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**Abstract:** In this case study, a straight rectangular fin with insulated tip in a situation that its thermal conductivity coefficient is dependent upon the temperature has been analyzed in a simple and innovativemethod entitled 'Akbari-Ganji's Method'' or 'AGM''. Comparisons have been made between AGM and Numerical Solution and these results have been indicated that this approach is very efficient and easy, so it can be applied for other nonlinear equations. It is necessary to mention that there are some valuable advantages in this way of solving differential equations and also the answer of various sets of complicated differential equations can be achieved by this method which in the other methods so far they have not had acceptable solutions. The reasons of selecting AGM for solving differential equations in a wide variety of fields is: not only in fluid mechanics but also in different fields of sciences such as solid mechanics, Vibrations, Strength of materials, Chemical engineering, etc. in comparison with the other methods are as follows: Without any dimensionless procedure, we can solve many differential equation(s), that is, differential equations are directly solvable by this method which will be proved in this case study, the process of solving nonlinear equation(s) will be very easy and convenient in comparison with the other methods.

Keywords: AGM Method; Straight Fin; Temperature - Dependent; Thermal Conductivity.

# **1. INTRODUCTION**

Extended surfaces or fins are frequently utilized in many engineering applications such as air conditioning, refrigeration, pipe lines for carrying oil, automobile and chemical processing equipment to increase the surface area and consequently to improve the rate of heat transfer between the primary surface and surrounding fluid. The selection of any particular type of fin depends mainly on the geometry of the primary surface which was discussed completely by Kundu and Das [1]. In the past decades, different fin shapes have been introduced depending upon the application and the geometry of the primary surface. Kern and Kraus in 1972 have identified three main fin geometries which are longitudinal fins, radial or circumferential fins and pin fins or spines. For any of the above geometry, fins with straight profile or constant thickness are a common choice as they can be manufactured easily. It is notable that fin manufacturers always try to observe economic aspects of their production. Therefore, attempts have been made to produce fins with lower materials by narrowing down the section of the fins. As a result, a lot of researches have been done for the determination of optimum fin shapes in order to obtain minimum fin volume for a given rate of heat dissipation [2].

Recently, a design of absorber plate fin for saving in material has been proposed [3]. The results from this study reveal that approximately 20 % reduction in fin material is possible. Afterwards, the optimum dimensions of convective rectangular fins with a step change in cross-sectional area were presented by Aziz in 1994 [4]. For more information, it is better for the other geometries such as annular fins to say that the study of the optimization demonstrated that an annular fin with a step change in thickness is a better choice for transferring rate of heat in comparison with the concentric-annular disc fin for the same fin volume and identical surface conditions [5].

In addition, experimental work has also been done by some authors, for instance Dogan and Sivrioglu [6], who investigated the mixed convection heat transfer from longitudinal fins in a horizontal rectangular channel. Through an experimental parametric study the effects of fin spacing, fin height and magnitude of heat flux on mixed convection heat transfer from rectangular fin arrays heated from below in a horizontal channel were investigated. They were able to determine that the mixed convection heat transfer depends on the fin height and spacing. Experimental results indicated that to obtain the maximum amount of heat transfer from fin arrays, the fin spacing should be at an optimum value which is dependent mainly on modified Rayleigh number.

Obviously, there are many well documented mathematical models which describe the heat transfer in fins of different geometries and profiles with a variety of boundary conditions [7]. Khani and Aziz [8] applied the Homotopy Analysis Method to develop an analytical solution for the thermal performance of a straight fin of trapezoidal profile when both the thermal conductivity and the heat transfer coefficient are dependent on temperature. Moreover, they implemented the same method to obtain an analytical solution for the thermal performance of a radial fin of rectangular and various convex parabolic profiles mounted on a rotating shaft and losing heat by convection to its surroundings. Furthermore, exact solutions were constructed with both the thermal conductivity and the heat transfer coefficient assumed to be dependent on temperature. This assumption was also made in the work conducted by Moitsheki and Rowjee [9] in order to construct exact solutions for models describing heat transfer in a two-dimensional rectangular fin. Longitudinal fins with rectangular profile have been investigated by Khani et al. [10] where analytical approximate solutions were obtained and their efficiency with temperature-dependent thermal conductivity and heat transfer coefficient was also considered.

