

N-Order Rational Solutions to the Johnson Equation Depending on $2N - 2$ Parameters

P. Gaillard

Université de Bourgogne, Institut de mathématiques de Bourgogne,
9 avenue Alain Savary BP 47870, Dijon Cedex, France

***Corresponding Author:** P. Gaillard, Université de Bourgogne, Institut de mathématiques de Bourgogne,
9 avenue Alain Savary BP 47870, Dijon Cedex, France

Abstract: We construct rational solutions of order N depending on $2N - 2$ parameters. They can be written as a quotient of 2 polynomials of degree $2N(N + 1)$ in x , t and $4N(N + 1)$ in y depending on $2N - 2$ parameters. We explicitly construct the expressions of the rational solutions of order 4 depending on 6 real parameters and we study the patterns of their modulus in the plane (x, y) and their evolution according to time and parameters $a_1, a_2, a_3, b_1, b_2, b_3$.

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1. INTRODUCTION

Johnson introduced a new equation in a paper written in 1980, [1] to describe waves surfaces in shallow incompressible fluids [2, 3]. This equation was later derived for internal waves in a stratified medium [4]. The Johnson equation is a dissipative equation and there is no soliton-like solution with a linear front localized along straight lines in the (x, y) plane. We consider the Johnson equation (J) in the following normalization

$$(u_t + 6uu_x + u_{xxx} + \frac{u}{2t})_x - 3\frac{u_{yy}}{t^2} = 0, \quad (1)$$

where as usual, subscripts x , y and t denote partial derivatives.

Johnson constructed the first solutions in 1980 [1]. Golinko, Dryuma, and Stepanyantshter found other types of solutions in 1984 [5]. A new approach to solve this equation was given in 1986 [6] by giving a connection between solutions of the Kadomtsev-Petviashvili (KP) [9] and solutions of the Johnson equation. Another types of solutions were obtained by using the Darboux transformation [7]. More recently, in 2013, extension to the elliptic case has been considered [8].

In the following, we recall that the solutions can be expressed in terms of Fredholm determinants of order $2N$ depending on $2N - 1$ parameters. They can also be given in terms of wronskians of order $2N$ with $2N - 1$ parameters. These representations allow to obtain an infinite hierarchy of solutions to the Johnson equation, depending on $2N - 1$ real parameters. We use these results to build rational solutions to the equation, making go a parameter towards 0.

Here we construct rational solutions of order N depending on $2N - 2$ parameters without the presence of a limit.

That provides an effective method to build an infinite hierarchy of rational solutions of order N depending on $2N - 2$ real parameters. We present here only the explicit rational solutions of order 4, depending on 6 real parameters, and the representations of their modulus in the plane of the coordinates (x, y) according to the real parameters $a_1, b_1, a_2, b_2, a_3, b_3$ and time t .

2. RATIONAL SOLUTIONS TO THE JOHNSON EQUATION OF ORDER N DEPENDING ON $2N - 2$ PARAMETERS

2.1. Families of rational solutions of order N depending on $2N - 2$ parameters

We need to define some notations. First of all, we define real numbers λ_j such that $-1 < \lambda_\nu < 1$, $\nu = 1, \dots, 2N$ which depend on a parameter ε which will be intended to tend towards 0; they can be

written as

$$\lambda_j = 1 - 2\varepsilon^2 j^2, \quad \lambda_{N+j} = -\lambda_j, \quad 1 \leq j \leq N, \tag{2}$$

The terms $\kappa_\nu, \delta_\nu, \gamma_\nu$ and $x_{r,\nu}$ are functions of $\lambda_\nu, 1 \leq \nu \leq 2N$; they are defined by the formulas :

$$\begin{aligned} \kappa_j &= 2\sqrt{1 - \lambda_j^2}, \quad \delta_j = \kappa_j \lambda_j, \quad \gamma_j = \sqrt{\frac{1 - \lambda_j}{1 + \lambda_j}}, \\ x_{r,j} &= (r - 1) \ln \frac{\gamma_j^{-i}}{\gamma_j + i}, \quad r = 1, 3, \quad \tau_j = -12i\lambda_j^2 \sqrt{1 - \lambda_j^2} - 4i(1 - \lambda_j^2) \sqrt{1 - \lambda_j^2}, \\ \kappa_{N+j} &= \kappa_j, \quad \delta_{N+j} = -\delta_j, \quad \gamma_{N+j} = \gamma_j^{-1}, \\ x_{r,N+j} &= -x_{r,j}, \quad \tau_{N+j} = \tau_j \quad j = 1, \dots, N. \end{aligned} \tag{3}$$

$e_\nu, 1 \leq \nu \leq 2N$ are defined in the following way :

$$\begin{aligned} e_j &= 2i \left(\sum_{k=1}^{1/2 M-1} a_k (je)^{2k+1} - i \sum_{k=1}^{1/2 M-1} b_k (je)^{2k+1} \right), \\ e_{N+j} &= 2i \left(\sum_{k=1}^{1/2 M-1} a_k (je)^{2k+1} + i \sum_{k=1}^{1/2 M-1} b_k (je)^{2k+1} \right), \quad 1 \leq j \leq N, \\ a_k, b_k &\in R, \quad 1 \leq k \leq N. \end{aligned} \tag{4}$$

$\varepsilon_\nu, 1 \leq \nu \leq 2N$ are real numbers defined by :

$$\varepsilon_j = 1, \quad \varepsilon_{N+j} = 0 \quad 1 \leq j \leq N. \tag{5}$$

Let I be the unit matrix and $D_r = (d_{jk})_{1 \leq j, k \leq 2N}$ the matrix defined by :

$$d_{\nu\mu} = (-1)^{\varepsilon_\nu} \prod_{\eta \neq \mu} \left(\frac{\gamma_\eta + \gamma_\nu}{\gamma_\eta - \gamma_\mu} \right) \exp(\kappa_\nu x + \left(\frac{\kappa_\nu \gamma}{12} - 2\delta_\nu \right) y t + 4i\tau_\nu t + x_{r,\nu} + e_\nu). \tag{6}$$

Then we recall the following result¹ :

Theorem 2.1 *The function v defined by*

$$v(x, y, t) = -2 \frac{|\ln(x, y, t)|^2}{d(x, y, t)^2} \tag{7}$$

where

$$n(x, y, t) = \det(I + D_3(x, y, t)), \tag{8}$$

$$d(x, y, t) = \det(I + D_1(x, y, t)), \tag{9}$$

and $D_r = (d_{jk})_{1 \leq j, k \leq 2N}$ the matrix

$$d_{\nu\mu} = (-1)^{\varepsilon_\nu} \prod_{\eta \neq \mu} \left(\frac{\gamma_\eta + \gamma_\nu}{\gamma_\eta - \gamma_\mu} \right) \exp(\kappa_\nu x + \left(\frac{\kappa_\nu \gamma}{12} - 2\delta_\nu \right) y t + 4i\tau_\nu t + x_{r,\nu} + e_\nu). \tag{10}$$

is a solution to the Johnson equation (1), depending on $2N - 1$ parameters $a_k, b_k, 1 \leq k \leq N - 1$ and ε .

We recall another result on the solutions to the Johnson equation obtained recently by the author in terms of wronskians. We need to define the following notations :

$$\phi_{r,\nu} = \sin \Theta_{r,\nu}, \quad 1 \leq \nu \leq N, \quad \phi_{r,\nu} = \cos \Theta_{r,\nu}, \quad N + 1 \leq \nu \leq 2N, \quad r = 1, 3, \tag{11}$$

with the arguments

$$\Theta_{r,\nu} = \frac{-i\kappa_\nu x}{2} + i \left(\frac{-\kappa_\nu \gamma}{24} + \delta_\nu \right) y t - i \frac{x_{r,\nu}}{2} + 2\tau_\nu t + \gamma_\nu w - i \frac{\varepsilon_\nu}{2}, \quad 1 \leq \nu \leq 2N. \tag{12}$$

We denote $W_r(w)$ the wronskian of the functions $\phi_{r,1}, \dots, \phi_{r,2N}$ defined by

$$W_r(w) = \det[(\partial_w^{\mu-1} \phi_{r,\nu})_{\nu, \mu \in [1, \dots, 2N]}]. \tag{13}$$

We consider the matrix $D_r = (d_{\nu\mu})_{\nu, \mu \in [1, \dots, 2N]}$ defined in (10).

Then we have the following statement :

Theorem 2.2 *The function v defined by*

¹ The proof of this result is submitted to a review

$$v(x, y, t) = -2 \frac{|W_3(\phi_{3,1}, \dots, \phi_{3,2N})(0)|^2}{(W_1(\phi_{1,1}, \dots, \phi_{1,2N})(0))^2}$$

is a solution to the Johnson equation depending on $2N - 1$ real parameters a_k, b_k and ε , with ϕ_v^r defined in (11)

$$\begin{aligned} \phi_{r,v} &= \sin\left(\frac{-i\kappa_v x}{2} + i\left(\frac{-\kappa_v y}{24} + \delta_v\right)yt - i\frac{x_{r,v}}{2} + 2\tau_v t + \gamma_v w - i\frac{e_v}{2}\right), \quad 1 \leq v \leq N, \\ \phi_{r,v} &= \cos\left(\frac{-i\kappa_v x}{2} + i\left(\frac{-\kappa_v y}{24} + \delta_v\right)yt - i\frac{x_{r,v}}{2} + 2\tau_v t + \gamma_v w - i\frac{e_v}{2}\right), \quad N + 1 \leq v \leq 2N, \quad r = 1,3, \end{aligned}$$

$\kappa_v, \delta_v, x_{r,v}, \gamma_v, e_v$ being defined in(3), (2) and (4).

From those two preceding results, we can construct rational solutions to the Johnson equation as a quotient of two determinants.

We use the following notations :

$$\begin{aligned} X_v &= \frac{-i\kappa_v x}{2} + i\left(\frac{-\kappa_v y}{24} + \delta_v\right)yt - i\frac{x_{3,v}}{2} + 2\tau_v t + \gamma_v w - i\frac{e_v}{2}, \\ Y_v &= \frac{-i\kappa_v x}{2} + i\left(\frac{-\kappa_v y}{24} + \delta_v\right)yt - i\frac{x_{1,v}}{2} + 2\tau_v t + \gamma_v w - i\frac{e_v}{2}, \end{aligned}$$

for $1 \leq v \leq 2N$, with $\kappa_v, \delta_v, x_{r,v}$ defined in (3) and parameters e_v defined by (4).

We define the following functions :

$$\begin{aligned} \varphi_{4j+1,k} &= \gamma_k^{4j-1} \sin X_k, \quad \varphi_{4j+2,k} = \gamma_k^{4j} \cos X_k, \\ \varphi_{4j+3,k} &= -\gamma_k^{4j+1} \sin X_k, \quad \varphi_{4j+4,k} = -\gamma_k^{4j+2} \cos X_k, \end{aligned} \tag{14}$$

for $1 \leq k \leq N$, and

$$\begin{aligned} \varphi_{4j+1,N+k} &= \gamma_k^{2N-4j-2} \cos X_{N+k}, \quad \varphi_{4j+2,N+k} = -\gamma_k^{2N-4j-3} \sin X_{N+k}, \\ \varphi_{4j+3,N+k} &= -\gamma_k^{2N-4j-4} \cos X_{N+k}, \quad \varphi_{4j+4,N+k} = \gamma_k^{2N-4j-5} \sin X_{N+k}, \end{aligned} \tag{15}$$

for $1 \leq k \leq N$.

We define the functions $\psi_{j,k}$ for $1 \leq j \leq 2N, 1 \leq k \leq 2N$ in the same way, the term X_k is only replaced by Y_k .

$$\begin{aligned} \psi_{4j+1,k} &= \gamma_k^{4j-1} \sin Y_k, \quad \psi_{4j+2,k} = \gamma_k^{4j} \cos Y_k, \\ \psi_{4j+3,k} &= -\gamma_k^{4j+1} \sin Y_k, \quad \psi_{4j+4,k} = -\gamma_k^{4j+2} \cos Y_k, \end{aligned} \tag{16}$$

for $1 \leq k \leq N$, and

$$\begin{aligned} \psi_{4j+1,N+k} &= \gamma_k^{2N-4j-2} \cos Y_{N+k}, \quad \psi_{4j+2,N+k} = -\gamma_k^{2N-4j-3} \sin Y_{N+k}, \\ \psi_{4j+3,N+k} &= -\gamma_k^{2N-4j-4} \cos Y_{N+k}, \quad \psi_{4j+4,N+k} = \gamma_k^{2N-4j-5} \sin Y_{N+k}, \end{aligned} \tag{17}$$

for $1 \leq k \leq N$.

