International Journal of Advanced Research in Physical Science (IJARPS)
Volume 5, Issue 10, 2018, PP 29-34
ISSN No. (Online) 2349-7882
www.arcjournals.org

Matlab Modeling of Polarizing Beam Splitter as Hadamard Gate

A. M. Gomaa[1], Kh. M. Haroun[2,5], Yousif H. Alsheikh[3], S. I. Babiker[4]

[1]*Institute of laser- Sudan University of Science & Technology, Sudan*

[2]*Department of Physics, Faculty of Radiobiological Sciences & Medical Imaging, Al-Zaiem Al-Azhari University, Sudan*

[3]*Department of Applied Physics& Mathematics, Omdurman Ahlia University, Sudan*

[4]*Faculty of Science, Al-Baha University, KSA*

[5]*Department of Basic Sciences, Gezira College of Technology, Sudan*

Corresponding Author:** ***A. M. Gomaa, *Institute of laser- Sudan University of Science & Technology, Sudan.*

Abstract: *Polarizing beam splitter is an important device in quantum information and quantum cryptography. Polarizing beam splitter in the case of single photon input works as single bit gate, that takes a single qubit and transform it into another one. The polarizing beam splitter transformation is similar to Hadamard Gate transformation in quantum information, the two tern basis states into superposition states. This work aimed to simulate a polarizing beam splitter using MATLAB code, by treating the polarizing beam splitter as a Hadamard Gate. And then using it for calculation the Bell's inequality. The obtained results show an excellent average value of Bell's inequality (above 2.7), and the suitable number of entangled photon pairs used to calculate the Bell's inequality is above 600.*

Keywords: *CHSH Bell's inequality, Beam splitter, Hadamard gate, entangled*

1. Introduction

Polarizing beam splitters (PBS) are essential components for numerous optical information processing applications such as free-space optical switching networks, read-write magneto-optic data storage systems, and polarization based imaging systems [1,2]. And beam splitter has played important roles in many aspects of optics. For example, in quantum information the beam splitter plays essential roles in teleportation, bell measurements, entanglement and in fundamental studies of the photon [3].

2. Beam Splitter

The optical field consisted of two orthogonally polarized components traveling along the same path [4]. These two components can be separated with a polarizing beam splitter into distinguishable spatial paths. From the fundamental point of view, there is no conceptual difference between the field before and after this transformation. Before we needed two complex numbers to describe the horizontal and the vertical components of the electric field, now we also need two complex numbers to describe the amplitudes of the fields traveling along separate spatial paths. An elementary optical device that combines two spatially separate modes is a beam splitter which partly reflects and partly transmits each of the incident beams, If a beam with an amplitude E_1 enters through the upper port, a fraction R_1 E_1 will get reflected into the upper output port, and a fraction T_1 E_1 will get transmitted [5].

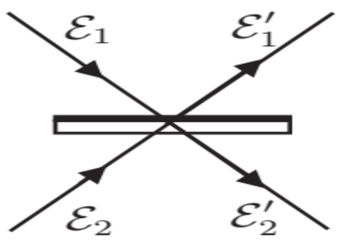

Figure1. *Beam-Splitter*

Similarly, a beam with an amplitude E_2 entering through the lower port will be split respectively into T_2 E_2 into the upper output port and R_2 E_2 into the lower. Where R and T represent reflection and transmission index respectively. We can describe the input modes entering the beam splitter with a two-element complex vector $\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$ which is transformed by the beam splitter into $\begin{pmatrix} \varepsilon_1' \\ \varepsilon_2' \end{pmatrix}$.

The dependence between the amplitudes of the incoming and outgoing modes is linear and can be written in the matrix form [6]:

$$\begin{pmatrix} \varepsilon_1' \\ \varepsilon_2' \end{pmatrix} = B \begin{pmatrix} \varepsilon \\ \varepsilon \end{pmatrix} \qquad B = \begin{pmatrix} R_1 & T_2 \\ T_1 & R_2 \end{pmatrix}$$

Matrix B is not arbitrary due to the energy conservation constraint. Since the intensity of the light beam is proportional to $|\varepsilon|^2$, the energy is conserved if:

$$|\varepsilon_1'|^2 + |\varepsilon_2'|^2 = |\varepsilon_1|^2 + |\varepsilon_2|^2 \tag{1}$$

This equality should be satisfied for arbitrary input fields E1, E2 which leads to the following constraints on the entries of the B matrix:

$$|R_1|^2 + |T_1|^2 = |R_2|^2 + |T_2|^2 \tag{2}$$

Note that these conditions imply $|R_1| = |R_2|$, $|T_1| = |T_2|$ which equivalent to the condition that B is a unitary matrix $B^{\perp}B = 1$.