The research undertaken in this paper is to solve the governing equation on a straight rectangular fin with insulated tip while its thermal conductivity is assumed to be a linear function of temperature by utilizing a new method for the following reasons:

As everyone knows, classic analytical methods are not able to solve a wide variety of nonlinear differential equations such as the presented problem here. Therefore, the equations should be solved by utilizing particular methods such as Adomian Decomposition Method[11], Variational Iteration Method [12], Differential Transformation Method (DTM)[13], EXP-function Method [14-17], Homotopy Perturbation Method[18] and so on. It is citable that the afore-mentioned methods do not have the ability to gain the solution of the presented problem in high precision. That means the obtained charts by the above methods are not completely overlapped the Numerical chart. Consequently, it is required to discover new approaches in order to accurately solve complicated nonlinear differential equations such as AGM [19-23].

# Nomenclature

$A_{c}$	cross-sectional area of the fin $(m^2)$
K	thermal conductivity of the fin $(W / m.k)$
Р	fin perimeter $(m)$
h	heat transfer coefficient ( $W / m^2 . k$ )
$T_{b}$	temperature of the fin base $(k)$
$T_a$	fluid temperature ( $k$ )
λ	the slope of the thermal conductivity temperature curve ( $k^{-1}$ )
k <sub>a</sub>	thermal conductivity at the ambient fluid temperature ( $W / m.k$ )
	dimensionless temperature
$ heta _{\xi }$	dimensionless coordinate
β	dimensionless parameter describing variation of the thermal conductivity
Ψ	thermo geometric fin parameter

### 2. THE ANALYTICAL METHOD

Boundary conditions and initial conditions are required for analytical methods of each linear and nonlinear differential equation according to the physic of the problem. Therefore, we can solve every

differential equation with any degrees. In order to comprehend the given method in this paper, two differential equations governing on engineering processes will be solved in this new manner.

In accordance with the boundary conditions, the general manner of a differential equation is as follows:

The nonlinear differential equation of p which is a function of u, the parameter u which is a function of x and their derivatives are considered as follows:

$$p_{k}:f\left(u,u',u'',\ldots,u^{(m)}\right)=0 \quad ; \ u=u\left(x\right)$$
<sup>(1)</sup>

Boundary conditions:

$$\begin{cases} u(0) = u_0, u'(0) = u_1, \dots, u^{(m-1)}(0) = u_{m-1} \\ u(L) = u_{L_0}, u'(L) = u_{L_1}, \dots, u^{(m-1)}(L) = u_{L_{m-1}} \end{cases}$$
(2)

To solve the first differential equation with respect to the boundary conditions in x=L in Eq.(2), the series of letters in the *n*th order with constant coefficients which is the answer of the first differential equation is considered as follows:

$$u(x) = \sum_{i=0}^{n} a_i x^i = a_0 + a_1 x^1 + a_2 x^2 + \ldots + a_n x^n$$
(3)

The more precise answer of Eq.(1), the more choice of series sentences from Eq.(3). In applied problems, approximately five or six sentences from the series are enough to solve nonlinear differential equations. In the answer of differential Eq.(3) regarding the series from degree (n), there are (n+1) unknown coefficients that need (n+1) equations to be specified. The boundary conditions of Eq.(2) are used to solve a set of equations which is consisted of (n+1) ones.

The boundary conditions are applied on the functions such as follows:

a) The application of the boundary conditions for the answer of differential Eq.(3) is in the form of:

If 
$$x = 0$$
:

$$\begin{aligned}
 & (u(0) = a_0 = u_0 \\
 & u'(0) = a_1 = u_1 \\
 & u''(0) = a_2 = u_2 \\
 & \vdots & \vdots \\
 & \vdots & \vdots
 \end{aligned}$$
(4)

and when x = L:

$$\begin{cases} u(L) = a_0 + a_1 L + a_2 L^2 + \ldots + a_n L^n = u L_0 \\ u'(L) = a_1 + 2a_2 L + 3a_3 L^2 + \ldots + na_n L^{n-1} = u L_1 \\ u''(L) = 2a_2 + 6a_3 L + 12a_4 L^2 + \ldots + n(n-1)a_n L^{n-2} = u L_{m-1} \\ \vdots \qquad \end{cases}$$
(5)

**b**) After substituting Eq.(5) into Eq.(1), the application of the boundary conditions on differential Eq.(1) is done according to the following procedure:

