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Muon as a Composition of Massless Preons: A Confinement Mechanism beyond the Standard Model

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Abstract: *The observed parameters of muon namely its mass and magnetic moment are evaluated within the framework of a massless preon gauge model with a confinement mechanism. The radius of muon is also estimated.*

1. Preliminary Remarks

The present paper continues a discussion (initiated in [1]) on the puzzle of origin of masses for fundamental fermions in particle physics. Namely the question is how to find a possible solution for this problem beyond the standard model. One of the ways is the consideration of composite models from light preons with a confinement mechanism to generate masses of the observed fundamental fermions (leptons and quarks).

In the previous paper [2] we borrowed a confinement mechanism proposed earlier by us in quantum chromodynamics (QCD) to analyse τ-lepton as a possible composition of massless preons. In the present paper we want to consider muon from this point of view. Under the circumstances we refer for more details of the general ideology to [2] and here we would like just to make several remarks about the choice of a gauge group to insert dynamics into the static preon model [3] underlying our considerations.

One should say that our choice of gauge group as SU(3) is dictated only for reasons of convenience so long as we can under this situation use the main relations of QCD based on the gauge group SU(3). Though we might write out the similar formulas for any group SU(N) with $N > 2$ (see, e. g.,[4]) or even for any semisimple compact Lie group [1] but it would only entail some modification of the relations used in the paper without changing merits of case. It is clear, however, that at present there exist no explicit physical considerations for choice of a gauge group for describing a preon interaction because the existence of preons themselves is the open question.

Section 2 contains a short review of the main relations necessary to us to obtain for numerical results for muon μ^+ in section 3. Section 4 is devoted to concluding remarks. At last Appendix gives a concise description of the compatible nonperturbative solutions for the Dirac-Yang-Mills (DYM) system directly derived from the QCD-Lagrangian which allowed one to build a confinement mechanism.

Throughout the paper we employ the Heaviside-Lorentz system of units with $\hbar = c = 1$, unless explicitly stated otherwise, so the (preon) gauge coupling constant g and the (preon) strong coupling constant α_{ps} are connected by relation $g^2/(4\pi) = \alpha_{ps}$. Further we shall denote $L_2(F)$ the set of the modulo square integrable complex functions on any manifold F furnished with an integration measure, then $L_2^n(F)$ will be the n-fold direct product of $L_2(F)$ endowed with the obvious scalar product while $\dagger$ and $*$ stand, respectively, for Hermitian and complex conjugation.

When calculating we apply the relations $1\ GeV^{-1} \approx 0.1973269679\ fm$, $1\ s^{-1} \approx 0.658211915 \times 10^{-24}\ GeV$.

2. Specification of Main Relations

If the interaction among preons is decribed by a QCD-like theory based on, e.g., SU(3)-group then such a theory should also manifest the similar confinement mechanism to generate masses for particles composed from preons. This signifies that preons might possess small masses or be just massless and, as a result, mass paradox would be removed. Recently we have explored one such a possibility by the example of τ-lepton [2] and here we shall also adduce our results for muon μ^+ when taking into account the constructions of Ref. [3]. Clearly, the results obtained will hold true also for μ^- with obvious changes.

It should be noted, however, that in accordance with Ref. [3] the main building blocks of that model are a trinity of preons $\alpha,\ \beta,\ \delta$ in notations of [3] so muon is represented by combination $\alpha\alpha\delta$, i.e., we deal with three-body problem. In this situation it is natural to use the confinement mechanism for baryons [13] for modelling muon when replacing a trinity of quarks by that of preons. Besides we take that an interaction law for a pair of preons is given by (A.1) (r is now a distance between preons) while the parameters $a_j,\ b_j,\ B_j$ in (A.1) now describe a pregluonic field and we may put $A_j = 0$ as in QCD.

Inasmuch as in Ref. [2] all the details of our approach can be found we shall here restrict ourselves to the main relations necessary for numerical computation.

