for np- scattering phases for the Reid93 potential of papers [7, 8]*

State	σ	χ^2	*R*	*a*	*b*	*c*
$^1S_0$ [8]	2.53466	6.42449	0.99674	61.73167	-38.2194	5.19865
$^1S_0$ [7]	2.52679	6.38469	0.99677	61.74440	-38.22495	5.19380
$^3P_0$ [8]	3.34895	11.21545	0.93174	3.96439	5.64195	-3.37986
$^3P_0$ [7]	3.35480	11.25466	0.93191	3.97839	5.63373	-3.38344
$^1P_1$ [8]	1.01931	1.03899	0.99625	-1.53013	-14.91513	2.14592
$^1P_1$ [7]	1.02139	1.04324	0.99624	-1.53073	-14.95112	2.15576
$^3P_1$ [8]	0.71160	0.50637	0.99846	-0.85135	-13.34335	1.37341
$^3P_1$ [7]	0.71104	0.50557	0.99847	-0.85390	-13.36223	1.37541
$^1D_2$ [3]	0.13709	0.01879	0.99956	-0.26844	4.52958	-0.40765
$^1D_2$ [7]	0.13693	0.01875	0.99956	-0.27077	4.55897	-0.41614
$^3D_2$ [8]	0.91072	0.82941	0.99723	-0.48671	20.00056	-3.76889
$^3D_2$ [7]	0.91915	0.84484	0.99718	-0.47793	20.02265	-3.77559

On Fig. 1-3 shows the phases of *np*- scattering from paper [7] (points) and the results of approximation (curve). The illustration only shows data for three states ($^1S_0$, $^3P_1$, $^3D_2$).