With regard to the choice of n; (n < m) sentences from Eq.(3) and in order to make a set of equations which is consisted of (n+1) equations and (n+1) unknowns, we confront with a number of additional

unknowns which are indeed the same coefficients of Eq.(3). Therefore, to remove this problem, we should derive m times from Eq.(1) according to the additional unknowns in the afore-mentioned set differential equations and then this is the time to apply the boundary conditions of Eq.(2) on them.

c) Application of the boundary conditions on the derivatives of the differential equation  $P_k$  in Eq.(7) is done in the form of:

$$p'_{k} :\begin{cases} f(u'(0), u''(0), u'''(0), \dots, u^{(m+1)}(0)) \\ f(u'(L), u''(L), u'''(L), \dots, u^{(m+1)}(L)) \end{cases}$$

$$p''_{k} :\begin{cases} f(u''(0), u'''(0), \dots, u^{(m+2)}(0)) \\ f(u''(L), u'''(L), \dots, u^{(m+2)}(L)) \end{cases}$$
(8)
(9)

(n+1) Equations can be made from Eq.(4) to Eq.(9) so that (n+1) unknown coefficients of Eq.(3) for example  $a_0, a_1, a_2, \ldots, a_n$  will be computed. The answer of the nonlinear differential Eq.(1) will be gained by determining coefficients of Eq.(3).

#### **3.** APPLICATION

Consider a straight fin with a temperature-dependent thermal conductivity, arbitrary constant crosssectional area  $A_c$ , perimeter P, and length b. The fin is attached to a base surface of temperature  $T_b$ , it extended into a fluid of temperature  $T_a$ , and we assume that the tip of the fin is insulated.

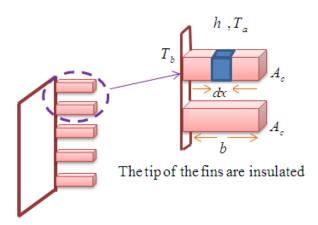


Fig1. The schematic diagram of the physical model.

It is notable that in the present paper, we are going to solve the mentioned problem in two manners:

A) Direct solution in which there is no need to convert any variables into new ones that is one of the benefits of AGM in comparison to other methods. B) Solution by utilizing dimensionless parameters like most of the other semi-analytical methods.

A: The differential equation governing on the system according to the Fig.1 is as follows:

$$f(x): A\frac{d}{dx}(k\frac{dT}{dx}) - ph(T_b - T_a) = 0$$
<sup>(10)</sup>

In Eq. (10), K is defined as a function of temperature as follows:

$$k = k_0 (1 + \lambda (T - T_a)) \tag{11}$$

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The boundary conditions for the mentioned problem are expressed as:

$$\begin{cases} T'(0) = 0 & at \ x = 0 \\ T'(b) = T_b & at \ x = b \end{cases}$$
(12)

It is notable that in Eq. (11),  $\lambda$  is constant and  $k_0$  is conductivity coefficient at the base of the fins.

# 4. SOLVING THE DIFFERENTIAL EQUATION WITH AGM

After substituting Eq.(11) into Eq.(10), the desired differential equation is obtained and in this step, its answer is considered as a finite series of polynomials with constant coefficients as follows:

$$T = \sum_{n=0}^{3} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$
(13)

#### 5. APPLYING BOUNDARY CONDITIONS

The boundary conditions are applied in order to compute constant coefficients of Eq.(13) in two ways in AGM:

a) applying the boundary conditions on Eq.(13) is explained as follows:

$$T = T \left( B \, . C \right) \tag{14}$$

It is notable that BC is the abbreviation of boundary conditions.

