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Trans-Planckian Effects on Scalar Field in Conformal Inflation

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Abstract: *The possible effects of trans-Planckian physics on the scalar field in conformal inflation are investigated in the lattice Schrödinger picture. For the massless conformally coupled scalar field during the slow-roll inflation, we consider the Corley-Jacobson type dispersion relations with quartic or sextic correction to obtain the time evolution of the vacuum state wave functional. We then calculate explicitly the finite vacuum energy density due to fluctuations of the inflaton field, and evaluate the corresponding cosmological constant through the backreaction constraint on the magnitude of dispersion parameters. We also show how the cosmological constant reduces significantly during the slow-roll inflation at the grand unification phase transition. Finally, using the current astronomical observations which indicate that radiation and matter dominated epoch is sandwiched between two asymptotic de Sitter epochs, and knowing the fact that de Sitter geometry is invariant under time translation, we propose the possibility that similar reduction mechanism may reappear during the late time acceleration era, and thus yield a tiny current value of the cosmological constant.*

Keywords: *Trans-Planckian physics, conformal inflation, Schrödinger picture, cosmological constant.*

1. INTRODUCTION

Usually standard inflation is realized through a slow rolling scalar field (inflaton) minimally coupled to gravity [1]. Nevertheless, it is well known that the extension to the non-minimal coupling with the Ricci scalar curvature can soften the problem related to the small value of the self-coupling in the quartic potential of chaotic inflation [2]. Moreover, non-minimal coupling terms also can lead to corrections on power spectrum of primordial perturbations [3], a tiny tensor-to-scalar ratio [4] and non-Gaussianities [5]. It was even pointed out that inflation with a conformally coupled inflaton can be realized as the rapid roll inflation [6, 7]. Recently, models of chaotic inflation were also proposed in supergravity with an arbitrary inflaton potential, where the inflaton field is non-minimally coupled to gravity [8, 9].

However, the standard inflationary scenario suffers from several problems. One of these problems is the so-called trans-Planckian problem [10, 11] of whether the predictions of standard cosmology are insensitive to the effects of trans-Planckian physics. In fact, nonlinear dispersion relations such as the Corley-Jacobson (CJ) type were used to mimic the trans-Planckian effects on cosmological perturbations [10-12]. These CJ type dispersion relations can be obtained naturally from quantum gravity models such as Horava gravity [13, 14]. Moreover, in several approaches to quantum gravity, the phenomenon of running spectral dimension of spacetime from the standard value of 4 in the infrared to a smaller value in the ultraviolet is associated with modified dispersion relations, which also include the CJ type dispersion relations [15, 16]. These recent research results suggest that spacetime becomes effectively two-dimensional at super-Planckian energies, and all particles are conformally coupled to gravity [17].

In our previous work [18-22] we used the lattice Schrödinger picture to study the free scalar field theory in de Sitter space, derived the wave functionals for the Bunch-Davies (BD) vacuum state and its excited states, and found the trans-Planckian effects on the quantum evolution of the vacuum state wave functional of massless minimally coupled scalar field for the CJ type dispersion relation with sextic correction. In this paper we try to extend the study to the case of massless conformally coupled scalar field for the CJ type dispersion relations with quartic or sextic correction.

The organization of the paper is as follows. In Section 2, the theory of a generically coupled scalar field in de Sitter space is briefly reviewed in the lattice Schrödinger picture. In Section 3, we consider the massless conformally coupled scalar field during the slow-roll inflation, and use the CJ type dispersion relations with quartic or sextic correction to obtain the time evolution of the vacuum state wave functional. In Section 4, using the results of Section 3, we calculate the finite vacuum energy density and use the backreaction constraint to address the cosmological constant problem. Finally, conclusions are presented in Section 5. Throughout this paper we will set $\hbar$ =c=1.