In what follows we will adopt a notation

$$B(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \tag{3}$$

in which is a standard beam splitter with power transmission

$T = \sin^2(\theta/2)$ and reflection power $R = \cos^2(\theta/2)$. It is easy to convince oneself that the minus sign in the above definition is necessary to ensure unitarity of $B(\theta)$. In particular the balanced beam splitter with

$T = R = 50\%$ corresponds to $B(\pi/2)$ is:

$$B = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \tag{4}$$

2.1. Polarization of Photons

In order to predict the behavior of a quantum system, we need to know precisely the physical properties of all states. One of the most simple physical systems is the polarization of the photon. The dimension of its Hilbert space is just two, yet it is quite sufficient to show how amazing the world of quantum mechanics can be [4]. Suppose we can isolate a single particle of light, photon, from a polarized wave. The photon is a microscopic object and must be treated quantum-mechanically [7]. By notice that the state of the photon obtained from a horizontally polarized wave, whose state we denote as $|H\rangle$, is incompatible with its vertical counterpart, $|V\rangle$: an $|H\rangle$ photon can never be detected in a $|V\rangle$ state. If we prepare a horizontally polarized photon and send it through a polarizing beam splitter, it will always be transmitted and never reflected. This means that states $|H\rangle$ and $|V\rangle$ are orthogonal.

So that states $|H\rangle$ and $|V\rangle$ form an orthonormal basis in the corresponding Hilbert space, and any polarization state of the photon can be written as linear combination of these states. So:

$$|Q\rangle = \alpha|H\rangle + \beta|V\rangle \tag{5}$$

Suppose a single photon in state $|Q\rangle = \alpha|H\rangle + \beta|V\rangle$ hits a polarizing beam splitter (PBS) . If we were dealing with a classical wave, we would expected it to split: a part would be transmitted through the PBS, and the remainder reflected. But the photon is the smallest energy portion of light, and cannot be divided into parts. The experiment shows that the outcome will be random: the photon will go through the PBS with a probability $|\alpha|^2$, and be reflected with a probability $|\beta|^2$. If a large number N of photons are incident on the PBS, on average $|\alpha|^2N$ of them will be transmitted, and $|\beta|^2N$ reflected. This means that the total flux of energy in the transmitted and reflected channels will be proportional to $|\alpha|^2$ and $|\beta|^2$, respectively.

As in classical view, the part of the classical wave that is transmitted through the PBS will become horizontally polarized. The same happens with photons [4]. After the PBS, the photon state in the transmitted channel will become $|H\rangle$ (and in the reflected channel $|V\rangle$). If we place a series of additional PBS's in the transmitted channel of the first one, the photon will be transmitted through all of these PBS's.

The photon propagating through a PBS gives us an example of the photon polarization state measurement [8]. Into both output channels of the PBS, we can place single-photon detectors — devices that generate a macroscopic electric pulse whenever a photon hits their sensitive areas. for the two detectors, only one will click — thus providing us with some information about the photon's initial polarization [4].

2.2. Hadamard Gate

The most Important single-qubit gates in quantum computation are the Pauli operators σ_x, σ_y, and σ_z. Beyond that, there are one of important gate, that is often used in quantum computing:

The Hadamard gate H turns basis states into superposition states:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{6}$$

The Hadamard gate transformations is analogous to polarizing beam splitter transformations, turns basis states into superposition states:

$$\begin{aligned} H|0\rangle &\to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ H|1\rangle &\to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{aligned} \tag{7}$$

And so for general Qubit

$$|Q\rangle = \alpha|0\rangle + \beta|1\rangle \tag{8}$$

$$H|Q\rangle \to \frac{1}{\sqrt{2}}((\alpha+\beta)|0\rangle + (\alpha-\beta)|1\rangle) \tag{9}$$

Where $(\alpha+\beta)$ is now the probability amplitude to find the photon in the outgoing upper beam and $(\alpha-\beta)$ is the probability amplitude for finding it in the lower outgoing beam.

For the specific case of either $\alpha = 0$ or $\beta = 0$, we find that the photon will be found with equal probability in either of the outgoing beams.

For another specific case, $(\alpha = \beta)$, we find that the photon will definitely be found in the upper beam and never in the lower beam [8].

3. Modeling Program

With above arguments we built a model program using MATLAB to simulate the PBS, and we trade its transformation as a Hadamard gate. And we used the model to calculate CHSH Bell's inequality for quantum prediction, since quantum predict that the correlation is about (2.83).

The process of model begin with using random function to generates zeros and ones to simulate the source of entangled photon pairs that propagate toward two observers (Alice and Bob). And we used other rand function to generates random numbers in the range (0,1), for compare and serve the probability purpose.