The following ratio

$$q(x, t) := \frac{W_3(0)}{W_1(0)}$$

can be written as

$$q(x, t) = \frac{\Delta_3}{\Delta_1} = \frac{\det(\varphi_{j,k})_{j,k \in [1,2N]}}{\det(\psi_{j,k})_{j,k \in [1,2N]}}. \tag{18}$$

The terms λ_j depending on ε are defined by $\lambda_j = 1 - 2j\varepsilon^2$. All the functions $\varphi_{j,k}$ and $\psi_{j,k}$ and their derivatives depend on ε . They can all be prolonged by continuity when $\varepsilon = 0$.

We use the following expansions

$$\varphi_{j,k}(x, y, t, \varepsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} \varphi_{j,1}[l] k^{2l} \varepsilon^{2l} + O(\varepsilon^{2N}), \quad \varphi_{j,1}[l] = \frac{\partial^{2l} \varphi_{j,1}}{\partial \varepsilon^{2l}}(x, y, t, 0),$$

$$\varphi_{j,1}[0] = \varphi_{j,1}(x, y, t, 0), \quad 1 \leq j \leq 2N, \quad 1 \leq k \leq N, \quad 1 \leq l \leq N - 1,$$

$$\varphi_{j,N+k}(x, y, t, \varepsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} \varphi_{j,N+1}[l] k^{2l} \varepsilon^{2l} + O(\varepsilon^{2N}), \quad \varphi_{j,N+1}[l] = \frac{\partial^{2l} \varphi_{j,N+1}}{\partial \varepsilon^{2l}}(x, y, t, 0),$$

$$\varphi_{j,N+1}[0] = \varphi_{j,N+1}(x, y, t, 0), \quad 1 \leq j \leq 2N, \quad 1 \leq k \leq N, \quad 1 \leq l \leq N - 1.$$

We have the same expansions for the functions $\psi_{j,k}$.

$$\psi_{j,k}(x, y, t, \varepsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} \psi_{j,1}[l] k^{2l} \varepsilon^{2l} + O(\varepsilon^{2N}), \quad \psi_{j,1}[l] = \frac{\partial^{2l} \psi_{j,1}}{\partial \varepsilon^{2l}}(x, y, t, 0),$$

$$\psi_{j,1}[0] = \psi_{j,1}(x, y, t, 0), \quad 1 \leq j \leq 2N, \quad 1 \leq k \leq N, \quad 1 \leq l \leq N - 1,$$

$$\psi_{j,N+k}(x, t, \varepsilon) = \sum_{l=0}^{N-1} \frac{1}{(2l)!} \psi_{j,N+1}[l] k^{2l} \varepsilon^{2l} + O(\varepsilon^{2N}), \quad \psi_{j,N+1}[l] = \frac{\partial^{2l} \psi_{j,N+1}}{\partial \varepsilon^{2l}}(x, y, t, 0),$$

$$\psi_{j,N+1}[0] = \psi_{j,N+1}(x, t, 0), \quad 1 \leq j \leq 2N, \quad 1 \leq k \leq N, \quad N + 1 \leq k \leq 2N..$$

Then we get the following result :

Theorem 2.3 *The function v defined by*

$$v(x, y, t) = -2 \frac{|\det((n_{jk})_{j,k \in [1,2N]})|^2}{\det((d_{jk})_{j,k \in [1,2N]})^2} \tag{19}$$

is a rational solution to the Johnson equation (1), where

$$n_{j1} = \varphi_{j,1}(x, y, t, 0), \quad 1 \leq j \leq 2N \quad n_{jk} = \frac{\partial^{2k-2} \varphi_{j,1}}{\partial \varepsilon^{2k-2}}(x, y, t, 0),$$

$$n_{jN+1} = \varphi_{j,N+1}(x, y, t, 0), \quad 1 \leq j \leq 2N \quad n_{jN+k} = \frac{\partial^{2k-2} \varphi_{j,N+1}}{\partial \varepsilon^{2k-2}}(x, y, t, 0),$$

$$d_{j1} = \psi_{j,1}(x, y, t, 0), \quad 1 \leq j \leq 2N \quad d_{jk} = \frac{\partial^{2k-2} \psi_{j,1}}{\partial \varepsilon^{2k-2}}(x, y, t, 0), \tag{20}$$

$$d_{jN+1} = \psi_{j,N+1}(x, y, t, 0), \quad 1 \leq j \leq 2N \quad d_{jN+k} = \frac{\partial^{2k-2} \psi_{j,N+1}}{\partial \varepsilon^{2k-2}}(x, y, t, 0),$$

$$2 \leq k \leq N, \quad 1 \leq j \leq 2N$$

The functions φ and ψ are defined in (14),(15), (16), (17).

Proof : In each column k (and $N + k$) of the determinants appearing in $q(x, t)$, we can successively eliminate the powers of ε strictly inferior to $2(k - 1)$; then each common term in the numerator and denominator is factorized and simplified; in the end, we take the limit when ε goes to 0.

First of all, the components j of the columns 1 and $N + 1$ are respectively equal by definition to $\varphi_{j1}[0] + 0(\varepsilon)$ for C_1 , $\varphi_{jN+1}[0] + 0(\varepsilon)$ for C_{N+1} of Δ_3 , and $\psi_{j1}[0] + 0(\varepsilon)$ for C'_1 , $\psi_{jN+1}[0] + 0(\varepsilon)$ for C'_{N+1} of Δ_1 .

So we can replace the columns C_k by $C_k - C_1$ and C_{N+k} by $C_{N+k} - C_{N+1}$ for $2 \leq k \leq N$, for Δ_3 ; the same changes for Δ_1 are made. Each component j of the column C_k of Δ_3 can be rewritten as $\sum_{l=1}^{N-1} \frac{1}{(2l)!} \varphi_{j,1}[l] (k^{2l} - 1) \varepsilon^{2l}$ and the column C_{N+k} replaced by $\sum_{l=1}^{N-1} \frac{1}{(2l)!} \varphi_{j,N+1}[l] (k^{2l} - 1) \varepsilon^{2l}$ for $2 \leq k \leq N$. For Δ_1 , we make the same reductions, each component j of the column C'_k can be rewritten as $\sum_{l=1}^{N-1} \frac{1}{(2l)!} \psi_{j,1}[l] (k^{2l} - 1) \varepsilon^{2l}$ and the column C'_{N+k} replaced by $\sum_{l=1}^{N-1} \frac{1}{(2l)!} \psi_{j,N+1}[l] (k^{2l} - 1) \varepsilon^{2l}$ for $2 \leq k \leq N$.

The term $\frac{k^2-1}{2} \varepsilon^2$ for $2 \leq k \leq N$ can be factorized in Δ_3 and Δ_1 in each column k and $N + k$, and so those common terms can be simplified in the numerator and denominator.

If we restrict the developments at order 1 in columns 2 and $N + 2$, we respectively get $\varphi_{j1}[1] + 0(\varepsilon)$ for the component j of C_2 , $\varphi_{jN+1}[1] + 0(\varepsilon)$ for the component j of C_{N+2} of Δ_3 , and $\psi_{j1}[1] + 0(\varepsilon)$ for the component j of C'_2 , $\psi_{jN+1}[1] + 0(\varepsilon)$ for the component j of C'_{N+2} of Δ_1 . We can extend this algorithm up to the columns C_N, C_{2N} of Δ_3 and C'_N, C'_{2N} of Δ_1 .

Then we take the limit when ε tends to 0, and $q(x, y, t)$ can be replaced by $Q(x, y, t)$ defined by :

$$Q(x, y, t) := \frac{\left\| \begin{matrix} \varphi_{1,1}[0] & \dots & \varphi_{1,1}[N-1] & \varphi_{1,N+1}[0] & \dots & \varphi_{1,N+1}[N-1] \\ \varphi_{2,1}[0] & \dots & \varphi_{2,1}[N-1] & \varphi_{2,N+1}[0] & \dots & \varphi_{2,N+1}[N-1] \\ \vdots & & \vdots & \vdots & & \vdots \\ \varphi_{2N,1}[0] & \dots & \varphi_{2N,1}[N-1] & \varphi_{2N,N+1}[0] & \dots & \varphi_{2N,N+1}[N-1] \end{matrix} \right\|^2}{\left(\begin{matrix} \psi_{1,1}[0] & \dots & \psi_{1,1}[N-1] & \psi_{1,N+1}[0] & \dots & \psi_{1,N+1}[N-1] \\ \psi_{2,1}[0] & \dots & \psi_{2,1}[N-1] & \psi_{2,N+1}[0] & \dots & \psi_{2,N+1}[N-1] \\ \vdots & & \vdots & \vdots & & \vdots \\ \psi_{2N,1}[0] & \dots & \psi_{2N,1}[N-1] & \psi_{2N,N+1}[0] & \dots & \psi_{2N,N+1}[N-1] \end{matrix} \right)^2} \quad (21)$$

So the solution to the Johnson equation takes the form :

$$v(x, y, t) = -2Q(x, y, t)$$

and we get the result. ■

3. EXPLICIT EXPRESSION OF RATIONAL SOLUTIONS OF ORDER 4 DEPENDING ON 6 PARAMETERS

We explicitly construct rational solutions to the Johnson equation of order 4 depending on 6 parameters.

Because of the length of the expression, we only give the expression without parameters in the appendix.

We give patterns of the modulus of the solutions in the plane (x, y) of coordinates in function of the parameters $a_1, a_2, a_3, b_1, b_2, b_3$, and time t .

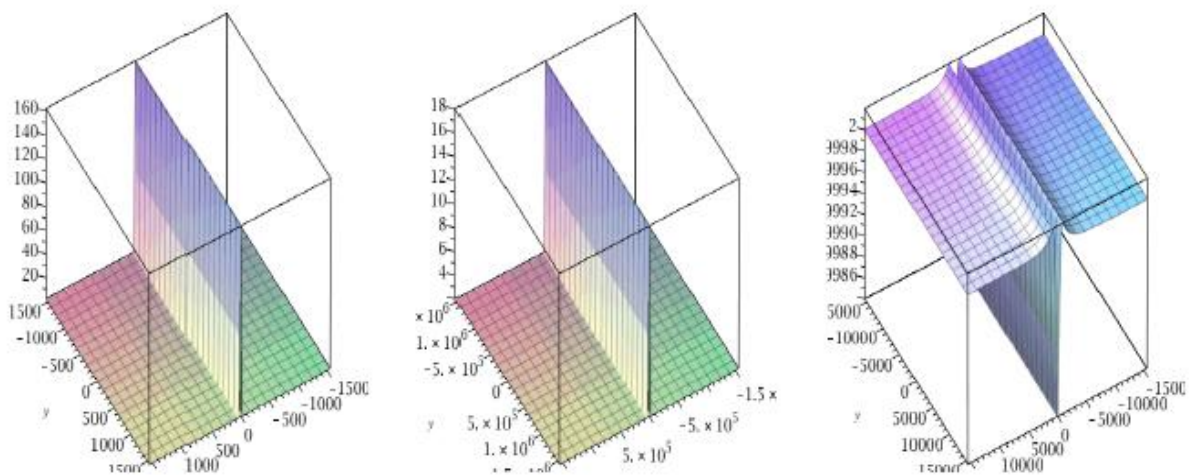


Figure1. Solution of order 4 to (1), on the left for $t = 0$; in the center for $t = 0, a_2 = 10^4$; on the right for $t = 0, a_1 = 10^3$; all other parameters not mentioned equal to 0.

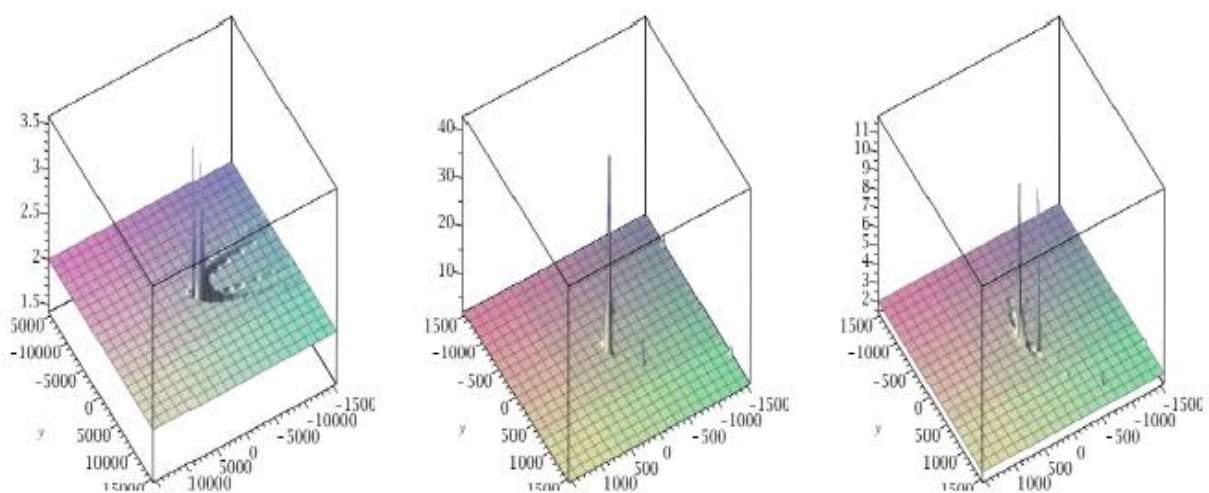


Figure2. Solution of order 4 to (1), on the left for $t = 0,01, a_1 = 10^2$; in the center for $t = 0,01, a_2 = 10$; on the right for $t = 0,01, b_1 = 10$; all other parameters not mentioned equal to 0.