2. De Sitter Scalar Field Theory in Schrödinger Picture

In this section, we review briefly the theory of a generically coupled scalar field in de Sitter space in the lattice Schrödinger picture (for the details see [18, 19]). We consider the following Lagrangian density for the scalar field

$$L = |g|^{\frac{1}{2}} \left\{ \frac{1}{2} \left[g^{\mu\nu}(x)\phi(x)_{,\mu}\phi(x)_{,\nu} \right] - V(\phi) \right\},\ V(\phi) = m^2\phi^2/2 + \xi R\phi^2/2, \tag{1}$$

where ϕ is a real scalar field, $V(\phi)$ is the potential , m is the mass of the scalar quanta, R is the Ricci scalar curvature, ξ is the coupling parameter, and g = det $g_{\mu\nu}$, μ,ν=0,1,…,d. For a spatially flat (1+d)-dimensional Robertson-Walker spacetime with scale factor $a(t)$, we have

$$ds^2 = dt^2 - a^2(t)d^2x^i,\ i = 1,2,...,d\ ,\ L = a^d \left\{ \frac{1}{2} \left[(\partial_0\phi)^2 - a^{-2}(\partial_i\phi)^2 \right] - V(\phi) \right\}. \tag{2}$$

In the (1+d)-dimensional de Sitter space we have $a(t) = \exp(ht)$, where $h \equiv \dot{a}/a$ is the Hubble parameter which is a constant.

For d=1, in the lattice Schrödinger picture, from (2) we can obtain the time-dependent functional Schrödinger equation in momentum space

$$H\psi = i\frac{\partial}{\partial t}\psi\ , \tag{3}$$

where

$$H = 2\sum_{l=1}^{N/2}\sum_{r=1}^{2} H_{rl}\ , \tag{4}$$

$$H_{rl} = \frac{1}{2}p_{rl}{}^2 + \frac{1}{2}hp_{rl}\phi_{rl} + \frac{1}{2}a^{-2}\omega_l{}^2\phi_{rl}{}^2 + \frac{1}{2}\left(m^2 + \xi R\right)\phi_{rl}{}^2, \tag{5}$$

$$\psi[\phi_{rl},t] = \prod_{l=1}^{N/2}\prod_{r=1}^{2}\psi_{rl}(\phi_{rl},t) \equiv \prod_{rl}\psi_{rl}(\phi_{rl},t), \tag{6}$$

Here $\omega_l \equiv (2/\varepsilon)\sin(l\pi/N)$ with $\varepsilon = W/N$, *i.e., W* is the overall comoving spatial size of lattice. Moreover, $\phi_l = \phi_{1l} + i\phi_{2l}$, $p_l = p_{1l} + ip_{2l}$, and p_l is the conjugate momentum for ϕ_l (the subscripts 1 and 2 denote the real and imaginary parts respectively).

For each real mode ϕ_{rl}, we have

$$H_{rl}\psi_{rl} = i\frac{\partial}{\partial t}\psi_{rl},\ r=1,2 \tag{7}$$

$$-\frac{1}{2}\frac{\partial^2\psi_{rl}}{\partial\phi_{rl}{}^2} + \frac{1}{2}\left[a^{-2}\omega_l{}^2 + \left(m^2 + \xi R\right) - \frac{1}{4}h^2\right]\phi_{rl}{}^2\psi_{rl} = i\frac{\partial\psi_{rl}}{\partial t}. \tag{8}$$

Here (8) arises from the field quantization of the Hamiltonian (5) through the functional Schrödinger representation $\bar{\phi}_{rl} \to \phi_{rl}$, $\bar{p}_{rl} \to -i\partial/\partial\phi_{rl}$, where operators $\bar{\phi}_{rl}$ and $\bar{p}_{rl}$ satisfy the equal time commutation relations $[\bar{\phi}_{rl}, \bar{p}_{rl}] = i$. Therefore (8) governs the time evolution of the state wave functional $|\psi_{rl}\rangle$ of the Hamiltonian operator H_{rl} in the $\{|\phi_{rl}\rangle\}$ representation. In terms of the conformal time τ defined by

$$d\tau = dt/a, \quad \tau = -h^{-1}\exp(-ht) = -h^{-1}a^{-1}, \ -\infty < \tau < 0, \tag{9}$$