Then a two observers Alice and Bob chose randomly and independently there measurements angles to determine the photon polarization, and the model calculate the probability amplitudes as in Hadamard transformation and guess the Alice and Bob measurement results (±1) [9].

By using different choice of angles for Alice and Bob, and the statistic correlation relation, the model calculated the CHSH Bell's inequality. And the process is repeated for the different number of photon pairs.

4. Results and Discussion

The results obtained for quantum correlation of entangled photon pairs modeled using matlab is represented in table 1.

Table1. *Quantum correlation of entangled photon pairs (CHSH Bell's inequality)*

n	**1000**	**600**	**300**
S	2.84	2.84	2.9
"	2.76	2.70	2.53
"	2.78	2.76	2.61
"	2.76	2.70	2.59
"	2.70	2.64	2.57
"	2.73	2.64	2.49
"	2.74	2.67	2.55
"	2.74	2.68	2.55
"	2.73	2.67	2.58

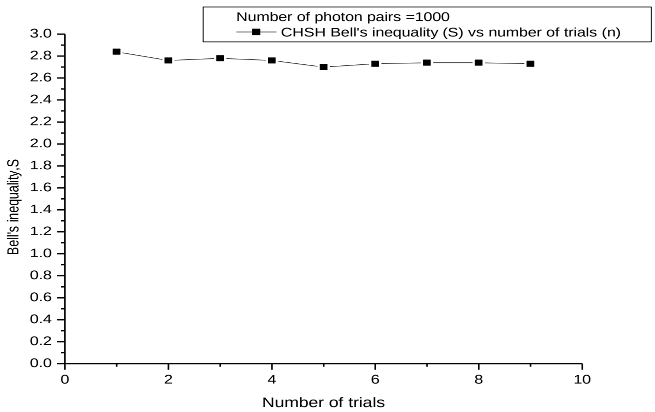

Figure2. *Relation between CHSH bell's value (S) of (1000) photon pairs and number of trial (n)*

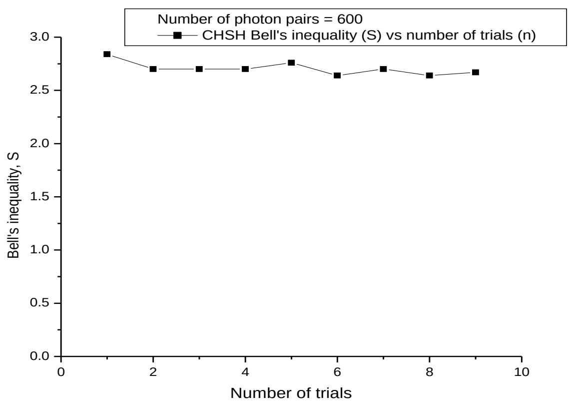

Figure3. *Relation between CHSH bell's value (S) of (600) photon pairs and number of trial (n)*

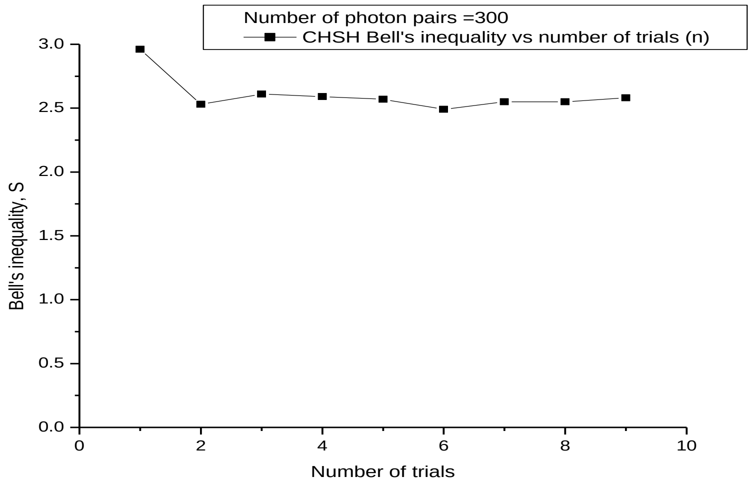

Figure4. *Relation between CHSH bell's value (S) of (300) photon pairs and number of trial (n).*