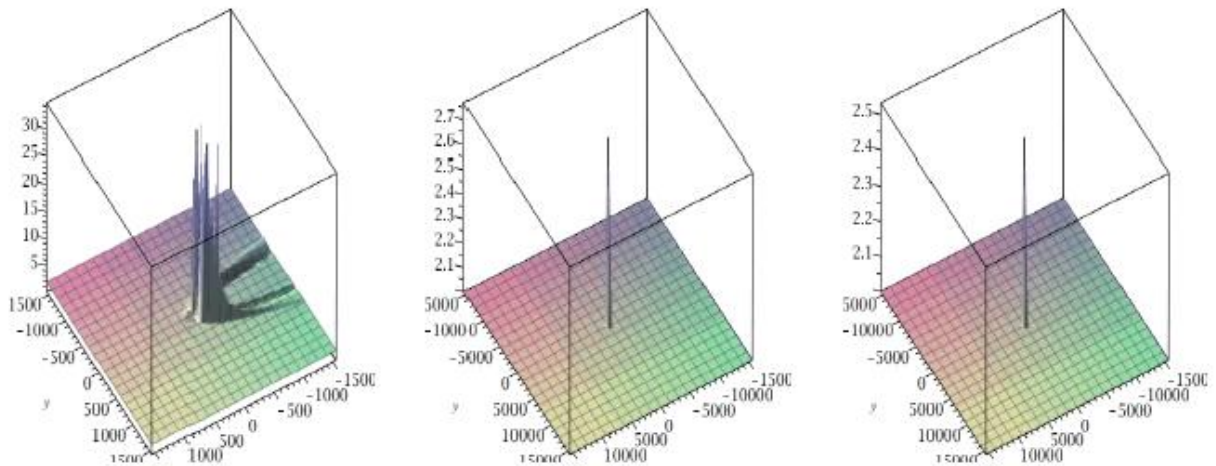


Figure3. Solution of order 4 to (1), on the left for $t = 0,1$, $a_1 = 10^2$; in the center for $t = 0,1$, $b_1 = 10$; on the right for $t = 0,1$, $a_2 = 10^2$; all the other parameters to equal to 0.

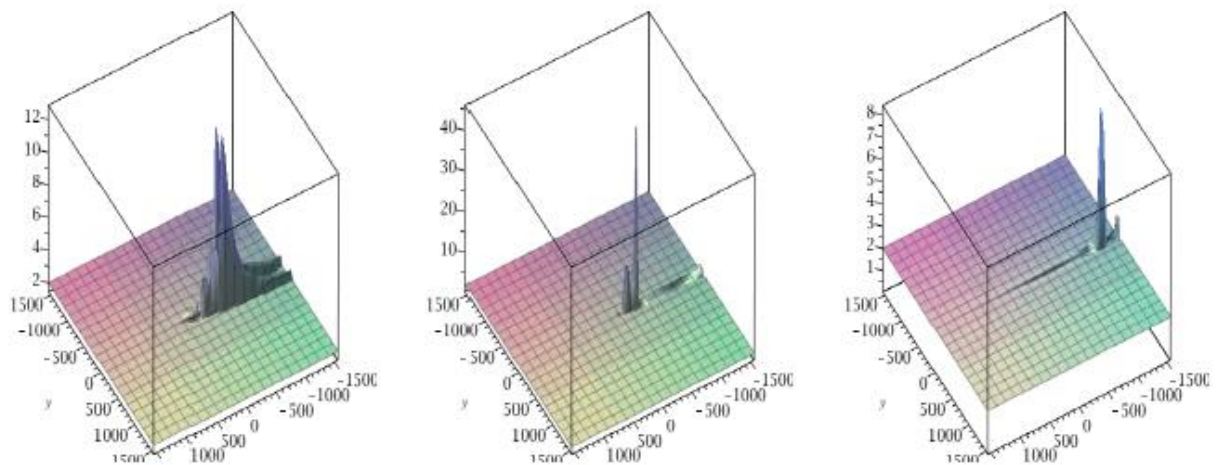


Figure4. Solution of order 4 to (1), on the left for $t = 1$, $a_2 = 10^2$; in the center for $t = 10$, $a_1 = 10^2$; on the right for $t = 10$, $a_1 = 10^3$; all the other parameters to equal to 0.

In these constructions, we note that the initial rectilinear structure becomes deformed very quickly as time t increases. The heights of the peaks also decrease very quickly according to time t and of the various parameters. Because of the structure of the polynomials, one notices that the modulus of these solutions tend towards value 2 when time t and variables x and y tend towards the infinite.

4. CONCLUSION

From the previous results giving the solutions to the Johnson equation in terms of Fredholm determinants and wronskians, we succeed in obtaining rational solutions to the Johnson equation depending on $2N - 2$ real parameters. These solutions can be expressed in terms of a ratio of two polynomials of degree $2N(N + 1)$ in x, t and $4N(N + 1)$ in y . That gives a new approach to find explicit solutions for higher orders and try to describe the structure of those rational solutions.

In the (x, y) plane of coordinates, different structures appear.

It will be relevant to go on this study for higher orders to try to understand the structure of those rational solutions.

REFERENCES

- [1] R.E. Johnson, Water waves and Kortewegde Vries equations, J. Fluid Mech., V. 97, N. 4, 701719, 1980
- [2] R.E. Johnson, A Modern Introduction to the Mathematical Theory of Water Waves, Cambridge University Press, Cambridge, 1997
- [3] M. J. Ablowitz, Nonlinear Dispersive Waves : Asymptotic Analysis and Solitons, Cambridge University Press, Cambridge, 2011

[4] V.D. Lipovskii, On the nonlinear internal wave theory in fluid of finite depth, *Izv. Akad. Nauka.*, V. 21, N. 8, 864871, 1985

[5] V.I. Golinko, V.S. Dryuma, Yu.A. Stepanyants, Nonlinear quasicylindrical waves: Exact solutions of the cylindrical Kadomtsev- Petviashvili equation, in *Proc. 2nd Int. Workshop on Nonlinear and Turbulent Processes in Physics*, Kiev, Harwood Acad., Gordon and Breach, 13531360, 1984

[6] V.D. Lipovskii, V.B. Matveev, A.O. Smirnov, connection between the Kadomtsev-Petvishvili and Johnson equation, *Zap. Nau. Sem.*, V. 150, 7075, 1986

[7] C. Klein, V.B. Matveev, A.O. Smirnov, Cylindrical Kadomtsev-Petviashvili equation: Old and new results, *Theor. Math. Phys.*, V. 152, N. 2, 1132-1145, 2007

[8] K. R. Khusnutdinova, C. Klein, V.B. Matveev, A.O. Smirnov, On the integrable elliptic cylindrical K-P equation *Chaos*, V. 23, 013126-1-15, 2013

[9] B.B. Kadomtsev, V.I. Petviashvili, On the stability of solitary waves in weakly dispersing media, *Sov. Phys. Dokl.*, V. 15, N. 6, 539-541, 1970

[10] M.J. Ablowitz, H. Segur On the evolution of packets of water waves, *J. Fluid Mech.*, V. 92, 691-715, 1979

[11] D.E. Pelinovsky, Y.A. Stepanyants, Y.A. Kivshar, Self-focusing of plane dark solitons in nonlinear defocusing media, *Phys. Rev. E*, V. 51, 5016-5026, 1995

[12] P. Gaillard, Fredholm and Wronskian representations of solutions to the KPI equation and multi-rogue waves, *Jour. of Math. Phys.*, V. 57, 063505-1-13, doi: 10.1063/1.4953383, 2016

APPENDIX

Because of the length of the complete expression, we only give in this appendix the explicit expression of the rational solution of order 4 to the Johnson equation without parameters. They can be written as

$$v(x, y, t) = -2 \frac{\ln \frac{(x,y,t)^2}{(d(x,y,t))^2}}{(d(x,y,t))^2} \quad \text{with} \quad n(x, y, t) = A(x, y, t) + iB(x, y, t), \quad d(x, y, t) = C(x, y, t) + iD(x, y, t)$$

$$A(x, y, t) = \sum_{k=0}^{20} a_k(y, t)x^k, \quad B(x, y, t) = \sum_{k=0}^{20} b_k(y, t)x^k, \quad C(x, y, t) = \sum_{k=0}^{20} c_k(y, t)x^k, \quad D(x, y, t) = 0.$$

a20 = 3833759992447475122176, a19 = (6389599987412458536960 y2+1840204796374788058644480)t, a18 = (5058433323368196341760 y4+2760307194562182087966720 y2 + 419566693573451677370941440)t2 + 28753199943356063416320, a17 = (2529216661684098170880 y6 + 1955217596148212312309760 y4+563102667690685145945210880 y2+60417603874577041541415567360)t3+(43129799915034095124480 y2+17941996764654183571783680)t, a16 = (895764234346451435520 y8+868985598288094361026560 y6+354699474501240398303723520 y4+72077141464407698680986992640 y2+6162595595206858237224387870720)t4+(30550274939815817379840 y4+20242252760122668645089280 y2+4786372675370823740534292480)t2 -161736749681377856716800, a15 = (238870462492387049472 y10 +271557999465029487820800 y8 + 139242162925692296437432320 y6+40101742922599381373980508160 y4+6486942731796692881288829337600 y2+4732873417118867126188329884712960)t5+(13577899973251474391040 y6+10044451180212384820101120 y4+4306079223517004057228083200 y2+756809985376280836150363422720)t3+(-215648999575170475622400 y2-45545068710276004451450880)t, a14 = (49764679685913968640 y12+63363533208506880491520 y10+38145911924852377465651200 y8+13808896791996409592068177920 y6+3163974518694955596510973132800 y4+435922551576737761622609331486720 y2+2839724050713202757129979308277760)t6+(4243093741641085747200 y8+2747527994587357170892800 y6+1531970492982011058821529600 y4+556477930423735908934090752000 y2+81086784147458661016110366720000)t4+(-134780624734481547264000 y4-72458063857257279809126400 y2-3229559417637753042921062400)t2 + 1186069497663437615923200, a13 = (8294113280985661440 y14 + 11440637940424853422080 y12 + 7737805584756487288258560 y10+3293353183912079028954071040 y8+948485716966678760338772459520 y6+184839709369548119511547584184320 y4+22667972681990363604375685237309440 y2+1363067544130233732342239006797332480)t7+(990055206382920007680 y10+382044832580703250022400 y8+198588767608779211328716800 y6+143756798692798443141306777600 y4+49192649049458254349773622476800 y2+6320876997862697543527835306557440)t5+(-52414687396742823936000 y6-51324461898890573198131200 y4-8984799918299902696331673600 y2+417358447817801931700568064000)t3+(1383747747274010551910400 y2 + 398519351214915038950195200)t, a12 = (1123161173466808320 y16 + 1634376848632121917440 y14 + 1203285919851743406981120 y12+576801970063697457048453120 y10+195462873061337238013099376640 y8+47842262604963321877426895585280 y6+8278254729452363179048963481272320 y4 +932625161773317816865742478335016960 y2 +53159634221079115561347321265095966720)t8 + (17875996781913834720 y12 - 8075744428535190650880 y10 - 39248117922681026563276800 y8 + 1648823497551810100545454080 y6 + 9283906805902660747383747379200 y4+3166849124689830209434838378741760 y2+374021549252840999472198806415605760)t6+(-14195644503284514816000 y8-21804509957045014757376000 y6-6831185242542533546449305600 y4-417358447817801931700568064000 y2+110683460361281072286990650572800)t4+(749530029773422382284800 y4+470977415072172318759321600 y2+72602979984971794368744652800)t2-4953187958992196861952000, a11 = (124795685940756480 y18+187272347239097303040 y16+146517077489138458951680 y14+76751875048798451945963520 y12+29343905682992370383144878080 y10+8451044836701802670345724887040 y8+183343896692569173883327482429440 y6+290302440061792555740071800854282240 y4+30776630338519487956569501785055559680 y2+1701108295074531697963114280483070935040)t9+(25537138259876904960 y14-16487978208259347578880 y12-23548870753608615937966080 y10-8817341281829796982995025920 y8-761138147057354337664295239680 y6+437525208016358121040339512852480 y4+154008873221758058606199508524072960 y2+17253565492806379612016235849197813760)t7+(-2839128900656902963200 y10-6268796612650441742745600 y8-2644719330789890713033113600 y6-423155092926382514085298176000 y4+16527394533584956495342495334400 y2+11875684217586863285380643920281600)t5+(-249843343257807460761600 y6+255112766497426672661299200 y4+82167444414129755303549337600 y2+9121600742862404440611304243200)t3+(-4953187958992196861952000 y2-1462747164118381336146739200)t, a10 = (11439604544569344 y20+17340032151768268800 y18+14144570226764761006080 y16+7918844251066506946805760 y14+