According to the above explanations, applying the boundary conditions on Eq. (13) is done in the following form:

$$x = 0 \rightarrow T'(0) = 0$$
 so  $a_1 = 0$  (15)

$$x = b \rightarrow T(b) = T_b$$
 therefore  $a_0 + ba_1 + b^2 a_2 + b^3 a_3 = T_b$  (16)

b) applying the boundary conditions on the main differential equation (Eq.(10)) and its derivatives is done after substituting Eq.(13) into the main differential equation which in this case is Eq.(10) as follows:

$$f(T(x)) = 0 \tag{17}$$

Based on the above formulae, we have:

$$f(T(B.C)) = 0$$
 ,  $f'(T(B.C)) = 0$  (18)

Therefore, after doing the above procedure, applying the boundary conditions on Eq.(10) and its derivative is expressed as:

$$x = b \rightarrow f(T(b)) = 0$$
  
Then  $A\lambda k_0 (a_1 + 2a_2b + 3a_3b_2)^2 + k_0 A\{1 + \lambda (a_0 + a_1b + a_2b^2 + a_3b^2 - T_a\}$   
 $(2a_2 + 6a_3b) - ph(T_b - T_a) = 0$  (19)

And

$$f'(T(0)) = 0$$
Afterwards  $6k_0\lambda a_1a_2 + 6k_0(1 + \lambda(a_0 - T_a)a_3 = 0$ 
(20)

By solving a set of equations which is consisted of four equations with four unknowns from Eqs. (15-16) and Eqs.(19-20), the constant coefficients of Eq.(13) can be gained.

To simplify, the following new variables are considered in the form of:

$$\Omega = A\lambda k_0 (T_a - T_b) + Ak_0 \tag{21}$$

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And the other parameter is defined as:

$$\psi = \{A^2 k_0^2 \lambda^2 (T_a - T_b)^2 - 2A_0^2 k_0^2 \lambda (T_a - T_b) - 4A k_0 \lambda b^2 ph(T_a - T_b) + A^2 k_0^2\}^{0.5}$$
(22)

With regards to the above variables, the constant coefficients are acquired in the forms of:

$$a_{0} = \frac{1}{4Ak_{0}\lambda}(\Omega - \psi) + T_{b} , \quad a_{1} = 0$$

$$a_{2} = -\frac{1}{4Ak_{0}\lambda b^{2}}(\Omega - \psi) , \quad a_{3} = 0$$
(23)

By substituting the constant values of Eq.(23) into Eq.(13), the differential equation is answered in the following:

$$T(x) = T_b + \frac{1}{4Ak_0\lambda}(\Omega - \psi) - \frac{1}{4Ak_0\lambda b^2}(\Omega - \psi)x^2$$
(24)

By choosing physical values bellow:

$$T_{a} = 20 (^{\circ}C) , \quad h = 2 (W/m^{2} \circ C) , \quad p = 0.07 (m^{2})$$

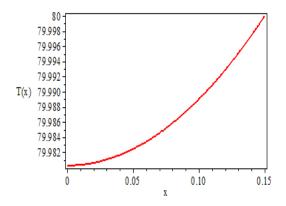
$$A = 0.05 (m^{2}) , \quad \lambda = 0.78 , \quad b = 0.15 (m) , \quad T_{b} = 80 (^{\circ}C)$$
(25)

Eq. (24) is rewritten in the form of:

$$T(x) = 79.9802 + 0.8781x^{2}$$

(26)

Therefore, the solution of Eq. (10) which is defined as temperature profile, Eq. (26), is depicted as follows:



**Fig2.***The result of temperature distribution in the specified domain.* **6. SOLVING THE DIFFERENTIAL EQUATION WITH NUMERICAL METHOD** 

In accordance with the given physical values in Eq. (25) and  $b \in \{0, 15\}$  which b is the length of the fin in terms of centimeter, the numerical solution of the mentioned problem is obtained as follows:

Table1.	Obtained	values o	on the	basis	ofthe	numerical	solution	of Eq.	(26).
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<i>x(cm)</i>	0	2	4	6
Т	79.9801	79.9805	79.9815	79.9833
$\frac{dT}{dx}$	0.0	0.0352	0.0705	0.10578
<i>x(cm)</i>	8	10	12	14
Т	79.9858	79.9889	79.993	79.99
$\frac{dT}{dx}$	0.141	0.1763	0.2115	0.246

#### 7. COMPARING THE CHARTS OF AGM AND NUMERICAL SOLUTION

Due to table 1 and the obtained solution by AGM, a comparison between Numerical Solution and the acquired solution by AGM is illustrated as follows:

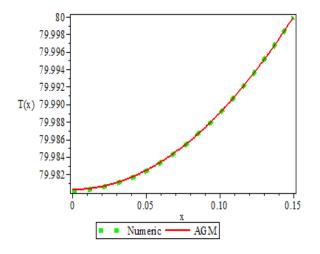


Fig3. Comparison between AGM and numerical solution for the profile of temperature distribution.