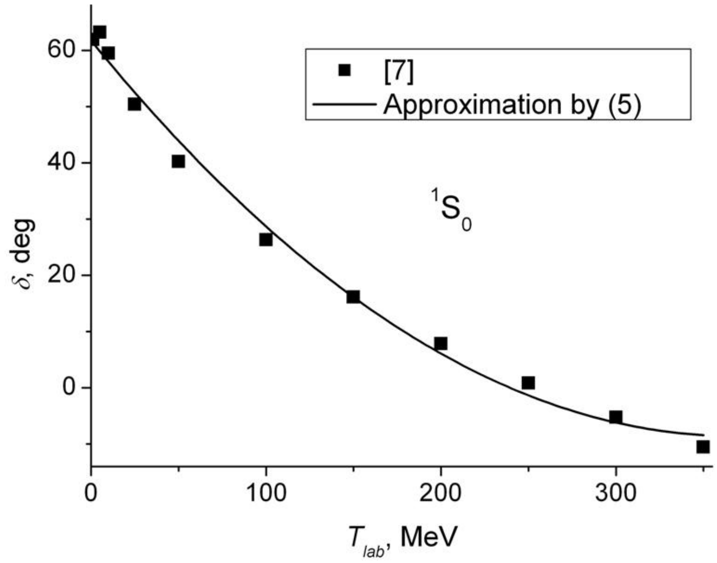

Fig1. *Phases of np- scattering for $^1S_0$- state*

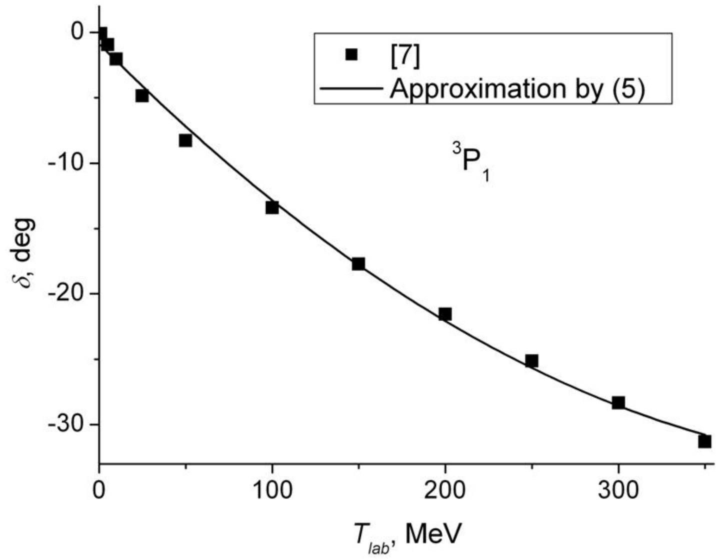

Fig2. Phases of *np*- scattering for $^3P_1$- state

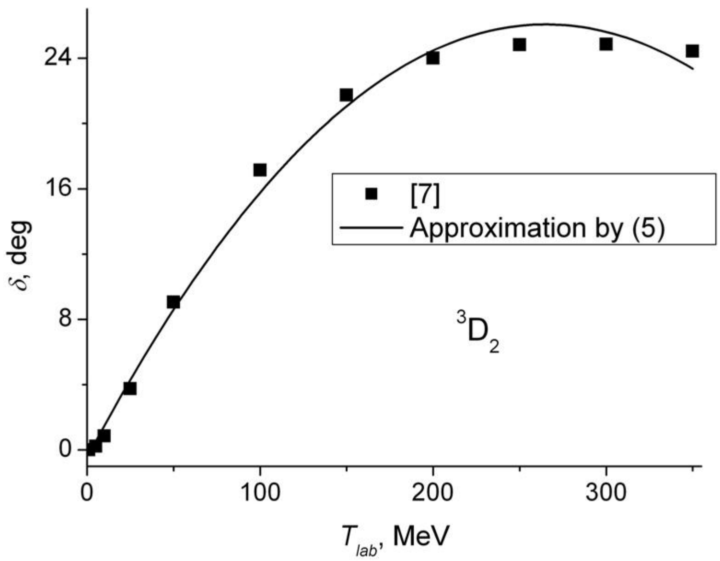

Fig3. *Phases of np- scattering for $^3D_2$- state*

4. CONCLUSIONS

The paper considers the variable phase approach for the problem of single-channel nucleon-nucleon scattering and the final results of the application of this method for the search of scattering phases for a specific Reid93 interaction potential.

For the approximation of the scattering phases, we use a parabolic-type quadratic function in the form (5), which was proposed in paper [19]. The results of the approximation of the pp- and np- scattering phases obtained by different methods are compared.
The form of the recording of the phase function in a convenient form (5) allows it to be used for further calculation or recalculation of the scalar scattering amplitude, the full cross-section and the partial scattering amplitude accordingly [1]

$$F(\theta)=\frac{1}{k}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l P_l(\cos\theta), \qquad (6)$$

$$\sigma=\frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l\ , \qquad (7)$$

$$f_l=\frac{1}{k}e^{i\delta_l}\sin\delta_l\ , \qquad (8)$$

For calculations, it will not be necessary to calculate the scattering phases separately for each value. And, having a specific approximate function for the phase function, it will be possible to calculate the values of (6)-(8) for any phase value within the limits of the approximation carried out (in the interval T_{lab}=0-350 MeV).

In further researches one can obtain and compare coefficients and parameters for approximation for other modern phenomenological potentials.

REFERENCES

[1] V.V. Babikov, *The phase-function method in quantum mechanics* (Moscow, Science, 1988).

[2] I. Haysak and V. Zhaba, Visnyk Lviv Univ. Ser. Phys. 44, 8 (2009).

[3] V.I. Zhaba, Calculation of phases of nucleon-nucleon scattering for potentials NijmI, NijmII and Nijm93 on the phase-function method, J. Phys. Stud. 19, 4201 (2015).

[4] V.I. Zhaba, The phase-functions method and full cross-section of nucleon-nucleon scattering, Mod. Phys. Lett. A 31, 1650049 (2016).

[5] V.I. Zhaba, The phase-functions method and scalar amplitude of nucleon-nucleon scattering, International Journal of Modern Physics E 25, 1650088 (2016).

[6] V.I. Zhaba, Calculation of phases of np-scattering up to Tlab=3 GeV for Reid68 and Reid93 potentials on the phase-function method, Probl. Atom. Sci Tech. 5(105), 29 (2016).

[7] V. Zhaba, Calculation of phases of nucleon-nucleon scattering for potential Reid93 on the phase-functions method, Scientific notes NaUKMA, Physics and mathematics 178, 44 (2016).

[8] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen, J.J. de Swart, Construction of high quality NN potential models, Phys. Rev. C 49, 2950 (1994).

[9] V.V. Babikov, The phase-function method in quantum mechanics, Sov. Phys. Usp. 10, 271 (1967).

[10] P.M. Morse and W. P. Allis, The Effect of Exchange on the Scattering of Slow Electrons from Atoms, Phys. Rev. 44, 269 (1933).

[11] G.F. Drukarev, about determination of phase of wave function at dispersion of particles, ZhETF 19, 247 (1949).

[12] G.I. Kynch, The Two-Body Scattering Problem with Non-Central Forces I - Non-Relativistic, Proc. Phys. Soc. A 65, 83 (1952).

[13] F. Calogero, A novel approach to elementary scattering theory, Nuovo Cimento 27, 261 (1963).

[14] R.F. Dashen, Some extensions of the born approximation for phase shifts, Nuovo Cimento 28, 229 (1963).

[15] R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Accurate nucleon-nucleon potential with charge-independence breaking, Phys. Rev. C 51, 38 (1995).

[16] R. Machleidt, High-precision, charge-dependent Bonn nucleon-nucleon potential, Phys. Rev. C 63, 024001 (2001).

[17] E. Epelbaum, H. Krebs, U.-G. Meissner, Improved chiral nucleon-nucleon potential up to next-to-next-to-next-to-leading order, Eur. Phys. J. A 51, 53 (2015).

[18] R.N. Perez, J.E. Amaro, E.R. Arriola, Partial-wave analysis of nucleon-nucleon scattering below the pion-production threshold, Phys. Rev. C 88, 024002 (2013).

[19] M.A. Dolgopolov, L.A. Minin, V.A. Rabotkin, Approximation Properties of the Paris Potential of Nucleon-Nucleon Interaction, Bull. Russ. Acad. Sci.: Phys. 81, 1225 (2017).

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