3318042598263440768740884480 y12+1079072537972782120301972422656 y10+275211725270362831122443922309120
y8+54479329302934823452533159477903360 y6+
8071322172111728459198374242649374720 y4+820710142360519678841853380934814924800 y2+44909258989967636826226217004753072685056)t10+
(2926130425610895360 y16-4142023680111799173120 y14-5552074294617943572480000 y12-2774875482533482493430988800
y10-713283621327627974199245537280 y8-
66272811660597455181323981291520 y6+15584231218886374978008283600650240 y4+5783190341388465874191981544577433600
y2+630920921939649503109674786593638973440)t8+
(-433755804267026841600 y12-1299185384940598795960320 y10-624531483269670095762227200 y8-139312704109553329979680358400 y6-
9508816636115587343911275724800 y4+3613990271343910486981558979788800
y2+808546584414229807682547419751383040)t6+(57255766163247543091200 y8+
8394736334623306815897600 y6+42080020585102165249150156800 y4+8142354162519524723084230656000
y2+829736418158311427016176015769600)t4+
(-2270211147871423561728000 y4-1611437149325461379088384000 y2-236992212361124247935700172800)t2-22289345815464885878784000, a9 =
(866636707921920 y22+1300502411382620160 y20+1089362019887559475200 y18+640868004293042629509120 y16+288170959432301880016896000
y14+
102896891394092649087164743680 y12+29634304721498682937103446179840 y10+6883790993036019975202969485312000
y8+1269787444522250115855195947831132160 y6+
179027457524721204453247428684310118400 y4+1772339074987225062984033028192002375680
y2+979838377962930258026753825558248858583040)t11+
(270938002371379200 y18-638011182995944243200 y16-840742678899288598118400 y14-482979213715195386711244800
y12-165537155961891038805413068800 y10-
34306864410623318785786694860800 y8-3075319634693734127133359131852800 y6+429548557965447646452665295883468800
y4+169724064366835411525199458373468160000 y2+
18465978203111692773941701071033335808000)t9+(-51637595746074624000 y14-201262693179900454502400 y12-96196093393826585405030400
y10-
21999073276661724119826432000 y8-2454299538973018581694729420800 y6+374509647175174266712643076096000
y4+257034039786313243415566487440588800 y2+
39036534804372144650795875425897676800)t7+(9542627693874590515200 y10+1873825074433559557120000 y8+12926719864534302210392064000
y6+
3607606490492998564718837760000 y4+475997309736203103104497876992000
y2+55341062407124027660404604377497600)t5+(-630614207742062100480000 y6-
784176138142669388906496000 y4-252289921092987691135401984000 y2-29497677796289438610295357440000)t3+(-18574454846220738232320000
y2-
10580009480407332497129472000)t, a8 = (54164794245120 y24+78818327962583040 y22+67014124257127956480 y20+40891467820678187581440 y18+
19425182661732300509675520 y16+7463870654096140365862010880 y14+2362145763180161945293931151360
y12+61908328753330266506058630430720 y10+
13363956777602318193811111988510720 y8+23333915812210853614131125536381992960 y6+3171803314887420170582927478083372974080 y4+
309422645672504292008448576492078586920960 y2 + 176370908033327446448156886004879454494720)t12 + (20320350177853440 y20 -
70380130498353561600 y18 - 90716928206703899443200 y16 - 56579907888537320344780800 y14 - 22667297643345319033621708800 y12 -
6200400762543581883101180067840 y10-1108170150645827689051348323532800 y8-94291146870464117513999699096371200
y6+9334823540175947633885970210540748800 y4+
3910442443011887881540595520924706406400 y2+434319807337187014043108809190704058204160)t10+(-4841024601194496000
y16-23794604119791186739200 y14-
9667015512205934827929600 y12-1247888877541653152268288000 y10+133395295561210652128601702400 y8+87700921834780779248012702515200
y6+
35422713956708994030381333636710400 y4+11227899642729111839359348784706355200 y2+1400223531026392145082895531581112320000)t8+
(1192828461734323814400 y12+2998120119093689529139200 y10+2657739139139241630498816000 y8+993899211034013967444030259200 y6+
1587440246210335388970264821760000 y4+16018551114005492380213162750771200
y2+2732705521761071046765909548571033600)t6+(-118240163951636643840000 y8-
22423002683265800798208000 y6-111183502517217986932113408000 y4-19902780980311429617970839552000
y2-2644826568724399453807231107072000)t4+
(-6965420567332776837120000 y4-4992813462664134436847616000 y2-2017300427220980848585900032000)t2+3436274146550836572979200, a7 =
(2777681756160 y26+3831446498181120 y24+3282551541029928960 y22+2056323812821460582400 y20+1019791990583597664829440 y18+
415426232781608404238991360 y16+141700676533649013283781345280 y14+40809794841690915825729027440640
y12+9923648674129265514585283016785920 y10+
2020570627637843037872649962174545920 y8+337939470383743397170174921561394380800 y6+44744941408523904971809965718217246638080 y4+
4331917039415060088118280070889100216893440 y2 + 2604862641722989978261893247022544657894604800)t13 + (1231536374415360 y22 -
5839510827580784640 y20-7233853412961209548800 y18-4745742273984199152107520 y16-2102050603858950609489100800
y14-674935593575973765845153218560 y12-
157935856359995234100947409960960 y10 - 25267023525214695048671830789324800 y8 - 2033787506344472196225039663586344960 y6 +
162935101792161995064191480038529433600 y4+70561761416125621329132523622019146711040
y2+8199700110321363738223887277040082553405440)t11+
(-358594414903296000 y18-2168779021335134208000 y16-565579436385805900185600 y14+159340649686098184765440000
y12+131438927837064705573755289600 y10+
35609950231032233705874024038400 y8+5868393663076549401255347434291200 y6+1220180190795761185032562539705139200
y4+338142559007772089115589690144063488000 y2+
38126813819365906845020806329015887462400)t9+(113602710641364172800 y14+353944736281893902745600 y12+385814030989944578428108800
y10+
180974749011478909253635276800 y8+38153131661670154550801281843200 y6+2486454688719336788299304298086400
y4+232673661904083780266067730681036800 y2+
100717451187943441215338447819400806400)t7+(-1576535519355152512000 y10-42069338471810709061632000
y8-27507647263309807944204288000 y6-
6748381777345656755022987264000 y4-1050160804386218368894400200704000
y2-160370818290886027859806679728128000)t5+(-1547871237185061519360000 y6-
683539938340923166949376000 y4-432710500097558317860126720000 y2-217430708861580500105630318592000)t3+(-22908494310338910486528000
y2+
873742359662235264483328000)t, a6 = (115736739840 y28+147363326853120 y26+127113872057303040 y24+81006695656602992640 y22+
41569376769959987773440 y20+17751728676140123695349760 y18+6433560435325856251694284800 y16+1998374079032793841861090344960 y14+
533625236747667820940530758451200 y12+122126694449577041922820433887887360 y10+23720751286551219224444970455751720960 y8+
3834076900353743269712530019169274429440 y6+49901985503605643191907753264185564200960
y4+47984311821212973283771717708310033171742720 y2+
3125835170067587973914271896427053589473525760)t14+(598663515434080 y24-370523414294691840 y22-430542798309493309440 y20-
291744895412915530629120 y18-138469820027213815511777280 y16-49524818283235879426279342080 y14-13619488192715145406768018882560
y12-
2835633460501622596437267563151360 y10-415370085395046272449637485652213760 y8-31519354449252989928372084030245437440 y6+
2331782345647829440474206958773621227520 y4+987422266337256343615196192629133137674240
y2+123459635623329212511370982020339356181463040)t12+
(-20918007536025600 y20-15261778298284277600 y18-5827898498356976025600 y16+4053402492014778384384000
y14+24162751572399267891904512000 y12+)

N-Order Rational Solutions to the Johnson Equation Depending on $2N - 2$ Parameters

7239108040266923752379174092800 $y_{10} + 1261591116063651679144477143859200 y_8 + 136289903625315364883774623344230400 y_6 + 21266609891186321329512414651088896000 y_4 + 7281945703172778394791942300183507763200 y_2 + 793819815931413239952740890747715400499200)t_{10} + (8283530984266137600 y_{16} + 31230417907225932595200 y_{14} + 40591681076284200950169600 y_{12} + 22155842112352888790030745600 y_{10} + 6300107888948758383937191936000 y_8 + 454846511064014274099094644326400 y_6 - 78988258173722430869013430429286400 y_4 - 3826374302722400517019162614379315200 y_2 + 2780384807264807078700889281031412121600)t_8 + (-1532742866039734272000 y_{12} - 5456150563470108209971200 y_{10} - 4291111835140239881404416000 y_8 - 1441396187968442857696316620800 y_6 - 269899766642536214133813018624000 y_4 - 48362662216231252241598426120192000 y_2 - 653418656335068331129178120886681600)t_6 + (-225731222089488138240000 y_8 + 57787192854908963389440000 y_6 + 67006726709246439148486656000 y_4 - 19218052276860348323774595072000 y_2 - 15847908895634586337913239240704000)t_4 + (6681644173848848891904000 y_4 + 7578377577258061198786560000 y_2 + 896566646081259569588207616000)t_2 - 4179252340399661022720000, quada5 = (3857891328 y_3 + 4385813299200 y_2 + 3848783360163840 y_2 + 2466550028005539840 y_2 + 1302817744347023278080 y_2 + 577974816639164721659904 y_2)t_2 + 220177096099583437905592320 y_{18} + 72754336829473620686568161280 y_{16} + 20953249006888402757731630448640 y_{14} + 5259562288958548418008737424343040 y_{12} + 1145173671492696884932731450395983872 y_{10} + 214107105012357704780544390863372943360 y_8 + 33621905126178979442094494014253637304320 y_6 + 4351518473982549047793023419626154969006080 y_4 + 411294101324682628146614723214085998614937600 y_2 + 30008017632648844549577010205699714458945847296)t_{15} + (2302551982080 y_2 - 17973991660584960 y_2 - 18985944294516326400 y_2 - 13215834966248694743040 y_2 - 657997161111499633786880 y_8 - 2544709311786910912012615680 y_{16} - 782469801056930880840102051840 y_{14} - 191477872770691706976343582310400 y_{12} - 36350384925842578740010820283924480 y_{10} - 4942438604770984889775777234592727040 y_8 - 352236909515363310127374629781299527680 y_6 + 27870789775648364537525698985499725660160 y_4 - 10511269286815954625581120760245610820403200 y_2 + 1463521510546995685155037390103456011738152960)t_{13} + (-950818524364800 y_{12} - 8233327766179676160 y_{20} + 2172486335901519052800 y_{18} + 4175121043858325608857600 y_{16} + 2408895169254465399580262400 y_{14} + 815491356025478265158447923200 y_{12} + 176179057845720042982170018447360 y_{10} + 22573415952052762398729604536729600 y_8 + 1313537217912846578771656647376896000 y_6 + 93383841848829441809809044649122201600 y_4 + 112650822685227077818227155583561734553600 y_2 + 12589633895800018381093751456021447505346560)t_{11} + (460196165792563200 y_{18} + 2060340070268377497600 y_{16} + 3121294634476035165388800 y_{14} + 1808945009603036649514598400 y_{12} + 678493284511365098283781324800 y_{10} + 80242958238081001951818940416000 y_8 - 20977214656425318317543058623692800 y_6 - 4735465141532653193132430692293017600 y_4 - 264684480566484991132043281208180736000 y_2 + 57275866103067817213654694758672288972800)t_9 + (-109481633288552448000 y_{14} - 500350969880191696896000 y_{12} - 444190889078066898586828800 y_{10} - 202317254947934074063709798400 y_8 - 45890179567660134469726529126400 y_6 - 12465349100586244744583791509504000 y_4 - 1918590106883270395196604181118976000 y_2 - 179830937883178874359044899896452710400)t_7 + (-22573122208948813824000 y_{10} + 31060616159513567821824000 y_8 + 31231501144098798613561344000 y_6 + 7686079696238387527352844288000 y_4 - 55189133498157152312217305088000 y_2 - 81617908545888055000791996301312000)t_5 + (1113607362308141481984000 y_6 + 2514981186178287956656128000 y_4 + 155133846875635605716336640000 y_2 + 22265095007217660082947883008000)t_3 + (-20896261701998330511360000 y_2 - 12080825431981968146300928000)t, quada4 = (100465920 y_3 + 97462517760 y_3 + 89780178124800 y_2 + 56587517511598080 y_2 + 30856840431324364800 y_2 + 14080785157445973442560 y_2 + 5595869677864999798702080 y_2 + 1946527740165331368699494400 y_{18} + 596668003176561280134886195200 y_{16} + 161452796880273245045410863513600 y_{14} + 38497944554933860359769263314042880 y_{12} + 8034924470662430422227935197963223040 y_{10} + 146046568319687705909077633041550540800 y_8 + 222149591764874876313757952353287190609920 y_6 + 29234139554940677000617222973550230293708800 y_4 + 263228248477968820138334228570150391135600640 y_2 + 225060132244866334121827576542747858442093854720 y_{16} + (68528332800 y_2 - 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6315314647715050998988800000 y_2 - 28218311802378545522540544000000)t_2 - 35262441622122182737920000, quada3 = (1969920 y_3 + 1522851840 y_3 + 1582332641280 y_3 + 931237157928960 y_2 + 539245755110522880 y_2 + 247334053010693160960 y_2 + 102706903501370629816320 y_2 + 37213030326690158519255040 y_2 + 12022671336315281983143936000 y_{18} + 3462529344858801211145453568000 y_{16} + 888940105176092690367673930874880 y_{14} + 20349890407723475742507059098741760 y_{12} + 40647264969233471547741319236755128320 y_{10} + 73505379628083598780287557763955532042240 y_8 + 1052877056767978405386214160733226684907520 y_6 + 148388460085767850154857076610572203421859840 y_4 + 11845270118150859690622504028565676760110202880 y_2 + 1270927805618068710335026314594340847673000591360)t_{17} + (1522851840 y_3 - 17586251366400 y_2 - 13262699416780800 y_2 - 10510299208404172800 y_2 - 5313368173689018777600 y_2 - 2322087128758810973306880 y_2 - 823576841044216489023897600 y_{18} - 246121257499139986883503718400 y_{16} - 61583557633560107255372709888000 y_{14} - 12721530671830941866960461863321600 y_{12} - 2106377769218796864160448289743831040 y_{10} - 244592553739282808441350380290939289600 y_8 - 16674831489704149723305973752008382873600 y_6 +$

