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Kochen-Specker Theorem in a Two-Dimensional White Noise State

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We present the Kochen-Specker (KS) theorem in a two-dimensional white noise state. We consider whether we can simulate the state by a realistic theory of the KS type. It turns out that we cannot simulate the state by the realistic theory of the KS type.

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1. INTRODUCTION

The quantum theory (cf. [1—5]) gives approximate but frequently remarkably accurate numerical predictions.

Kochen and Specker present the no-hidden-variables theorem (the KS theorem) [6]. The KS theorem says the non-existence of a real-valued function which is mul-tiplicative and linear on commuting operators. Green-berger, Horne, and Zeilinger discover [7, 8] the so-called GHZ theorem for four-partite GHZ state. Then, the KS theorem becomes very simple form (see also Refs. [9—13]).

In this paper, we present the KS theorem in a two-dimensional white noise state. We consider whether we can simulate the state by a realistic theory of the KS type. It turns out that we cannot simulate the state by the realistic theory of the KS type.

2. THE KS THEOREM IN A TWO-DIMENSIONAL WHITE NOISE STATE

In this section, we present the KS theorem in the two-dimensional white noise state.

A. A Wave Function Analysis

Let σ_x a single Pauli observable. Here,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{1}$$

We assume that a source of a spin-carrying particle emits them in a state V_{noise}. Here,

$$V_{noise} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

We consider a quantum expected value $Tr[V_{noise}\sigma_x]$. If we consider only a wave function analysis, the possible value of the square of the quantum expected value is

$$(Tr[V_{noise}\sigma_x])^2 = 0 \tag{3}$$

We define $\|E_{QM}\|^2$ as

$$\left\| E_{QM} \right\|^2 = (Tr[V_{noise}\sigma_x])^2 \tag{4}$$

$\left\| E_{QM} \right\|^2_{\max}$ and $\left\| E_{QM} \right\|^2_{\min}$ are the maximal and minimal possible values of the product, respectively. We have

$$\left\| E_{QM} \right\|^2 \leq 0 \tag{5}$$

Thus,

$$\left\| E_{QM} \right\|^2_{\max} = 0 \tag{6}$$

We have

$$\left\| E_{QM} \right\|^2 \geq 0 \tag{7}$$

Thus,

$$\left\| E_{QM} \right\|^2_{\min} = 0 \tag{8}$$

Hence we have

$$\left\| E_{QM} \right\|^2_{\max} = 0 \text{ and } \left\| E_{QM} \right\|^2_{\min} = 0 . \tag{9}$$

B. The Realistic Theory of the KS Type

A mean value E satisfies the realistic theory of the KS type if it can be written as

$$E = \frac{\sum_{l=1}^{m} r_1(\sigma_x)}{m} \tag{10}$$

Where l denotes a notation and r is the result of the measurement of the Pauli observable σ_x. We assume the values of r are either 1 or −1 (in $\bar{h}/2$ unit). Assume the quantum mean value with the system in the two dimensional white noise state admits the realistic theory of the KS type. One has the following proposition concerning the realistic theory of the KS type

$$Tr[V_{noise}\sigma_x](m) = \frac{\sum_{l=1}^{m} r_1(\sigma_x)}{m} \tag{11}$$

We can assume the following by Strong Law of Large Numbers [14],

$$Tr[V_{noise}\sigma_x](+\infty) = Tr[V_{noise}\sigma_x] \tag{12}$$

We define $\left\| E_{QM} \right\|^2 (m)$ as

$$\left\| E_{QM} \right\|^2 (m) = (Tr[V_{noise}\sigma_x](m))^2 \tag{13}$$

We can assume the following by Strong Law of Large Numbers,

$$\left\| E_{QM} \right\|^2 (+\infty) = \left\| E_{QM} \right\|^2 = (Tr[V_{noise}\sigma_x])^2 \tag{14}$$