2.1. Interaction Energy of Two Preons

This is the quantity ω_j of (A.2) for j-th colour component of a di-preon and is quantized according to relation (at $j = 1,2,3$)

$$\omega_j = \omega_j(n_j, l_j, \lambda_j) =$$

$$\frac{\Lambda_j g^2 a_j b_j \pm (n_j + \alpha_j)\sqrt{(n_j^2 + 2n_j\alpha_j + \Lambda_j^2)\mu_0^2 + g^2 b_j^2 (n_j^2 + 2n_j\alpha_j)}}{n_j^2 + 2n_j\alpha_j + \Lambda_j^2} \quad (1)$$

with the gauge coupling constant g while μ_0 is a mass parameter and one should consider it to be the reduced mass which is equal to $m_{q_1}m_{q_2}/(m_{q_1} + m_{q_2})$ with the current preon masses m_{q_k}, $a_3 = -(a_1 + a_2)$, $b_3 = -(b_1 + b_2)$, $B_3 = -(B_1 + B_2)$, $\Lambda_j = \lambda_j - gB_j$, $\alpha_j = \sqrt{\Lambda_j^2 - g^2 a_j^2}$, $n_j = 0,1,2,\ldots$, while $\lambda_j = \pm(l_j + 1)$ are the eigenvalues of Euclidean Dirac operator $\mathcal{D}_0$ on a unit sphere with $l_j = 0,1,2,\ldots$.

In line with the above we should have $\omega = \omega_1 = \omega_2 = \omega_3$ in energy spectrum $\varepsilon = m_{q_1} + m_{q_2} + \omega$ for the premeson and this at once imposes two conditions on parameters a_j, b_j, B_j when choosing some value for ε at the given current preon masses m_{q_1}, m_{q_2}.

2.2. Chiral Limit

There is an interesting limit for relation (1) – the chiral one, i.e., the situation when $m_{q1}, m_{q2} \to 0$ which entails $\mu_0 \to 0$ and (1) reduces to (at $j = 1,2,3$)

$$(\omega_j)_{chiral} = \frac{\Lambda_j g^2 a_j b_j + (n_j + \alpha_j) g |b_j| \sqrt{n_j^2 + 2n_j\alpha_j}}{n_j^2 + 2n_j\alpha_j + \Lambda_j^2}, \quad (1')$$

which mathematically signifies that the Dirac equation in the field (A.1) possesses a nontrivial spectrum of bound states even for massless fermions. In our situation when we know nothing about the possible preon masses it is natural to put $m_{q_k} = \mu_0 = 0$. Then we obtain the lepton mass as

$m = 3(m_q + \omega) = 3\omega$ (2)

with using $(1')$ at $n_j = 0$ (the ground state), where ω obeys the equations

$$\omega = \frac{g^2 a_1 b_1}{\Lambda_1} = \frac{g^2 a_2 b_2}{\Lambda_2} = \frac{g^2 a_3 b_3}{\Lambda_3}.$$

2.3. Lepton Radius and Magnetic Moment

They are given by expressions

$$< R >= \frac{1}{\sqrt{3}} \sqrt{\sum_{j=1}^{3} \frac{2\alpha_j^2 + 3\alpha_j + 1}{6\beta_j^2}}, \quad (3)$$

$$\mu_a = F_M(Q^2=0) = \frac{2}{3}\frac{|q|}{q}\sum_{j=1}^{3}\frac{2\alpha_j+1}{6\beta_j}\cdot\frac{\mathcal{P}_j\mathcal{Q}_j}{\mathcal{P}_j^2+\mathcal{Q}_j^2},$$
$$\mathcal{P}_j = P_j(1-gb_j/\beta_j)\,,\mathcal{Q}_j = O_j(1+gb_j/\beta_j)(4)$$

while q is electric charge of ith preon pair in a lepton, $i=1,2,3$, $\beta_j=\sqrt{\mu_0^2-\omega_j^2+g^2b_j^2}$, $P_j = gb_j+\beta_j$, $Q_j=\mu_0-\omega_j$ and the definition of magnetic form factor $F_M(Q^2)$ can be found in [13]. It should be noted that, as follows from (3), at $|b_j|\to\infty$ we have $<R>\sim\sqrt{\sum_{j=1}^{3}\frac{1}{(g|b_j|)^2}}$, so in the strong magnetic colour field when $|b_j|\to\infty$, $<R>\to 0$, while the wave functions of (A.2) behave as $\Psi_j\sim e^{-g|b_j|r}$ [13], i. e., just the magnetic colour field of (A.1) provides two preons with confinement.