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y2+90813737572489924294772530885670188494178222080)t15+
(-846526464000 y26-9930771254476800 y24+11050169090546073600 y22+9908640750155857920000 y20+593528389802968551923200 y18+
2385487934633911093139865600 y16+701909315481235409208764006400 y14+149173183882256139321598594252800
y12+20668035518281890646539277801881600 y10+
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y14+364514121596358623232000 y12+
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y10+43469383910947144335360000 y8-
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(-11754147207374060912640000 y2-13415400012682928188293120000)t, quadra2 = (27360 y36+14929920 y34+20424130560 y32+9906778275840 y30+
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y20+
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y14+748350808542089107952601807653437440 y12+
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y8+3225836088821040220757762535012439204823040 y6+
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y16+16959700283972124672000 y14+
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y2-6488318011726583573480063631360000)t4+
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=

(240 y38 + 69120 y36 + 179159040 y34 + 51597803520 y32 + 59440669655040 y30 + 17118912860651520 y28 + 11503909442357821440 y26 +
3313125919399052574720 y24+1431270397180390712279040 y22+412205874387952525136363520 y20+118715291823730327239272693760 y18+
34190004045234334244910535802880 y16 + 656448077668499217502282874152960 y14 + 1890570463685277746406572987756052480 y12 +
233350411517725710413611294488747048960 y10+67204918517105004599120052812759150100480
y8+4838754133231560331136643802518658807234560 y6+
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y2+12843059930456273283385529073795444355432427028480)t19+
(233280 y34 - 3725015040 y32 - 1221148016640 y30 - 1976402266030080 y28 - 634113063879966720 y26 - 441462524850481397760 y24 -
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y6+53226295465547163642503081827705246879580160 y4+
2380667033549927682919228750839180133159403520 y2+128765053990251698283943455575845332510803230720)t17+(-167961600 y30-
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y14+1736211142922056035874363146240000 y12+
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(155196518400 y26 + 1214934781132800 y24 + 2941978794157670400 y22 -946301404975202304000 y20 + 189005637166238269440000 y18 +
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3855360284018691979345920000)t, quada0 = (y40+829440 y36+309588621120 y32+68475651442606080 y28+9939377758197157724160 y24+
989294098531086060327272448 y20+68380008090468668489821071605760 y16+3240977937746190422411267979010375680
y12+10080737775657506898680079219138725150720 y8+
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y22–
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y16–1641120194171248043755705718538240 y14–
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y4+1806055302720413430476090026002484362482685050880)t18+
(–874800 y32–15049359360 y30+45341569843200 y28+12482540627558400 y26+14211372504475238400 y24+6729787023779325542400 y22+
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y14–
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y18+
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30424060574661885413669252560532275200 y8–6263696267497285833563277130059271372800 y6+706702328964795641398326007471916502220800
y4+
631810240208933424487477396505951959882137600)t14+(–555532992000 y24–6338790162432000 y22–5205512001237811200
y20–4605158748643865395200 y18–
818820547887787973811200 y16+3066049553959868228960256000 y14+3951158349376199817236054016000
y12–1614156983515783293181485907968000 y10–
57696604451008326948067084966297600 y8–50173659986361159332734986738047385600 y6–4161632080138855959182837978065010688000 y4–
1246720149785699163687533499262085942476800)t12+(–359985378816000 y20+4792125362798592000 y18–3626164399526903808000 y16+
8600376623682218360832000 y14–4328431092566720772046848000 y12+57280917292756328583291281856000
y10–472556276283621954886362464256000 y8–
210418773919405564301611999690752000 y6+53585689503145760702242004750303232000 y4–321144871993668872478695711100679225344000)t10+
(79916754097152000 y16+1074910581378514944000 y14–1458295355403518607360000 y12–320673057493272596840448000
y10+768365404933504106294673408000 y8–
478062364556134935579057979392000 y6 +241881297033312248568491437719552000 y4 +616452403926593975672827819151327232000)t8 +
(–13996231528366800000 y12–110357486354860867584000 y10–238759138335795739361280000 y8+86581725523183601146920960000 y6–
248379393351328948833453342720000 y4+685480261675989201024754277941248000)t6+(–1700542130696478720000
y8+25539875413553515069440000 y6–
211969356898215287114956800000 y4–3201850261210208958960359178240000)t4+(68021685227859148800000
y4+2670542245153866393518080000)t2+
734634200460878807040000
b20 = 0, b19 = 0, b18 = 153350399697899004887040 ty, b17 = (230025599546848507330560 y3+66247372669492370111201280 y2)t2, b16 =
(162934799679017692692480 y5+88329830225989826814935040 y3+13514464024576443502685061120 y2)t3+1035115197960818282987520 ty, b15 =
(72415466524007863418880 y7+55206143891243641759334400 y5+15899369440678168826688307200 y3+1729851395145784768343687823360 y2)t4+
(1380153597281091043983360 y3+48581406624944047482142720 y2)t2, b14 = (22629833288752457318400 y9+21469055957705860684185600 y7+
87004882772599979412711101440 y5+1780729377355954908589090406400 y3+155686625563120629150931904102400 y2)t5+(86259598300681902489600
y5+
515257342984940656420454400 y3+103345901364408097373473996800 y2)t3–517575989804091414937600 ty, b13 = (5280294434042240040960 y11+
5814535988545337268633600 y9+2936966855014161741596590080 y7+845846454244078581579817943040 y5+138896891433764482869949051699200
y3+
10462141237841706278942623955681280 y2)t6+(335453999339154073190400 y7+246894143513617397868134400 y5+89654777679379674217159065600
y3+
13355470330169661814418178048000 y2)t4+(–6038171988104773317427200 y3–1738993532574174715419033600 y2)t2, b12 = (953386495035404451840
y13+
1162907197709067453726720 y11+683789432252931662791311360 y9+245674701135219704981272657920 y7+56716230668787163838562529443840
y5+
8000460946584834213309065377873920 y3+544031344367768726505016445695426560 y2)t7+(90852124821020894822400
y9+69774431862544047223603200 y7+
33620541629767377831434649600 y5+9645617460678089088190906368000 y3+1180623577186998104394566939443200
y2)t5+(–3270676493556752213606400 y5–
1932215036193527461576704000 y3–2156351980391976644711960166400 y2)t3+33209945934576253245849600 ty, b11 = (136198070719343493120 y15+
177666377427774194319360 y13+116290719770906745372672000 y11+48743344646375386097374986240 y9+14038083258156111196043996037120 y7+
2777937828675289657398981033984000 y5+352020281649732705385598876626452480 y3+2238300388255962760477819480040407040 y2)t8+
(18170424964204178964480 y11+12598161308514897415372800 y9+6827159794550463697571020800 y7+2901104943942409871909726453760 y5+
717633939074449828161403433779200 y3+76004378992555925026436121089802240 y2)t6+(–1090225497852250737868800
y7–9902602060491828240580608000 y5–
223364058183971774558266982400 y3–10016602747627246360813633536000 y2)t4+(33209945934576253245849600
y3+11303457961732135650223718400 y2)t2, b10 =
(15606028936591441920 y17 + 21150759217592165990400 y15 + 149239757039303232826240 y13 + 6990324619829028238770831360 y11 +

N-Order Rational Solutions to the Johnson Equation Depending on $2N - 2$ Parameters

2351428410446075179640385699840 y9+579805485267098918236607836323840 y7+102672582147838705737466339016048640
y5+12069266799419407041791961484335513600 y3+
738639128124467710957668042841333432320 y1+2776037147308971786240 y13+1421331019422193554554880 y11+698638862623678216431206400
y9+
432816168107350151393181696000 y7+177434843042042940349997949911040 y5+39113364627748078376177652958494720
y3+3696212957322193406548788204577751040 y1+7+
(-249843343257807460761600 y9-3099594538937836327929600 y7-101937224592783197114581647360 y5-10721474792830645178796815155200 y3+
396657468806038955888219888025600 y1)+15221225220014116071014400 y5+9854296684586990054041190400
y3+1972018665939114127285184102400 y1+3-
118876511015812724686848000 ty, b9 = (1445002679314022400 y19 + 1982883676649265561600 y17 + 1472092841544414752931840 y15 +
747838782526754147165798400 y13+281845766612762539061988556800 y11+81171580784475611249852704358400
y9+17864277113634939642965754957004800 y7+
2916739476526356701766390692047749120 y5+325870203584323990128382960077058867200 y3+19697043416652472292204481142435558195200
y1)+
(330480612774877593600 y15 + 74027657261572580966400 y13 - 4919991990307593073459200 y11 + 20137974488328097321766092800 y9 +
20614415778141505782069539635200 y7+810944158447901865371471710745600 y5+1613426290894608233017328184537907200
y3+138796568193323180980607557069858406400 y1)+
(-41640557209634576793600 y11-66112392369758281924608000 y9-27088581354635369718271180800 y7-4326615909044546691962555596800 y5+
58986660624916006347013619712000 y3+86735766512253851687557415514931200 y1)+6+(-4228118116670587797504000
y7+3894620932327578789740544000 y5+
1359313277962146569219211264000 y3+195416499900468593354021535744000 y1)+(-99063759179843937239040000
y3-38674491583811073098121216000 y1)+ b8 =
(108375200948551680 y21+146880272344390041600 y19+112627792833678283898880 y17+60726758280368005935267840
y15+24925573966896504254339481600 y13+
8052853962043040531063997726720 y11+2067426807110263648871933961830400 y9+417782312287594859138952512919306240
y7+64268845706908342497542194904086609920 y5+
6951897676465578456072169814977255833600 y3 + 425456137799693401511616792676608057016320 y1)+ (30982557447644774400 y17 -
5287689804398041497600 y15-12914978974557431817830400 y13-4315280247498877997521305600 y11+429466995377948037126448742400 y9+
785301655413976114687788869222400 y7 + 283477871040049173808114479975628800 y5 + 51294383897532479927615836308425932800 y3 +
4073377544804049876604787000963235840000 y1)+(-5205069651204322099200 y13-10147483480009410714009600
y11-4641676988855874910735564800 y9-
895002004764841920202329292800 y7-584301826944922704380795289600 y5+36604673080929009100957342393958400
y3+6168816954871517841973595698574131200 y1)+7+
(792772146875735212032000 y9+920821228185977930907648000 y7+423662299373383315043962060800 y5+117834201767226078716793716736000 y3+
1142393543366887447507949047808000 y1)+(-37148909692441476464640000 y5-27534064265757766327468032000
y3-6054958106231704587249647616000 y1)+3-
445786916309297717575680000 ty, b7 = (6568193996881920 y23+8568015886756085760 y21+6697740418904185896960
y19+3771993859235821356318720 y17+
1652503907144559676662743040 y15+582859106384564870542818017280 y13+167863422638754682716331588976640
y11+39474801813369127052480992908410880 y9+
7473661364255863591263483842223144960 y7+1100717132107049922211426887371398840320 y5+116791880964621718062012452891617898004480
y3+
7426143496140103008202765835809886086103040 y1)+2+(2295004255381094400 y19-1542242859616095436800 y17-2128091773585427162726400
y15-
998311102033322521814630400 y13-208335719013577672079337062400 y11+15086735002599062419990904832000
y9+23847527821550948135825098722508800 y7+
7620658844505202523182744911583641600 y5+1267273013939037739388155955855228928000 y3+93850618632285309156974292502192953753600
y1)+
(-495720919162316390400 y15-1153123122728342126592000 y13-532924586950136104275148800 y11-100153146042697840091725824000 y9+
5731722683364479862021134745600 y7+8296418169101393919116572203417600 y5+226936151850242893550936813916160000
y3+269469591425498684144624370832952524800 y1)+8+
(105702952916764694937600 y11+144664537215010194063360000 y9+79558954115268493230420787200 y7+30453254742438947616418116403200 y5+
6721140443657882308105948102656000 y3+384445226736131817125115906018508800 y1)+6+(-8255313264986994769920000
y7-8687419947885742610448384000 y5-
3023675004763346286434844672000 y3-477666743527474310831300149248000 y1)+(-297191277539531811717120000
y3-119827523103939226484342784000 y1)+2, b6 =
(319287208181760 y25+389455267579822080 y23+308448571923219087360 y21+179184893474165426749440 y19+82566411037343789214597120 y17+
31190244495355074401984839680 y15+9817095104423477338647633592320 y13+2587043639422731291198230542417920
y11+568032727207523229144943641829048320 y9+
102248352577645438118155489087806504960 y7+14598985120577714757751556611452237250560
y5+1528911896264138854629981201490270664785920 y3+
10396600894561442114838721701338405205442560 y1)+3+(133875248230563840 y21-171360317735121715200 y19-200728839882341036851200 y17-
106391099523742376279408640 y15-32811873855360457078996992000 y13-4877219905758254277413453168640
y11+61910024686075365552807097958400 y9+
557980866018224334876939944303001600 y7+156459475951704274747537716088280186880 y5+24331641867629524596252594352420395417600 y3+
1693482273987014911899180566928459521064960 y1)+(-3614631702225236800 y17-98229006750932847820800 y15-4076697908635552972800000
y13-
4733926838674142280862924800 y11+2483669207523160199110695321600 y9+1455746265988493137771581407232000
y7+415985505467860490466045875296665600 y5+
78355283670706237849116169531608268800 y3+8115421416186530138774152563457523712000 y1)+(-10276675978018789785600
y13+15850201468775029958246400 y11+
10137084406029896937740697600 y9+4397206165033483561892157849600 y7+1614425947848821432204137385164800
y5+242770397473692044597767873285324800 y3+
5584167885698010726385625536344883200 y1)+(-1203899851143936737280000 y9-1593825814360155790245888000
y7-656198340807286240271400960000 y5-
126355270076839534822346981376000 y3-25203859305269240853465604816896000 y1)+(-86680789282363445084160000
y5-43746556053819082684760064000 y3-
10385052002341399628643041280000 y1)+3 + 5498038634481338516766672000 ty, b5 = (12280277237760 y27 + 13522752346521600 y25 +
10904747492235018240 y23+6453693197162737827840 y21+3096116171678414631075840 y19+1235186352766677184696811520
y17+417152344880607955437999882240 y15+
120139875325615091166143966085120 y13 + 29505973491037230453357792397885440 y11 + 6134507285812125187558827975895941120 y9 +
1060610030127440815734087446131145441280 y7+148644212136791277533470394589331870187520
y5+15289118962641388546299812014902706647859200 y3+
1151623483709111358810521224999440796121825280 y1)+4+(6085238555934720 y23-12138022506237788160 y21-12575211009177393561600 y19-
7137715652605333890662400 y17-2611829929788016318123868160 y15-613568444026467333665477099520 y13-61910024686075365655280709795840
y11+
16895560332353300172693289775923200 y9+9711328749008360298924388589450035200 y7+2409768377274847147513478094518558392320 y5+
360108299640916964024538396415821852180480 y3+23698134392094152246764708623099195304181760 y1)+2+(-2008128723458457600 y19-
627277624632157401600 y17-1931034735084741549096960 y15+21603237554415223134617600 y13+430304288560298565693131980800 y11+)