the wave functionals of normalized ground and excited states are found to be

$$\psi_{rl(n_{rl})}(\phi_{rl},\tau) = R_{(n_{rl})}(\phi_{rl},\tau)\exp\left(i\Theta_{(n_{rl})}(\phi_{rl},\tau)\right), \quad n_{rl} = 0,1,2,... \tag{10}$$

where the amplitude $R_{(n_{rl})}(\phi_{rl},\tau)$ and phase $\Theta_{(n_{rl})}(\phi_{rl},\tau)$ are

$$R_{(n_{rl})}(\phi_{rl},\tau) = \left[\frac{\sqrt{2h/\pi}}{\sqrt{\pi}\,2^n n_{rl}!\left|H_\nu^{(1)}\right|}\right]^{1/2} H_{(n_{rl})}(\eta_{rl})\exp(-\frac{1}{2}\eta_{rl}^{\ 2}), \tag{11}$$

$$\Theta_{(n_{rl})}(\phi_{rl},\tau) = -\frac{h\omega_l|\tau|}{2}\frac{\left(\left|H_\nu^{(1)}\right|\right)'}{\left|H_\nu^{(1)}\right|}\phi_{rl}^{\ 2} - (\frac{1}{2}+n_{rl})\int\frac{\frac{2}{\pi|\tau|}}{\left|H_\nu^{(1)}\right|^2}d\tau. \tag{12}$$

Here η_{rl} is defined by $\eta_{rl} \equiv \left(\sqrt{2h/\pi}/\left|H_\nu^{(1)}\right|\right)\phi_{rl}$, $H_{(n_{rl})}(\eta_{rl})$ is the *n*th-order Hermite polynomial, and $H_\nu^{(1)}(\omega_l|\tau|)$ is the Hankel function of the first kind of order ν with $\nu^2 = 1/4 - (m^2 + \xi R)/h^2$. The prime in (12) denotes the derivative with respect to $\omega_l|\tau|$. From (10) we can write the complete wave functionals as $\psi_{[n]}[\phi_{rl},t] = \prod_{rl}\psi_{(n_{rl})}(\phi_{rl},t)$, where $[n] \equiv (n_i, n_j, \cdots)$ means that mode i is in the n_i excited state, mode j is in the n_j excited state, etc. For $n_{rl} = 0$, the ground state wave functional corresponds to the BD vacuum.

Note that extending from d=1 to d=3, we have $\nu^2 = 9/4 - (m^2 + \xi R)/h^2$, $R = 12h^2$, and the mode index l in ω_l carries labels $(l_i, i = 1,2,3)$ which will be suppressed below. Moreover, in the continuum limit ($\omega_l \to k$), from equations (3)-(8) we obtain

$$i\frac{\partial\psi}{\partial t} = \sum_{rk}\{-\frac{1}{2}\frac{\partial^2}{\partial\phi_{rk}^{\ 2}} + \frac{1}{2}\left[a^{-2}k^2 + (m^2 + \xi R) - \frac{9}{4}h^2\right]\phi_{rk}^{\ 2}\}\psi. \tag{13}$$

3. TRANS-PLANCKIAN EFFECTS

In this section, we consider the massless conformally coupled ($\nu = 1/2$) scalar field in the slow-roll inflation. To investigate the possible effects of trans-Planckian physics, we focus on the following CJ type dispersion relations

$$\omega^2(k/a) = k^2\left[1 + b_s(\frac{k}{aM})^{2s}\right], \tag{14}$$

where M is a cutoff scale, s is an integer, and b_s is an arbitrary coefficient [10-12].

3.1. CJ type Dispersion Relation with Quartic Correction

We first consider the CJ type dispersion relation (14) with $s = 1$ and $b_1 > 0$ to obtain the evolution of vacuum wave functional. Note that this CJ type dispersion relation can be obtained from theories based on quantum gravity models [13-16].