In what follows, we show that we cannot accept the relation (11) concerning the realistic theory of the KS type. Assume the proposition (11) is true. By changing the notation l into l', we have same quantum mean value as follows

$$Tr[V_{noise}\sigma_x](m) = \frac{\sum_{l'=1}^{m} r_{l'}(\sigma_x)}{m} \tag{15}$$

We introduce an assumption that Sum rule and Product rule commute with each other [15]. We do not pursue the details of the assumption. To pursue the details is an interesting point. It is suitable to the next step of researches. We have the following

$$\begin{aligned} \|E_{QM}\|^2(m) &= \frac{\sum_{l=1}^{m} r_l(\sigma_x)}{m} \times \frac{\sum_{l'=1}^{m} r_{l'}(\sigma_x)}{m} \\ &\leq \frac{\sum_{l=1}^{m}}{m} \cdot \frac{\sum_{l'=1}^{m}}{m} |r_l(\sigma_x) r_{l'}(\sigma_x)| \\ &= \frac{\sum_{l=1}^{m}}{m} \times \frac{\sum_{l'=1}^{m}}{m} = 1 \end{aligned} \tag{16}$$

Clearly, the above inequality can have the upper limit, since the following case is possible:

$$\|\{l | l \in N \wedge r_l(\sigma_x) = 1\}\| = \|\{l' | l' \in N \wedge r_{l'}(\sigma_x) = 1\}\| \tag{17}$$

And

$$\|\{l | l \in N \wedge r_l(\sigma_x) = -1\}\| = \|\{l' | l' \in N \wedge r_{l'}(\sigma_x) = -1\}\| \tag{18}$$

And we have the following

$$\begin{aligned} \|E_{QM}\|^2(m) &= \frac{\sum_{l=1}^{m} r_l(\sigma_x)}{m} \times \frac{\sum_{l'=1}^{m} r_{l'}(\sigma_x)}{m} \\ &\geq \frac{\sum_{l=1}^{m}}{m} \cdot \frac{\sum_{l'=1}^{m}}{m} (-1) \\ &= (-1) \frac{\sum_{l=1}^{m}}{m} \times \frac{\sum_{l'=1}^{m}}{m} = -1 \end{aligned} \tag{19}$$

Clearly, the above inequality can have the lower limit since the following case is possible:

$$\|\{l | l \in N \wedge r_l(\sigma_x) = 1\}\| = \|\{l' | l' \in N \wedge r_{l'}(\sigma_x) = -1\}\| \tag{20}$$

And

$$\|\{l | l \in N \wedge r_l(\sigma_x) = -1\}\| = \|\{l' | l' \in N \wedge r_{l'}(\sigma_x) = 1\}\| \tag{21}$$

Thus we derive a proposition concerning the quantum mean value under the assumption that the realistic theory of the KS type is true (in a spin-1/2 system), that is

$$-1 \leq \|E_{QM}\|^2(m) \leq 1 \tag{22}$$

From Strong Law of Large Numbers, we have

$$-1 \leq \|E_{QM}\|^2 \leq 1 \tag{23}$$

Hence we derive the following proposition concerning the realistic theory of the KS type

$$\|E_{QM}\|^2_{\min} = -1 \text{ and } \|E_{QM}\|^2_{\max} = 1 \tag{24}$$

We cannot accept the two relations (9) (concerning a wave function analysis) and (24) (concerning the realistic theory of the KS type), simultaneously. Hence we are in the KS contradiction.

3. Conclusion

In conclusion, we have presented the KS theorem in the two-dimensional white noise state. We have considered whether we can simulate the state by a realistic theory of the KS type. It has turned out that we cannot simulate the state by the realistic theory of the KS type.

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[14] In probability theory, the law of large numbers is a theorem that describes the result of performing the same experiment a large number of times. According to the law, the average of the results obtained from a large number of trials should be close to the expected value, and will tend to become closer as more trials are performed. The strong law of large numbers states that the sample average converges almost surely to the expected value.

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