N-Order Rational Solutions to the Johnson Equation Depending on $2N - 2$ Parameters

$193870727871513771384760172544000 y_9 + 54137734530375741130925526535372800 y_7 + 10968631957767807706446728633065144320 y_5 + 174826072604771797229229666373008998400 y_3 + 174766696876146681475006615204404186316800 y_1 + (734048284144199270400 y_15 + 123523946631657250758400 y_13 + 933400753161196208652288000 y_11 + 388777767074106004664745984000 y_9 + 199381405154737711705176932352000 y_7 + 61019808391512167863895213683507200 y_5 + 4897589866964648751830015839253299200 y_3 - 91832983265337612408459902745103564800 y_1 + (-120389985114393673728000 y_11 - 187303885412260481335296000 y_9 - 83372059725756657580376064000 y_7 - 1641522945073392226197967667200 y_5 - 41388046075265358226972999680000 y_3 - 1160703893189550053798362226884608000 y_1) + (-14446798213727240847360000 y_7 - 4398430907585070813413376000 y_5 - 1852571547670425501519839232000 y_3 - 461233254644648359770590281728000 y_1) + (274901931724066925838336000 y_3 + 181881061854193468770877440000 y_1) + (365484441600 y_29 + 346737239654400 y_27 + 292091450684866560 y_25 + 172558641635367321600 y_23 + 85934203534412926156800 y_21 + 35781759929509767806976000 y_19 + 127955573507926929677746176000 y_17 + 3957176394124344241309089792000 y_15 + 1061314708904149125519097882214400 y_13 + 246168029125687206563355857780736000 y_11 + 49036671401836891547420486869922611200 y_9 + 8167264403120399864476395307106146713600 y_7 + 1146683922198104140972485901117702998589440 y_5 + 112904263108736407726521688725435372168806400 y_3 + 9871058431792383075518753357138063966758502400 y_1) + (211293005414400 y_25 - 595001103246950400 y_23 - 537148688285093068800 y_21 - 321994425291595422105600 y_19 - 130168575965789526845030400 y_17 - 37499284406126236661710848000 y_15 - 6986889570875795301061361664000 y_13 - 346648652125292555538676265779200 y_11 + 288136259091212351848983540478771200 y_9 + 122296276869641404220675190145469644800 y_7 + 26983166942948126635587812967748259020800 y_5 + 4034628615141477533051339281710436476518400 y_3 + 25270462883582911013946898245894659689676800 y_1) + (-83672030144102400 y_21 - 296585165310787584000 y_19 - 39488966646743590502400 y_17 + 46814469241108612880793600 y_15 + 40616770239984775181893632000 y_13 + 17395087899171596560754540544000 y_11 + 5057642416977861097665639481344000 y_9 + 1089293528881165889042849670679756800 y_7 + 179299561114058347990502280962821324800 y_5 + 25948079614080437069430214256951820288000 y_3 + 2703619744445449867637451734005481629286400 y_1) + (38231681465843712000 y_17 + 68430286323525353472000 y_15 + 63841089249232759554048000 y_13 + 21938691556780676386652160000 y_11 + 12835221431674554545388650496000 y_9 + 6554197064530761535425270877056000 y_7 + 1565554943043148097209727667142656000 y_5 + 24093695850676635342388454580289536000 y_3 - 6352427533595639787169038748996337664000 y_1) + (-8360415632944005120000 y_1 - 14614962002458457407488000 y_11 - 7187626082715343446343680000 y_9 - 2094038045474735386483752960000 y_7 + 521469092290463481621513240576000 y_5 + 163838232275356326308374999203840000 y_3 - 46046162565198489484718500346855424000 y_1) + (-1504874813929920921600000 y_9 + 297191277539531811717120000 y_7 + 52147162832269848562630656000 y_5 + 4564858023007208627974963200000 y_3 - 13245392039557716554932153221120000 y_1) + (57271235775847276216320000 y_5 + 64193315948538871330897920000 y_3 + 3723212325015254537192079360000 y_1) + (-50151028084795932272640000 y_9 + 8121876480 y_31 + 6191736422400 y_29 + 5825185626193920 y_27 + 3275418660671324160 y_25 + 1725586416353673216000 y_23 + 735513954106589671587840 y_21 + 276712276788208871040614400 y_19 + 9068529236534955529999974400 y_17 + 26117364201220671992639992627200 y_15 + 6610067448745304620682703588556800 y_13 + 1457314732424068262855066678061957120 y_11 + 283585569552791661960985948163407872000 y_9 + 44647712070391519259137627678846935367680 y_7 + 6586082014676290450713765175650396709847040 y_5 + 580650495987787239736397256302239056868147200 y_3 + 63174773963471251683320021485683609387254415360 y_1) + (5417769369600 y_27 - 20284128519782400 y_25 - 15278629728131481600 y_23 - 9864602346821831884800 y_21 - 4211811325036045585612800 y_19 - 1377597757286126060568576000 y_17 - 328390679929068845025302937600 y_15 - 47881834368904565319839986483200 y_13 + 1111175131470115862959592413593600 y_11 + 3060689162129377601604391165073817600 y_9 + 1091804442778247898549795900429120307200 y_7 + 208265242279570196544148080331206741196800 y_5 + 33126424419056341850316259365622531070361600 y_3 + 1975824604402887135214129552695119012954112000 y_1) + (-2535516064972800 y_23 - 10092134097380966400 y_21 + 1170163288589834649600 y_19 + 3088972243914710423961600 y_17 + 2265879947536206046376755200 y_15 + 978559570552233129985179648000 y_13 + 306359649962614075731848095334400 y_11 + 72532668998462755336650830433484800 y_9 + 13628477512470857973363788676385996800 y_7 + 1879640603192097234435159987670587801600 y_5 + 250377699358159758345354493633472810188800 y_3 + 29475657230861523110895310663346083489382400 y_1) + (1415988202438656000 y_19 + 2638933914072121344000 y_17 + 3212437952004815742566400 y_15 + 827825925035845773950976000 y_13 + 344417330201496270026047488000 y_11 + 331892712336889825020107292672000 y_9 + 126779028020857209717000291483648000 y_7 + 26319977964082069016745804119094067200 y_5 - 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y15+11169748681227543549882610483200 y13+2952449307656173238440711893811200 y11+595012914149718773155958668394496000 y9+107673895939575584150811852193293926400 y7+12538026993852801354450091233174670540800 y5+1449689431553870975944560167011341041664000 y3+215664783828797278821363694479444910119321600 y)t13+(35399705060966400 y21+67405719388815360000 y19+112618779726057150873600 y17+23332689287367827757465600 y15–2609832114858618689421312000 y13+9985687307048149921428406272000 y11+3862142329784201786638901182464000 y9+1230327295567757517803105630866636800 y7+278374500086237270907047908849011916800 y5–32754133283347436956493877012873609216000 y3–1798201917255774288911513848006761145958400 y)t11+(–12441094691880960000 y17–25054705896822669312000 y15–21738991597799086227456000 y13–10219606089867328839745536000 y11–92967721376853159843987456000 y9+2378804033078865076716004442112000 y7 +283479874360598699257386642702336000 y5 +75092788009699864589631193928957952000 y3 –22828153705692387591171447220309327872000 y)t9+(–4180207816472002560000 y13+8805667482652794421248000 y11+1077593558189635717300224000 y9–560100833616598772924547072000 y7+600278830025447934578707660800000 y5–53522047348154919721280848527360000 y3–5081182175405359024928096765607936000 y)t7+(397716915110050529280000 y9+569616615284102639124480000 y7+322749727407931547524792320000 y5–66875170037055606399833210880000 y3–75487231183261006197370776453120000 y)t5+(–20896261701998330511360000 y5–2139771982846290443632640000 y3–6611911542699503747082485760000 y)t3–282099532976977461903360000 ty, b1 = (1244160 y35+358318080 y33+82564856320 y31+237762678620160 y29+239664780049121280 y27+69023456654146928640 y25 + 39757511032788630896640 y23 + 11450163177443125698232320 y21 + 4122058743879525251363635200 y19 + 1187152918237303272392726937600 y17+273520032361874673959284286423040 y15+78773769320219906100273874489835520 y13+11343422782111666478439437926536314880 y11+3266905761248159945790558122842458685440 y9+268819674068420018396480211251036600401920 y7+77420066131704965298186300840298540915752960 y5+27871223807413787507347068302507472967106560 y3+802691245653517080211595567112215272214526689280 y)t18+(1074954240 y31–6363729100800 y29–2427160677580800 y27–2576396385528053760 y25–949072528994520268800 y23–402544799206984887828480 y21–12165798376033210543718400 y19–28167401416510089217651507200 y17–4550752853242995877505453260800 y15–512850060678515013673658037043200 y13+285554913785797159613492795025653760 y11+42537835432918749294147892224511180800 y9+28585425410921399525667383574871513497600 y7+2430577886368630999668175243394789261967360 y5+661296398208313245255341319677550036988723200 y3+36232590949637923759551188793259717148572385280 y)t16+(–677221171200 y27–3315674854195200 y25+2858501803710873600 y23+1772321048463251865600 y21+1346475080680771210444800 y19+607246284137055351157555200 y17+220358390349892954062480998400 y15+60693192944882129801078164684800 y13+16044371689977153726387704561664000 y11+2503733996232510296754798536844902400 y9+494502336907680460544469247109942476800 y7+51215553861234174150154062238311461683200 y5+4091799465963320332102674048860179503513600 y3 + 952629719979963440192526748620860952674304000 y)t14 + (536359167590400 y23 + 1025128649038233600 y21+2438738927487339724800 y19+536959353108854262988800 y17–252932315501722171185561600 y15+16069588384342836722129216000 y13+120415641055580631405460193280000 y11+3385166546353670281227860862566400 y9+4902590155056264273213334005114470400 y7+170660909519651696941777155236508467200 y5–330965628585824328883117675452809242214400 y3–12146763983540814698442518920493616699801600 y)t12+(–230390642442240000 y19–480897679264579584000 y17–658823501306281328640000 y15–55559568640547393372160000 y13–10300785957704041254576128000 y11+19964901693725051525860350856000 y9+2233869356100003059234120507536000 y7–761742665751556831247155203145728000 y5+400442302253846396689131484338978816000 y3–22056199988076911867445426488170512384000 y)t10+(–99528757535047680000 y15+3497042424751435284848000 y13–41643469136712173617152000 y11+