Using $z = k|\tau| = k/ah$ which is the ratio of physical wave number $k_{phys} \equiv k/a$ to the inverse of Hubble radius, (13) becomes

$$i\frac{\partial \psi}{\partial t} = \sum_{rk}\{-\frac{1}{2}\frac{\partial^2}{\partial {\phi_{rk}}^2} + \frac{1}{2}\left[z^2\left(1+\sigma^2 z^2\right)h^2 - \frac{1}{4}h^2\right]{\phi_{rk}}^2\}\psi\,, \tag{15}$$

where $\sigma^2 \equiv b_1(h/M)^2$. Then the ground state wave functional of (15) can be expressed as

$$\psi_{(0)} = \prod_{rk} A_{k(0)}(\tau)\exp(-\frac{1}{2}B_k(\tau)a^{-1}{\phi_{rk}}^2)\,, \tag{16}$$

where $A_{k(0)}(\tau)$ and $B_k(\tau)$ satisfy

$$A_{k(0)}(\tau) = \exp\left[-i\frac{1}{2}\int B_k(\tau)d\tau + const\right], \tag{17}$$

$$B_k^{\ 2}(\tau) - i\left[\frac{dB_k(\tau)}{d\tau} + \frac{B_k(\tau)}{\tau}\right] - \left[k^2(1+\sigma^2 z^2) - \frac{1}{4\tau^2}\right] = 0\,. \tag{18}$$

In region I where $k_{phys} \equiv k/a > M$, *i.e.* $z > M/h$, the dispersion relation can be approximated by $\omega^2(k/a) \approx k^2\sigma^2 z^2$, and the corresponding wave functional for the initial BD vacuum state is [22]

$$\psi_{(0)}^{\ \mathrm{I}} = \prod_{rk} A_{k(0)}^{\ \mathrm{I}}(\tau)\exp(-\frac{1}{2}B_k^{\ \mathrm{I}}(\tau)a^{-1}\phi_{rk}^{\ \mathrm{I}2})\,,\ A_{k(0)}^{\ \mathrm{I}}(\tau) = \exp\left[-i\frac{1}{2}\int B_k^{\ \mathrm{I}}(\tau)d\tau + const\right], \tag{19}$$

$$B_k^{\ \mathrm{I}}(\tau) = \frac{\frac{4}{\pi|\tau|}}{\left|H_{1/4}^{\ (1)}\right|^2} - i\frac{k}{2}\frac{\left(\left|H_{1/4}^{\ (1)}\right|^2\right)'}{\left|H_{1/4}^{\ (1)}\right|^2}\sigma z\,, \tag{20}$$

where the prime in (20) denotes the derivative with respect to $\sigma z^2/2$.

On the other hand, in region II where $k_{phys} \equiv k/a < M$, *i.e.* $z < M/h$, linear relation is recoverd $\omega^2 \cong k^2$, and the corresponding wave functional for the non-BD vacuum state is [22]

$$\psi_{(0)}^{\ \mathrm{II}} = \prod_{rk} A_{k(0)}^{\ \mathrm{II}}(\tau)\exp(-\frac{1}{2}B_k^{\ \mathrm{II}}(\tau)a^{-1}\phi_{rk}^{\ \mathrm{II}2})\ A_{k(0)}^{\ \mathrm{II}}(\tau) = \exp\left[-i\frac{1}{2}\int B_k^{\ \mathrm{II}}(\tau)d\tau + const\right], \tag{21}$$

$$B_k^{\ \mathrm{II}}(\tau) = \frac{\frac{2}{\pi|\tau|}}{\left(\left|C_1^{\ \mathrm{II}}\right|^2 + \left|C_2^{\ \mathrm{II}}\right|^2\right)\left|H_{1/2}^{\ (1)}\right|^2 + 2\mathrm{Re}\left[C_1^{\ \mathrm{II}}C_2^{\ \mathrm{II}*}\left(H_{1/2}^{\ (1)}\right)^2\right]} - i\frac{k}{2}\frac{\left\{\left(\left|C_1^{\ \mathrm{II}}\right|^2 + \left|C_2^{\ \mathrm{II}}\right|^2\right)\left|H_{1/2}^{\ (1)}\right|^2 + 2\mathrm{Re}\left[C_1^{\ \mathrm{II}}C_2^{\ \mathrm{II}*}\left(H_{1/2}^{\ (1)}\right)^2\right]\right\}'}{\left(\left|C_1^{\ \mathrm{II}}\right|^2 + \left|C_2^{\ \mathrm{II}}\right|^2\right)\left|H_{1/2}^{\ (1)}\right|^2 + 2\mathrm{Re}\left[C_1^{\ \mathrm{II}}C_2^{\ \mathrm{II}*}\left(H_{1/2}^{\ (1)}\right)^2\right]}\,, \tag{22}$$