#### 8. THE ERROR CHART IN AGM FROM EXACT SOLUTION

In order to understand what percentages of errors exist in the differential equation in each method, the procedure bellow should be followed:

At beginning, we should substitute the solution of the differential equation which is Eq.(26) into the main differential equation which is Eq.(10), then it is necessary to take a plot from the yielded equation in Cartesian coordinates, after that the yielded error of the differential equation can be observed from the obtained chart.

To understand more, reading the following lines is recommended:

If Eq.(10) is considered as follows:

$$f(x) = f(u(x), u'(x), u''(x), ...)$$
(27)

And the answer of the above equation is assumed to be a function of x in the form of:

$$u = g\left(x\right) \tag{28}$$

Therefore, by substituting Eq. (28) into Eq.(27), we will have the following equation which is the desired computational error of the mentioned problem as follows:

$$f(x) = f(u(g(x)), u'(g(x)), ...)$$

Eventually, in regard to the given physical values and by substituting the yielded solution with AGM which is Eq.(26) into Eq.(10), the computational error of AGM is depicted in the form of:

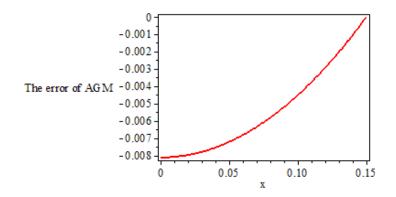


Fig4.The obtained computational error in AGM.

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(29)

Due to chart 4, the maximum error of AGM is approximately 0.8% which is very close to the exact solution of the differential equation.

**B**) In this step, we are going to solve Eq.(10) by introducing some dimensionless parameters as follows:

$$\theta = \frac{T - T_a}{T_b - T_a} \quad , \quad \xi = \frac{x}{b} \quad , \quad \beta = \lambda (T_b - T_a) \quad , \quad \psi = (\frac{hpb^2}{k_a A_c})^{\frac{1}{2}}$$
(30)

In regard to the above parameters, Eq.(10) is rewritten in the following form:

$$f(\theta):\frac{d^2\theta}{d\xi^2} + \beta\theta\frac{d^2\theta}{d\xi^2} + \beta(\frac{d\theta}{d\xi})^2 - \psi^2\theta = 0$$
(31)

Also the appearance of the boundary conditions in Eq.(12) is changed in the forms of:

$$\begin{cases} \frac{d\theta}{d\xi} = 0 & \text{at} \quad \xi = 0\\ \theta = 1 & \text{at} \quad \xi = 1 \end{cases}$$
(32)

#### 9. SOLVING THE DIFFERENTIAL EQUATION WITH AGM

Like the previous section, the answer of Eq.(10) is considered as a finite series of polynomials with constant coefficients in the following:

$$\theta = \sum_{n=0}^{3} a_n \xi^n = a_0 + a_1 \xi + a_2 \xi^2 + a_3 \xi^3$$
(33)

With regard to Eq.(14) and Eq.(17), the boundary conditions are applied on the answer of Eq.(31), which in this case study is considered as Eq.(33), and also on the derivatives of Eq.(31) in the following procedure:

Applying boundary conditions on Eq.(33) is done in the form of:

$$\begin{cases} \theta'(0) = 0 \rightarrow & a_1 = 0\\ \theta(1) = 1 \rightarrow & a_0 + a_1 + a_2 + a_3 = 1 \end{cases}$$
(34)

After substituting Eq.(33) into Eq.(31), the boundary conditions are applied on the achieved equation and its derivatives as follows:

$$\begin{cases} f(1): 2a_{2} + 6a_{3} + \beta(a_{0} + a_{1} + a_{2} + a_{3})(2a_{2} + 6a_{3}) + (a_{1} + 2a_{2} + 3a_{3})^{2} - \\ \psi^{2}(a_{0} + a_{1} + a_{2} + a_{3}) = 0 \\ \text{Then} \\ f'(0): 6a_{3} + 2\beta a_{1}a_{2} + 6\beta a_{0}a_{3} + 4a_{1}a_{2} - \psi^{2}a_{1} = 0 \end{cases}$$
(35)