N-Order Rational Solutions to the Johnson Equation Depending on $2N - 2$ Parameters

3388924028323804024406016000 y9+12622919304572790715547910144000 y7+24672631560938481961399398432768000
y5-3579993921618463612832957507567616000 y3-
56339784478414114970886795731927040000 y)8+(-13257230503668350976000 y11+17026583609035676712960000 y9+297191277539531811717120000
y7+
20490506450773607728822616064000 y5-7371789221354341213316768071680000 y3-2056184422601395518237715759890432000
y)6+(-1160903427888796139520000 y7-
408638006616856241111040000 y5+802416449356735891636224000000 y3-510465248422781104823300259840000
y)4+(-47016588829496243650560000 y3+
34604209378509235326812160000 y)2, b0 = (5760 y37+4299816960 y33+1426576071720960 y29+276093826616587714560
y25+34350489532329377094696960 y21+
2849167003769527853742544650240 y17+157547538640439812200547748979671040 y13+5600409876425417049926671067729929175040 y9+
116130099197557447947279451260447811373629440 y5+1070254994204689440282127422816287029619368919040 y)19+(5598720 y33-40131624960
y31-
6191736422400 y29-16643387503411200 y27-3714804090761379840 y25-2484844439549289431040 y23-775271465139378302484800 y21-
114501631774431256982323200 y19-80380145505650742401590886400 y17+9497223345898426179141815500800
y15-4239560501609057446368906439557120 y13+
1417927847763958309804929740817039360 y11-85075670865837498588295784449022361600 y9+65338115224963198915811162456849173708800 y7+
120968853307890082784160950629664701808640 y5+1083880925843869514174608211764179572820541440
y3+57136008805198264390061490020140323195825684480 y)17+
(-4031078400 y29-21671077478400 y27+29698044576399360 y25+10784915102210457600 y23+10461367649144143872000
y21+5118779545471536227942400 y19+
1422324957198013270327296000 y17+708994103947278343234545254400 y15+97791721639797857063350881484800
y13+42053704975638231121239959037542400 y11+
3901763261642142224029190345824665600 y9+964190936479491650667352223755586764800 y7+108420434951423308200924147701834097623040 y5+
2352172148098675160969201848446570253516800 y3 + 1923404767959545231626815911501166875875737600 y)15 + (3724716441600 y25 +
7021429103001600 y23+24322230412797542400 y21+7297792552495742976000 y19-1013350623003694596096000
y17-5313591349532200519335936000 y15+
5782475531648496516678549504000 y13-595653976725566916923050740940800 y11-78984251532623379970469104975872000
y9+9046675070369004841203327773442048000 y7+
4766896183201457342775448172409284198400 y5-1289406867642633128604210909109382912409600
y3-35147822358828640816774167204131302381977600 y)13+
(-1919922020352000 y21-4072428879740928000 y19-9168020408371249152000 y17+6345671487498997884518400
y15-5454621801754369877606400000 y13+
702746608663585937754472448000 y11+1096620914931491736708012048384000 y9+35365285434289264484579335471104000
y7-89745553058608866740546961198284800 y5+
1440082970385270158395631768017305600000 y3 - 896365244086607557391777037684691697664000 y)11 + (-1036757890990080000 y17 +
5414364409906593792000 y15-2484237788074790092800000 y13+2793912496995979487084544000 y11+488925881306813757445373952000 y9+
474895947165572792451399155712000 y7 + 179374817853821821017860345954304000 y5 - 43207297080876965830122143584616448000 y3 -
275883202703976214599374696519565312000 y)9+ (184128201439838208000 y13+171985693020562391040000 y11-670744202780193325056000000
y9+
176767484951767238867681280000 y7+194428715344934533487023620096000 y5-16117089443532754162876668706816000
y3-13110155381711314195003218151342080000 y)7+
(-24185488081016586240000 y9+26829768111207733002240000 y7-81044061385030325055258624000 y5+51868199286419408035365519360000 y3-
13176234440509157344218332528640000 y)5+ (-1959024534562343485440000 y5+2006041123391839729090560000
y3+3196960870312128581593989120000 y)3+
3918049069124686970880000 ty
182011033506237424016535060480000 y4-29291050584748975170609267867648000
y2+6390981197172791115357897764949196800)6+(-62344813719953866752000 y8-
47880816936924569665536000 y6+2817373311074761570782976000 y4+31908357580820388309544992768000
y2+6334379587045722980523177934848000)4+
(3173136036229376114688000 y4+1872305048499050413817856000 y2+674029817459658148974428160000)2-30647850496264218083328000, c5 =
(3857891328 y30+4385813299200 y28+3848783360163840 y26+2466550028005539840 y24+1302817744347023278080 y22+577974816639164721659904
y20+
220177096099583437905592320 y18+72754336829473620686568161280 y16+20953249006888402757731630448640
y14+5259562288958548418008737424343040 y12+
1145173671492696884932731450395983872 y10+214107105012357704780544390863372943360 y8+33621905126178979442094494014253637304320 y6+
4351518473982549047793023419626154969006080 y4+411294101324682628146614723214085998614937600
y2+30008017632648844549577010205699714458945847296)15+
(-767517327360 y26-5127376931389440 y24-4576099394062909440 y22-2301500882811711651840 y20-644920667248021827747840 y18+
3309812793479653522145280 y16-107482681746658620929306787840 y14+64828442276742069533246163517440
y12+24516369775685844799491161079152640 y10+
6817008146563092967755732091593031680 y8+1441441817906043951775861492352255262720 y6+224293498670693981278183006121402554122240
y4+
25035932301325273744565942174403182135869440 y2 +1175615639619717845452407083853595812707696640)13 + (570491114618880 y22 +
870189113498664960 y20+1401364906093471334400 y18+1552057132359106953216000 y16-1139537235755106386627788800
y14+572444467339481757524919582720 y12+
208068953111869602136819645808640 y10 + 55716351123401795157389755180646400 y8 + 10831008643984728202527402498824601600 y6 +
1467887649275016528376962812153639731200 y4+ 105351330124938220439351377277835615928320
y2+10105152064370792742320032003599757675069440)11+
(-376524135648460800 y18-375365599846465536000 y16-239069213070699213619200 y14+98688702972249466365542400
y12+135782386220508072513424588800 y10-
28645860797583522028859267481600 y8-31765763589136371131230062850867200 y6-3546935125910194648943728191445401600
y4+526087233594688164691588824637322035200 y2-
4664539516690996602286404806744054169600)19+(-140762099942424576000 y14+118287937755253466726400 y12+3219572173344927960268800 y10+
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y4-117075855902761828369534695228044800 y2+
216828838748133134846736519590313984000)17+(-6234481371995386675200 y10-9803184502172056289280000 y8+1248203365666033609211904000
y6+
3920071827257440409273499648000 y4+2439368830334592146617260834816000
y2+345345167495700170274520867169894400)15+(528856006038229352448000 y6+
345484860139705731121152000 y4+300638696358990380733038592000 y2+73847990667199117579064967168000)13+(-15323925248132109041664000
y2-
762295626888899070544128000)11, c4 = (100465920 y32+97462517760 y30+89780178124800 y28+56587517511598080 y26+30856840431324364800 y24+
14080785157445973442560 y22-5595869677864999798702080 y20+1946527740165331368699494400 y18+596668003176561280134886195200 y16+
161452796880273245045410863513600 y14 +38497944554933860359769263314042880 y12 +8034924470662430422227935197963223040 y10 +
1460465683196877059099077633041550540800 y8+222149591764874876313757952353287190609920
y6+29234139554940677000617222973550230293708800 y4+
263228248477968820138334228570150391135600640 y2+22506013224486633412182756542747858442093854720)16+(-22842777600 y28-
180592312320000 y26-158892340071628800 y24-86279320817683660800 y22-30391890758029069516800 y20-4820598212725621496217600 y18+

1733626268584748250247987200 y16+1747752907404918706578181324800 y14+814535296025569707770459770060800
y12+269759131916898897192344127484723200 y10+
68619338118785308829535445356380160000 y8+13838975794176233103696114270374304153600
y6+203058611222580941630544378329265726668800 y4+
237569386957966191257889386693103595605196800 y2+10899293685104089645885290165173278963295846400)t14+(19080719257600 y24+
30426192779673600 y22+55918661376304742400 y20+63738848254063804416000 y18+51355910711328618892492800
y16+2815666688531236287309414400 y14+
11488564162199475581042766643200 y12+3580156413174147343718366262067200 y10+855137409770433276315381522235392000
y8+151061011072977954307603322127148646400 y6+
20087163475838600643245899981393389158400 y4+1068890728758382332263348235817516951142400
y2+121118489282174749890218823305557379226009600)t12+
(-15688505652019200 y20-13902429623943168000 y18-12208860924299182080000 y16+6118929432390125223936000
y14+17529428280466945389625344000 y12+
3663866151630166772641746124800 y10-2607771514867798080912843866112000 y8-1067582876312442086797331480444928000
y6-67731786982523553783996470916022272000 y4+
1471764795737461018803356669136863232000 y2-26495754245279265966592368369342401740800)t10+(7331359372001280000
y16+5232368967556792320000 y14+
2327539712211611025408000 y12+1863866283828174819164160000 y10+95717605536435875358965760000 y8-1470087165987121498591403704320000
y6-
540990454022630308918568838758400000 y4-13242109098007181724937396552728576000 y2+5237739272366113455008487262503567360000)t8+
(-432950095277457408000 y12-1135105573935711780864000 y10-68106334436142706851840000 y8-7608096705012014379958272000 y6+
217572545521581081230856683520000 y4 + 103587949389930621646149292916736000 y2 + 1241172721927509188002765925777408000)t6 +
(55089167295648890880000 y8+30957424743701230387200000 y6+53717323415270374967869440000 y4+11126841431080071030688972800000 y2+
4618837469129268870000767139840000)t4+(-3192484426694189383680000 y4-2006041123391839729090560000
y2-1099310535618728171541626880000)t2+
6530081781874478284800000, c3 = (1969920 y34+1522851840 y32+1582332641280 y30+931237157928960 y28+539245755110522880 y26+
247334053010693160960 y24+102706903501370629816320 y22+37213030326690158519255040 y20+12022671336315281983143936000 y18+
3462529344858801211145453568000 y16+888940105176092690367673930874880 y14+203498904077234757425707509098741760
y12+40647264969233471547741319236755128320 y10+
7350537962808359878028755776395532042240 y8+1052877056767978405386214160733226684907520
y6+148388460085767850154857076610572203421859840 y4+
11845270118150859690622504028565676760110202880 y2+12709278056180687103350263145943408476730005913600)t17+(-507617280 y30-
4686800486400 y28 -3900793946112000 y26 -2221892231705395200 y24 -913122811987152076800 y22 -219908732900112114647040 y20 -
5367263989426465171046400 y18+2799564896884842332178022400 y16+17362111429220560358743631462400
y14+6823754974028019209713394437324800 y12+
20587852835878306708915328239062220800 y10+47855064862033592959163787525750784000 y8+96305659420128048401950827996293313331200 y6+
12838939641705268586956893422770862633779200 y4+1657273290631809413414300502362640641477836800
y2+75020044081622111373942525514249286147364618240)t15+
(507915878400 y26 +774741019852800 y24 +1638333457367040000 y22 +1776814763089172889600 y20 +1618902036162607369420800 y18 +
930661212166593117054566400 y16+41645674746757828551832268800 y14+145512795717691120898387686195200
y12+40320164926782209017728479526912000 y10+
8956669884719925456303600787940966400 y8+1412758319152318878467099267803643904000 y6+204564450596906265355557213707674268467200
y4+
7074892789203046382602677434780699590656000 y2+1056713337533329816065413930414621686392422400)t13+(-475409262182400 y22-
347560740598579200 y20 -507228038400835584000 y18+128286565140793393152000 y16+1065345992901263477440512000
y14+623080309268528915241605529600 y12-
17379114921539063400405506457600 y10 - 83318101654763435229247803752448000 y8 - 20139301351557360515081005078413312000 y6 -
670100146283761287166275064361385984000 y4+255274867662374529357843269725819659878400
y2+136291224727071672738449846687333823283200)t11+
(271531828592640000 y18+142657885800235008000 y16+175261610982858817536000 y14+108807067181482908844032000
y12+167630556644767029799157760000 y10-
49463497292156682061700136960000 y8-50354157609735197125696303398912000 y6-10981922787621579347097965027131392000
y4+369964584741586021642156625306320896000 y2+
87082094158230052876291236787346472960000)t9+(-20616671203688448000 y14-80737728112430678016000 y12-7017016275238945554432000 y10-
3166077433878122341597184000 y8-16227647599751459338318774272000 y6+6741747517018886278483663650816000
y4+2574562395921257318496747820941312000 y2+
29083069851938342078017879063599104000)t7+(3672611153043259392000 y10+1160903427888796139520000 y8+4829358260017391940403200000
y6+
5470697036947701590088744960000 y4-798051303872235248385722941440000
y2+164643838419256638487040020709376000)t5+(-354720491854909931520000 y6-
139308411346655536742400000 y4+163158011369202964632698880000 y2-111182823222869325145115197440000)t3+(2176693927291492761600000
y2-
1379153272331889813749760000)t, c2 = (27360 y36 + 14929920 y34 + 20424130560 y32 + 9906778275840 y30 + 6776236340674560 y28 +
2875977360589455360 y26+1311445676428791644160 y24+477090132393463570759680 y22+163164825278564541199810560
y20+4946470492655430301636622400 y18+
13533543267905257305277087088640 y16+3282240388342496087511411437076480 y14+748350808542089107952601807653437440
y12+13612107338539997741273255118435778560 y10+
26601946913020730987151687571717163581440 y8+3225836088821040220757762535012439204823040
y6+55161797118839787749577393487127104024739840 y4+
33445468568896545008816481963008969675605278720 y2 + 5083711222472274841340105258377363390692002365440)t18 + (-7931520 y32 -
84563066880 y30 -63465298329600 y28 -37447621882675200 y26 -17825068016153395200 y24 -4762618509136138076160
y22 -909453064875039931760640 y20+
200377855605254699719065600 y18+221045400140539541604374937600 y16+106843762641357294515345424384000
y14+35865314243450816622911152057221120 y12+
10752619512210017182687383867862548480 y10+2105622853929478090060320665113303449600 y8+473701335380983192139630927812156509388800
y6+
51915799554463616052820240797856443452620800 y4+8012976844631463908362282136970898984780431360
y2+361280738603601220563986372871253141183361187840)t16+
(9069926400 y28+13544423424000 y26+33351788239257600 y24+30107887993670860800 y22+35218140266268249292800
y20+19139514295628401842585600 y18+
9627529781033721900564480000 y16+3641838900217560559579771699200 y14+1136884411855692461952347406336000
y12+278349370433264023671377899605196800 y10+
63520582815519512563591937152096665600 y8+7890768472806427994064434007646824038400 y6+1478530083977389887965992437939559622246400
y4+
25579872110573092375540070101856451506995200 y2 + 6341792135866613758995963783676017199231795200)t14 + (-9904359628800 y24 -
5266071827251200 y22 -15657786899693568000 y20 -3999406017069711360000 y18+28283547704697947160576000
y16+36231715560623353137148723200 y14+
7293532097119708109213584588800 y12-1799649626990361929492849491968000 y10-134685243865145480016772292517632000
y8-207658118069337514186600875779162112000 y6-