where the prime in (22) denotes the derivative with respect to z, and the constants $C_1^{\ \mathrm{II}}$ and $C_2^{\ \mathrm{II}}$ satisfy $\left|C_1^{\ \mathrm{II}}\right|^2 - \left|C_2^{\ \mathrm{II}}\right|^2 = 1$. Let τ_c be the time when the modified dispersion relations take the standard linear form. Then $\sigma^2 {z_c}^2 = 1$ where $z_c = k|\tau_c| = M/b_1^{1/2}h >> 1$ for $b_1 \sim 1$. The values of $C_1^{\ \mathrm{II}}$ and $C_2^{\ \mathrm{II}}$ can be obtained by the following matching conditions at τ_c for the two wave functionals (19) and (21)

$$\psi_{(0)}^{\ \mathrm{I}}\Big|_{Z_C} = \psi_{(0)}^{\ \mathrm{II}}\Big|_{Z_C}\,, \tag{23}$$

$$\frac{d\psi_{(0)}^{\ \mathrm{I}}}{dz}\Big|_{Z_C} = \frac{d\psi_{(0)}^{\ \mathrm{II}}}{dz}\Big|_{Z_C}\,, \tag{24}$$

by requiring the conditions $A_{k(0)}^{\ \mathrm{I}} = A_{k(0)}^{\ \mathrm{II}}$, $B_k^{\ \mathrm{I}} = B_k^{\ \mathrm{II}}$ and $\phi_{rk}^{\ \mathrm{I}} = \phi_{rk}^{\ \mathrm{II}}$ when $z = z_c$.

Here we choose $C_1^{\text{II}} = \left|C_1^{\text{II}}\right|$ and $C_2^{\text{II}} = \left|C_2^{\text{II}}\right| \exp(i\theta)$, where θ is a relative phase parameter. Then, using $\left|H_{1/4}^{(1)}\left(\sigma z^2/2\right)\right|^2 = \left(4/\pi\sigma z^2\right)\left(1-3/8\sigma^2 z^4+\ldots\right) \approx 4/\pi\sigma z^2$ with $\sigma = z_c^{-1}$, z_c >>1, and $\left|H_{1/2}^{(1)}(z)\right|^2 = 2/\pi z$, we obtain from (23) and (24) (for the details of derivation see [23])

$$\left|C_2^{\text{II}}\right| = \frac{1}{4z_c},\ \left|C_1^{\text{II}}\right| = \sqrt{1+\left|C_2^{\text{II}}\right|^2} \cong 1+\frac{1}{32{z_c}^2} \cong 1. \tag{25}$$

3.2. CJ type Dispersion Relation with Sextic Correction

In this subsection, we consider the CJ type dispersion relation (14) with $s=2$ and $b_2>0$ to obtain the evolution of vacuum wave functional. For this case, only (15), (18), and (20) in the subsection 3.1. are respectively changed into

$$i\frac{\partial\psi}{\partial t} = \sum_{rk}\{-\frac{1}{2}\frac{\partial^2}{\partial{\phi_{rk}}^2}+\frac{1}{2}\left[z^2\left(1+\bar{\sigma}^2 z^4\right)h^2-\frac{1}{4}h^2\right]{\phi_{rk}}^2\}\psi, \tag{26}$$

$$B_k^2(\tau) - i\left[\frac{dB_k(\tau)}{d\tau}+\frac{B_k(\tau)}{\tau}\right]-\left[k^2(1+\bar{\sigma}^2 z^4)-\frac{1}{4\tau^2}\right]=0, \tag{27}$$