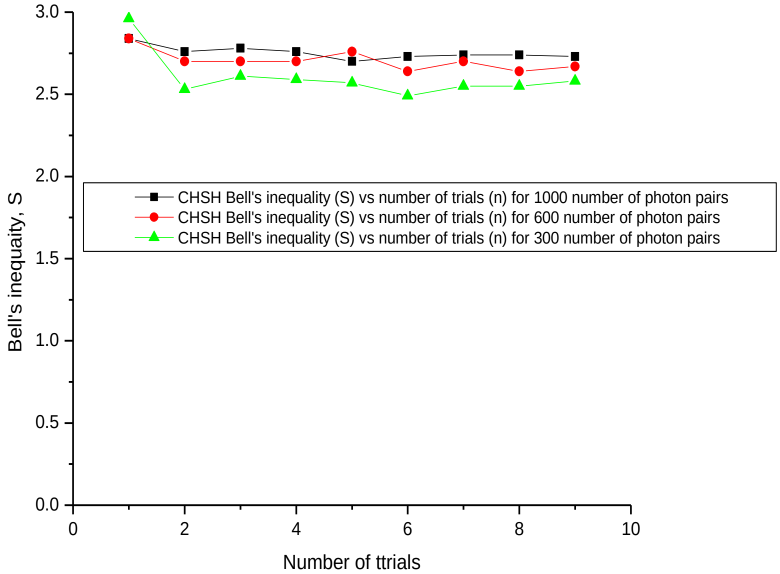

Figure5. *Relation between CHSH bell's value (S) of (1000, 600 and 300) photon pairs and number of trial (n)*

The results showed in figures (2 to 5) above explained the relation between CHSH bell's inequality value (S) and number of trial (n).

From figure 2 it was seen that the value of CHSH bell's inequality (S) it was above 2.7 and that this value was near to the value that predicted by quantum mechanics 2.83. And also it was find that as the results showed there is a stability of the value (S) for all trials. And from figure 3 find the value of CHSH bell's inequality (S) was found to be above 2.6 and this value is slightly different from the value predicted by quantum mechanics. And also it was found that there is some variations of the value (S) for the used trials. Figure 4 depicts the value of CHSH bell's inequality (S) as above 2.5 and the obtained value in this case is different also by small amount from the value that predicted by the quantum mechanics. Figure 5 showed the results of the three number of trails (300, 600, and 1000) and as a comparison of the values obtained for the three number of trials it was shown that the 1000 gave the best results among the other, and it was found instability of the value (S) for the trials. In other words figure 5 showed the value of CHSH bell's inequality (S) that has best and stable value when the number of photon pairs increase, and the best value can be obtained when use (1000) photon pairs, and the results agree with experiment.

5. Conclusion

In conclusion, by using Matlab we built a modeling program for polarizer beam splitter to measure single photon polarization and we obtained a good and stable value of CHSH Bell's inequality comparable to the value predicted by Quantum mechanics. So this model can be used to build an Ekert protocol of quantum cryptography effectively.

References

[1] Tyan Rong-Chung, Atul A. Salvekar, Pang-Chen Sun et al. (1997), Design, fabrication, and characterization of form-birefringent multilayer polarizing beam splitter J. Opt. Soc. Am. A, Vol. 14, No. 7, pp.1627-1636

[2] Lara Pajewski, Riccardo Borghi, Giuseppe Schettini, Fabrizio Frezza, and Massimo Santarsiero, Design of a binary grating with subwavelength features that acts as a polarizing beam splitter, APPLIED OPTICS Vol. 40, No. 32, pp. 5898-5905

[3] Anton Zeilinger (2017), Light for the quantum. Entangled photons and their applications: a very personal perspective, Physica Scripta, Volume 92, Number 7, visited [online] available from: http://iopscience.iop.org/article/10.1088/1402-4896/aa736d

[4] A. I. Lvovsky (2018), Quantum Physics : An Introduction Based on Photons, Springer-Verlag Berlin and Heidelberg GmbH & Co. KG

[5] K Banaszek (2012), Quantum Mechanics 1/2, [online] available from: http://www.fuw.edu.pl/~demko/Teksty/ik05/2012/qi12.pdf, visited 2/9/2018

[6] Stefan Rotter and Sylvain Gigan (2017), Light fields in complex media: mesoscopic scattering meets wave control, PACS numbers: 42.25.Bs, 42.25.Hz, 42.25.Fx from: https://arxiv.org/ pdf/1702.05395.pdf

[7] Peter Pickl (2014), Interpretations of Quantum Mechanics, EPJ Web of Conferences **71**, 00110, pp.1-12

[8] Sara Idris Babiker Mustafa (2007), simulation of quantum cryptography based on Ekert protocol, (PhD thesis: Institute of laser. Sudan University of science and technology), Sudan

[9] Steven M. Girvin (2013), Basic Concepts in Quantum Information, World Scientific Review Volume - 9.75in x 6.5in pp.1-10

Citation: *A. M. Gomaa et al., "Matlab Modeling of Polarizing Beam Splitter as Hadamard Gate", International Journal of Advanced Research in Physical Science (IJARPS), vol. 5, no. 10, pp. 29-34, 2018.*