1017584781996853205770944109308228403200 y4+2758748779166512566722667402328448119603200
y2+3782464326695135187235630576603534196736000)t12+
(6788295714816000 y20+1869455498674176000 y18+9367077923441344512000 y16-1639414944687011659776000
y14+11757278302463107484614656000 y12-
550802048754917422015709184000 y10-1758409395365365376116687110144000 y8-99862190526261103801858321809408000
y6-115792608891557806583758139003240448000 y4+
1408608493113690134914657006259273728000 y2+944828467064596160591508187083914084352000)t10+(-6442709751 15264000
y16-3514786980380540928000 y14+
5159570796168717312000 y12-1684083906057346933063680000 y10-3372949289557774333498097664000 y8-306088946636043362534481199104000
y6+
40812021857535488594237598990336000 y4+37179786344286775078009817977061376000 y2+4264415405846672431756069472613433344000)t8+
(153025464710135808000 y12-12898926976542179328000 y10+221345586917463797268480000 y8+750110784509778292774010880000) y6+
252536504941551919815752417280000 y4-89898853150536604412577973272576000
y2+4771925825949085060566951338704896000)t6+(-22170030740931870720000 y8+
4643613711555184558080000 y6+29756276663645622648176640000 y4+20156701207841205597901946880000
y2-6158963505366613470971670036480000)t4+
(272086740911436595200000 y4+104481308509991652556800000 y2-298398617104536159702220800000)t2-3265040890937239142400000, c1 =
(240 y38 + 69120 y36 + 179159040 y34 + 51597803520 y32 + 59440669655040 y30 + 17118912860651520 y28 + 11503909442357821440 y26 +
3313125919399052574720 y24+1431270397180390712279040 y22+412205874387952525136363520 y20+11871529182373032729272693760 y18+
34190004045234334244910535802880 y16 + 6564480776684992175022822874152960 y14 + 1890570463685277746406572987756052480 y12 +
233350411517725710413611294488747048960 y10+67204918517105004599120052812759150100480
y8+48387541332315600331136643802518658807234560 y6+
1393561190370689375367353415125373736483553280 y4+44593958091862060011755309284011959567473704960
y2+128430599045627328338529073795444355432427028480)t19+
(-77760 y34-948049920 y32-601974374400 y30-371504185344000 y28-214699698794004480 y26-44577649089136558080
y24-2236359955943604879360 y22+
119272533083658926899200 y20+1116390909800704755577651200 y18+816167631288145999769999769600
y16+438771718580507289476351876136960 y14+
86979370291076146319052403082526720 y12+38993015813508853519635567872468582400 y10+4423934885023549926591380791349162803200 y8+
1551313535769840522829687885761190381486080 y6+111291345064325887616142807457929152566394880
y4+23980865484295613001113206685282473048654479360 y2+
1086977728489137712786535663797791514457171558400)t17+(100776960 y30+145118822400 y28+422586010828800 y26+234047636766720000 y24+
514979696130549350400 y22+1874504524084995214540800 y20+143350675717598507276697600 y18+48240968736965068957365043200 y16+
16787084234449366586178404352000 y14+553984902954375720620660254310400 y12+90682147729175024717762141099786240
y10+302638975007119851748989691555636838400 y8+
19014412438514680934484107824356497817600 y6+6799247615597732887176599093165867139072000
y4+19657438666253213845242615447732051404390400 y2+
23429247513107215123363629291795345944629739520)t15+(-126978969600 y26-36569943244800 y24-306602404164403200
y22-264155524760390860800 y20+
20113866665622503424000 y18+976199092576883282490163200 y16+330891824948141577795010560000 y14+12530028066707786882832610099200
y12-
19076286046097836452624204614860800 y10-11657402410510570332685793218265088000 y8-1005288688942025129803104484212080640000 y6+
393368631659161340976326334616714444800 y4+17113992102713602891018756591334236304179200
y2+28079055017927934734069847065830932401356800)t13+
(102852965376000 y22-3730134210969600 y20+280260342646308864000 y18-335447932824409079808000 y16+335075448819066486128640000 y14-
1920474146216657069015040000 y12-103525473148950340129206671769600 y10-9054035047476789172334890057728000
y8-12281496861304233544442647469359104000 y6-
478325635656213231096914470061998080000 y4+176802863420658346919735085014244655104000
y2+6018922927957413120437426621226377910681600)t11+
(-11930943983616000 y18-86376743146487808000 y16+20064997519065612288000 y14+10181553026817293549568000
y12-155899939946626398830985216000 y10-
81240524632235791551591088128000 y8+3532834921165738901362383519744000 y6-2768098185909537159057208441307136000
y4+287555750217841197328558039951736832000 y2+
35491883326517857046563034080311508992000)y9+(3643463445479424000 y14-2328972926320115712000 y12+4385635172024340971520000 y10+
18995475822735074965585920000 y8+25344947654615336154134740992000 y6+2996874940684462536351843090432000
y4-2505338697728524963401994050994176000 y2+
104829056553375375606485906180014080000)t7+(-739001024697729024000 y10+870677570916597104640000 y8-984446106849699126312960000 y6+
3075929722534154251272192000000 y4-199684008143921582419979796480000
y2-153112459270177668627739906670592000)t5+(15115930050635366400000 y6+
46145911258579646545920000 y4-140506463684236774358384640000 y2-25958172136690406094431846400000)t3+(-544173481822873190400000 y2-
783609813824937394176000000)t, c0 = (y40 +829440 y36 +309586821120 y32 +68475651442606080 y28 +932937758197157724160 y24 +
98921098531086060327272448 y20+683800089046866848921071605760 y16+32409779374726190422411269790103755680
y12+10080737775657506898680079219138725150720 y8+
1858081587160919167156471220167164981978071040 y4+15411671916547527940062634888554533226518912434176)t20+(-360 y36-4976640 y34-
2418647040 y32-1651129712640 y30-1277974397583360 y28-293349690780124446720 y24+79515022065577261793280
y22-36497395128099963163115520 y20+
16488234975518101005454540800 y18-2552378774210202035644362915840 y16+1641120194171248043755705718538240
y14-88620490485247394362808108801064960 y12+
9074738225689331827515503412290519040 y10-350025617276588565620416941733120573440 y8+2688196740684200183964802112510366004019200
y6+
70161934931857624801481335136520552704901120 y4+33445468568896545008816481963008969675605278720
y2+1538491554169241070405558170298412605077842821120)t18+
(524880 y32+716636160 y30+2515392921600 y28+4175409839918284800 y24-103535184981220392960 y22+1492770297059235625697280 y20-
28625407943607814245580800 y18+236696341933707114043146240000 y16-3561458754711909817178180812800
y14+19432743549210064726451025786961920 y12-
236321307906659718300821623469506560 y10+82151194679824345743231168585872179200 y8-8167264403120399864476395307106146713600 y6+
149320928330192682093669867343349233662950400 y4-11613009919755744794279451260447811373629440
y2+40195530584754571670752100067522498711697489920)t16+
(-755827200 y28-2779315686604800 y24-4069308244584038400 y22-9395009168413084876800 y20+10769729941746545275699200 y18+
4318970241491608692326400000 y16+819259175346055643708522496000 y14-17353764260264203420109620101200
y12-30514578610371643313582653204070400 y10-
4629266715217649696675979542722969600 y8-1265500604129332791500899793679207628800 y6+253674381604210961415650955280872426700800
y4+
46749421443461168824262886737875583788646400 y2+78377736220573711613732761592880358804684800)t14+(714256704000 y24-292559545958400
y22+
3515102944690176000 y20-8241472623939158016000 y18+3734971133050075742208000 y16+1388779557264097863008256000
y14-2460124362217543653021607526400 y12-
849704701228349817215094305587200 y10 + 7583194317625521980334714388480000 y8 - 77027514863609562478715068873703424000 y6 +

281577761565235275682453576948383744000 y4+812387581097882274019636445703714530918400
y2+17077558446665307982248318921643942477824000)t12+
(-99424533196800) y20-921562569768960000 y18+51612769978889012000 y16+1742788355941698895872000 y14-1770000314481841211965440000
y12-
1692378844826005865408495616000 y10-762246513907490849854442176512000 y8+8550839878891125976681238495232000
y6-43296695260399539003892405263925248000 y4+
912652582481664962883231632980967424000 y2+127960954009264970947967361215101717708800)t10+(37952744223744000
y16-51186218160881664000 y14+
15048748139299209216000 y12-700050564870897156489216000 y10+1265054111102525062936264704000 y8-149575181220536202709979627520000
y6+
18822613842520643864361883926528000 y4-2672822263894656317122421321839412000 y2+986879996113586319150766693854216192000)t8+
(-10263903120801792000 y12+25797853953084358656000 y10-141243250393136863641600000 y8+117093363350575533816545280000 y6-
18933461909488492660874280960000 y4-22017588405210209231139080503296000
y2-1713239420935328354202115104571392000)t6+(31491520938823680000 y8+
2321806855777592279040000 y6+3507089255652053137489920000 y4-8516313249172823596565790720000
y2+184989088234701892457815080960000)t4+
(-2267389507595304960000 y4 + 104481308509991652556800000 y2 - 93155534667508557419642880000)t2 + 81626022273430978560000

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