$$B_k^{\text{I}}(\tau) = \frac{\dfrac{6}{\pi|\tau|}}{\left|H_{1/6}^{(1)}\right|^2} - i\frac{k}{2}\frac{\left(\left|H_{1/6}^{(1)}\right|^2\right)'}{\left|H_{1/6}^{(1)}\right|^2}\bar{\sigma}z^2, \tag{28}$$

where $\bar{\sigma}^2 \equiv b_2(h/M)^4$. The prime in (28) denotes the derivative with respect to $\bar{\sigma}z^3/3$. Then, using $\left|H_{1/6}^{(1)}\left(\bar{\sigma}z^3/3\right)\right|^2 = \left(6/\pi\bar{\sigma}z^3\right)\left(1-1/\bar{\sigma}^2 z^6+\ldots\right) \approx 6/\pi\bar{\sigma}z^3$ with $\bar{\sigma} = \bar{z}_c^{-2}$, $\bar{z}_c = k|\bar{\tau}_c| = M/b_2^{1/4}h$ >>1 for $b_2 \sim 1$, and $\left|H_{1/2}^{(1)}(z)\right|^2 = 2/\pi z$, we obtain from (23) and (24) (for the details of derivation see [23])

$$\left|C_2^{\text{II}}\right| = \frac{1}{2\bar{z}_c},\ \left|C_1^{\text{II}}\right| = \sqrt{1+\left|C_2^{\text{II}}\right|^2} \cong 1+\frac{1}{8{\bar{z}_c}^2} \cong 1. \tag{29}$$

4. Reduction of Cosmological Constant

Note that the vacuum energy density due to the fluctuations of the inflaton field with $k<k_{\max}$ is given by [24]

$$\rho_{vac} = \frac{1}{2\pi^2}\int_0^{k_{\max}}\left(\frac{k}{a}\right)^4\frac{dk}{k} = \frac{1}{8\pi^2}{k_{phys(\max)}}^4 = \frac{1}{8\pi^2}M^4, \tag{30}$$

where M is the momentum cutoff. Because having a non standard dispersion relation is equivalent to considering non-vacuum quantum states for the perturbations [25], the finite energy density due to the inflaton particles after the subtraction of zero-point energy is given by [26, 27]

$$\rho_{vac} = \frac{1}{2\pi^2}\int_0^{k_{\max}} < n_k > \left(\frac{k}{a}\right)^4\frac{dk}{k}, \tag{31}$$

where $<n_k>$ is the occupation number of the modes with the momentum k, which is equal to $\left|C_2^{\text{II}}\right|^2$ in our formulation (see Section 3). Therefore, for the case of $s=1$ and $b_1>0$, using $\left|C_2^{\text{II}}\right| \cong 1/4z_c$ in (25), we have $<n_k> \cong 1/16{z_c}^2$. From (30), (31), and $M = b_1^{1/2}hz_c$, we obtain

$$\rho_{vac(s=1)} = \frac{M^4}{128\pi^2{z_c}^2} = \frac{{b_1}^2}{128\pi^2}h^4{z_c}^2. \tag{32}$$

Thus, we see that there is no back reaction problem if the energy density due to fluctuations of the inflaton field is smaller than that due to the inflaton potential, i.e.

$$\rho_{vac(s=1)} = \frac{b_1^{\ 2}}{128\pi^2} h^4 z_c^{\ 2} < V(\phi). \tag{33}$$

Within the slow-roll approximation, substituting $V(\phi) \cong 3M_{Pl}^{\ 2}h^2/8\pi$ (here $M_{Pl} =$ 1.22×10^{19} GeV is the Planck mass) in (33) leads to $b_1 < 48\pi(M_{Pl}/M)^2$. For $M \sim M_{Pl}$, the constraint on the dispersion parameter b_1 is $b_1 < 1.5\times10^2$.