The constant coefficients  $a_0$  to  $a_3$  are obtained by solving the set of equations which is consisted of four equations with four unknowns from Eq.s (34-35) as:

$$a_{0} = \frac{1}{4} (5 + \beta - \sqrt{(1 + \beta)^{2} + 4\psi^{2}}, a_{1} = 0$$

$$a_{2} = -\frac{1}{4} (1 + \beta - \sqrt{(1 + \beta)^{2} + 4\psi^{2}}, a_{3} = 0$$
(36)

The answer of Eq.(31) is achieved by substituting the constant coefficients from Eq.(36) into Eq.(33) as follows:

$$\theta = \frac{1}{4} (5 + \beta - \sqrt{(1 + \beta)^2 + 4\psi^2} - \frac{1}{4} (1 + \beta - \sqrt{(1 + \beta)^2 + 4\psi^2}) \xi^2$$
(37)

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By choosing the following physical values as:

$$\beta = 1.1$$
 ,  $\psi = 0.5$  (38)

Therefore, Eq.(37) is rewritten in the form of:

$$\theta(\xi) = 0.94351 + 0.05648\xi^2$$

And the chart of Eq.(39) which is the solution of differential Eq.(31) is drawn as follows:

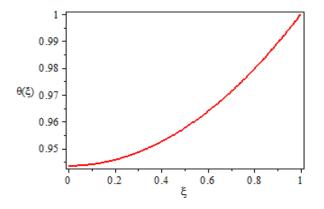


Fig5.The obtained dimensionless temperature profile by AGM.

### **10. NUMERICAL SOLUTION**

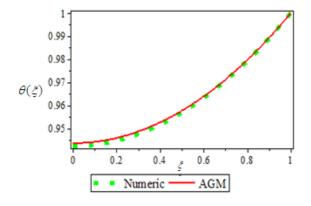
In regard to the domain of boundary conditions which is  $\xi = \{0, 1\}$  and the given physical values in Eq.(38), the numerical solution is acquired as the following table:

Table2.The result of numerical solution of Eq.(39) in the specified domain.

ξ	0	0.2	0.4	0.6
$\theta$	0.9424	0.9447	0.95165	0.9632
$\frac{d\theta}{d\xi}$	0.0	0.02313	0.0462	0.0691
ξ	0.7	0.8	0.9	1.0
$\theta$	0.97069	0.9793	0.989	1
$\frac{d\theta}{d\xi}$	0.0806	0.092	0.1034	0.1147



On the basis of Numerical Solution, which its results have been presented in table 2 and the obtained solution by AGM from Eq. (39), we have the following comparisons:



**Fig6.** Comparison between AGM and Numerical solution for  $\theta(\xi)$ 

(39)

#### 12. THE ERROR CHART WITH AGM

The error chart in AGM according to Eqs.(28-29) in part A is depicted as follows:

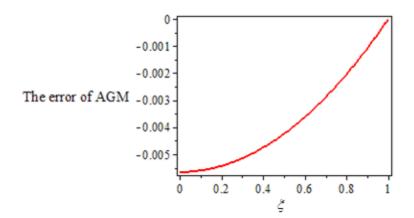


Fig7. The computational error of AGM for the obtained temperature profile.

In regard to the afore-mentioned chart, the maximum error in AGM is approximately 0.55% which is very negligible and really close to the exact solution.

### **13.** CONCLUSION

In this present paper, a new approach (AGM) has been offered in order to solve the yielded equation for straight fins with rectangular profile while the thermal conductivity is dependent upon the temperature. It is necessary to mention that the above proceeding has been done to indicate the ability of AGM for solving various differential equations. Consequently, according to the following reasons the afore-mentioned explanation will be proved: one of the significant benefits of AGM in comparison with other methods is defined that there is no need to convert variables into new ones which means differential equations are directly solvable by this method, that is, we can solve a wide variety of equations without any dimensionless procedure. In order to solve linear and nonlinear differential equations with regard to the order of differential equations, we need boundary conditions. If the numbers of boundary conditions are less than ideal limitation, solving differential equations are impossible. It is noteworthy that in AGM by deriving from the own differential equation(s) n times which n denotes to the number of the absence of boundary condition(s), the afore-mentioned difficulties are eliminated completely which in this paper, the issue has been explained thoroughly. Eventually, on the basis of the mentioned benefits, AGM is a very applicable tool to overcome difficulties which other analytical methods have not been able to conquer them so far.

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