3. Numerical Results

Now we should consider the equations (2)–(4) as a system which should be solved compatibly at $m=105.658367$ MeV, $m_q=0$ MeV, $\mu_a=0.4042487656\times10^{-3}$ 1/MeV [5], while the possible value of $<R>$ can be estimated from the following considerations. Classical radius of muon $r_\mu=m_er_e/m\sim0.0136$ fm, where classical electron radius $r_e\sim2.818$ fm and electron mass $m_e\approx 0.511$ MeV [5]. On the other hand, the Compton wave length for muon is $\lambda_c\approx1.867594337$ fm. But, as is known [6], from the point of view of quantum electrodynamics (QED) the photon condensate [huge number of (vitrual) photons] described by Coulomb law exists only at $r>>\lambda_c$ while at $r<\lambda_c$ one may only speak about single photons rather than about condensate, i.e., the field in classical sense. It is clear, however, the preon confinement should be realized at the scales $r<\lambda_c$ on the analogy of QCD. Indeed, e.g., for charged pions with masses of order 140 MeV we have $\lambda_c\sim1.41$ fm while the radius of pions (radius of confinement) is of order 0.6 fm [5]. So that it is reasonable to take $<R>\sim0.7-0.8$ fm for muon. Concerning a value of the gauge coupling constant g then the considerations similar to the ones of [2] give for muon $g\approx6.16465$.

While computing for distinctness we took all eigenvalues λ_j of the Euclidean Dirac operator $\mathcal{D}_0$ on the unit 2-sphere $\mathbb{S}^2$ equal to 1. The results of numerical compatible solving of equations (2)–(4) are gathered in Tables 1 and 2.

Table1. *Gauge coupling constant, reduced mass μ_0 and parameters of the confining SU(3)-pregluonic field for muon*

Particle	g	μ_0 (MeV)	a_1	a_2	b_1 (GeV)	b_2 (GeV)	B_1	B_2
$\mu^+=\alpha\alpha\delta$	6.16465	0	0.0163468	-0.00513040	0.103875	-0.0136027	-0.135	0.150

Table2. *Theoretical and experimental muon mass, magnetic moment and radius*

Particle	Theoret. (MeV)	Experim. (MeV)	Theoret.μ_a (1/MeV)	Experim.μ_a (1/MeV)	Theoret.$<R>$ (fm)	Experim. $<R>$ (fm)
$\mu^+=\alpha\alpha\delta$	$m=05.658$	105.658367	0.390919×10^{-3}	0.40424×10^{-3}	0.754968	–

4. Concluding Remarks

Of course, the Tables 1 and 2 correspond only to one of possible solutions for system (2)–(4). Further specification of such solutions requires knowledge of the exact preon masses, if any, and the (preon) gauge coupling constant g. At present the given knowledge is absent as well as the existence of preons themselves is the open question. But our results shows that the mechanisms of generating masses dynamically for fundamental leptons can be realized. The similar considerations can be conducted for both electron and quarks and also for all types of neutrinos if the latter would possess masses.The possibility of generating massive particles from *massless* ones should be specially emphasized. As is clear from the above, this remarkable feature is obligatory to the nonzero chiral limit for the preon interaction energy.

Finally, one should make a remark about the role of gravity. In accordance with [3] the preons are supposed to be the spin-1/2 particles. On the other hand, quanta of gravitational field, gravitons, should be the spin-2 particles when being massless themselves. Moreover, the effective dimensionless interaction constant of gravitons at the mass scale $m \leq m_P$ is (in usual units) $\alpha_g \sim Gm^2/(\hbar c) = (m/m_P)^2$ with the Planck mass $m_P \sim 1.22 \times 10^{19}$ GeV and at the muon mass $m = 105.658$ MeV we have $\alpha_g \sim 7.5 \times 10^{-41}$. At the same time, according to the Table 1 the preon strong coupling constant $\alpha_{ps} = g^2/(4\pi) \sim 3.024$ at the mass scale of muon. As a result, massless preons cannot be identified with gravitons but, as we have seen above, preons could dynamically generate masses even being massless themselves.