On the other hand, for the case of $s=2$ and $b_2 > 0$, using $\left|C_2^{\ \mathrm{II}}\right| \cong 1/2\bar{z}_c$ in (29), we have $< n_k > \cong 1/4\bar{z}_c^{\ 2}$. From (30), (31), and $M = b_2^{1/4}h\bar{z}_c$, we obtain

$$\rho_{vac(s=2)} = \frac{M^4}{32\pi^2\bar{z}_c^{\ 2}} = \frac{b_2}{32\pi^2} h^4 \bar{z}_c^{\ 2}. \tag{34}$$

There is also no back reaction problem if

$$\rho_{vac(s=2)} = \frac{b_2}{32\pi^2} h^4 \bar{z}_c^{\ 2} < V(\phi). \tag{35}$$

Substituting $V(\phi) \cong 3M_{Pl}^{\ 2}h^2/8\pi$ in (35) leads to $b_2 < 144\pi^2(M_{Pl}/M)^4$. For $M \sim M_{Pl}$, the constraint on the dispersion parameter b_2 is $b_2 < 1.4\times10^3$.

Moreover, if we consider standard linear dispersion relation with both quartic and sextic corrections, then the corresponding perturbation energy density due to fluctuations of the inflaton field can be rewritten as

$$\rho_{vac}(h) = \rho_{vac(s=1)}(h) + \rho_{vac(s=2)}(h) = c_2 M^2 h^2, \tag{36}$$

where $c_2 \equiv (b_1/128\pi^2) + (b_2^{\ 1/2}/32\pi^2)$. For $M \sim M_{Pl} >> h$, the usual parameter choice is $b_1 \sim 1$ and $b_2 \sim 1$, yielding $c_2 \sim 10^{-3}$.

Because the vacuum energy density before the beginning of inflation is $\rho_{vac}(h=0) = M^4/8\pi^2$, we expect that while z decreases from $z > z_c >> 1$ (near the beginning of inflation) to $z < z_c$, the corresponding cosmological constant $\Lambda = 8\pi\rho_{vac}/M_{Pl}^{\ 2}$ decreases as

$$(1/\pi)\left(M^4/M_{Pl}^{\ 2}\right) \to 8\pi c_2\left(M^2h^2/M_{Pl}^{\ 2}\right), \tag{37}$$

where the first and second term in (37) comes from (30) and (36) respectively. Such a significant reduction appears in the early universe during the inflationary era when the Hubble parameter h is close to the grand unification scale, i.e. $h \sim 10^{15}$ GeV.

Note that current astronomical observations indicate that radiation and matter dominated epoch is sandwiched between two asymptotic de Sitter epochs, and the present values of vacuum energy density and the cosmological constant are respectively $\rho_{vac,0} \sim 2.5\times10^{-47}$ GeV ^{4} and $\Lambda_0 \sim 4.2\times10^{-84}$ GeV^2 [28]. Thus, using the fact that de Sitter geometry is invariant under time translation, we propose the possibility that reduction mechanism similar to (37) may reappear during the late time acceleration era with the present value of Hubble parameter $h_0 \sim 1.43\times10^{-42}$ GeV such that a tiny current value of the cosmological constant may be obtained. For example, evaluating $\Lambda = 8\pi c_2\left(M^2h^2/M_{Pl}^{\ 2}\right)$ with $M \sim 9M_{Pl}$, $h = h_0 \sim 1.43\times10^{-42}$ GeV and $c_2 \sim 10^{-3}$ may yield the current value of the cosmological constant $\Lambda_0 \sim 4.2\times10^{-84}$ GeV^2.

5. Conclusions

In the lattice Schrödinger picture, we have considered the theory of a generically coupled free real scalar field in de Sitter space. To investigate the possible effects of trans-Planckian physics on the quantum evolution of the vacuum state of scalar field, we focus on the massless conformally coupled scalar field in the slow-roll inflation, and consider the CJ type dispersion relations with quartic or sextic correction.

We then calculate explicitly the finite vacuum energy density due to fluctuations of the inflaton field, and obtain the corresponding cosmological constant by using the backreaction to constraint the magnitude of dispersion parameters. We also show explicitly how the cosmological constant reduces

significantly during the slow-roll inflationary era at the grand unification phase transition.

Finally, using the current astronomical observations that radiation and matter dominated epoch is sandwiched between two asymptotic de Sitter epochs and the fact that de Sitter geometry is invariant under time translation, we propose the possibility that similar reduction mechanism may reappear during the late time acceleration era, and yield a tiny current value of the cosmological constant.

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