It should also be noted that so far gravitons have not been detected (see, e.g., recent review [7]). Under the situation, then perhaps the gravity is a statistical concept like entropy or temperature, only defined for gravitational effects of matter in bulk and not for effects of individual elementary particles [7]. As a consequence, then the gravitational field is not a local field like the electromagnetic field which implies that the gravitational field at a point in spacetime does not exist, either as a classical or as a quantum field [7].

APPENDIX

Let us write down arbitrary SU(3)-Yang-Mills field as the SU(3)-algebra Lie valued 1-form $A = A_\mu dx^\mu = A_\mu^a \lambda_a dx^\mu$ (λ_a are the known Gell-Mann matrices, $\mu = t, r, \vartheta, \varphi$, $a = 1, \ldots, 8$ and we use the ordinary set of local spherical coordinates r, ϑ, φ for spatial part of the flat Minkowski spacetime).

In fact in [?, ?, ?] (see also Appendix C in [1]) the following uniqueness theorem was proved:

The unique exact spherically symmetric (nonperturbative) solutions (depending only on r and r^{-1}) of SU(3)-Yang-Mills equations in Minkowski spacetime consist of the family of form:

Electric colour field part:

$$\mathcal{A}_{1t} \equiv A_t^3 + \frac{1}{\sqrt{3}} A_t^8 = -\frac{a_1}{r} + A_1,$$

$$\mathcal{A}_{2t} \equiv -A_t^3 + \frac{1}{\sqrt{3}} A_t^8 = -\frac{a_2}{r} + A_2,$$

$$\mathcal{A}_{3t} \equiv -\frac{2}{\sqrt{3}} A_t^8 = \frac{a_1 + a_2}{r} - (A_1 + A_2),$$

Magnetic colour field part:

$$\mathcal{A}_{1\varphi} \equiv A_\varphi^3 + \frac{1}{\sqrt{3}} A_\varphi^8 = b_1 r + B_1,$$

$$\mathcal{A}_{2\varphi} \equiv -A_\varphi^3 + \frac{1}{\sqrt{3}} A_\varphi^8 = b_2 r + B_2,$$

$$\mathcal{A}_{3\varphi} \equiv -\frac{2}{\sqrt{3}} A_\varphi^8 = -(b_1 + b_2) r - (B_1 + B_2) (\mathrm{A}.1)$$

with the real constants a_j, A_j, b_j, B_j parametrizing the family.

Now, having postulated interaction between two quarks as (A.1) (r is distance between quarks) and inserting solution (A.1) into the Dirac equation $\mathcal{D}\Psi = \mu_0 \Psi$ (where the explicit form of the Dirac operator $\mathcal{D}$ in arbitrary curvilinear coordinates on Minkowski spacetime can be found in [?, ?, ?]), we obtain that the meson wave functions $\Psi = (\Psi_1, \Psi_2, \Psi_3)$ are given by the unique nonperturbative modulo square integrable solutions of the mentioned Dirac equation in the confining SU(3)-field of (A.1) with the four-dimensional Dirac spinors Ψ_j representing the jth colour component of the meson, so Ψ may describe the relative motion (relativistic bound states) of two quarks in mesons and is at $j = 1,2,3$ (with Pauli matrix σ_1)

$$\Psi_j = e^{-i\omega_j t} \psi_j \equiv e^{-i\omega_j t} r^{-1} F_{j1}(r) \Phi_j(\vartheta, \varphi) F_{j2}(r) \sigma_1 \Phi_j(\vartheta, \varphi), (\mathrm{A}.2)$$

with the 2D eigenspinor $\Phi_j = \Phi_{j1} \Phi_{j2}$ of the Euclidean Dirac operator $\mathcal{D}_0$ on the unit sphere $\mathbb{S}^2$. An explicit form of Φ_j can be found, e.g., in [?, ?, ?].

The relations (A.1)–(A.2) give grounds to build a confinement mechanism for mesons and quarkonia which was successfully applied to the description of both the heavy quarkonia spectra (see [10] and references to early papers therein) and all the mesons from pseudoscalar nonet [1] (and references therein). Recently in the papers [?, ?] this confinement mechanism has been extended over vector mesons while [13] does so over baryons.

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