

Numerical Solution for Coupled MHD Flow Equations in a Square Duct in the Presence of Strong Inclined Magnetic Field

Muhim Chutia

Department of Mathematics
Mariani College, Jorhat
Assam, India
muhimchutia@gmail.com

P. N. Deka

Department of Mathematics
Dibrugarh University, Dibrugarh
Assam, India
pndeka@yahoo.co.in

Abstract: A steady two dimensional MHD flow through a square duct under the action of transverse magnetic field acting with inclination to the duct walls has been investigated. The problem is described by coupled partial differential equations for velocity and induced magnetic field. These equations are transformed into non-dimensional equations by introducing non-dimensional quantities, and then these equations have been solved numerically using finite difference method. The solutions for velocity and induced magnetic field distribution are analyzed for different values of parameters: inclination parameter(γ) and Hartmann number (H) and presented graphically.

Keywords: MHD flow, Insulated walls, Square duct, Inclined magnetic field, Finite difference method.

1. INTRODUCTION

The problems of fully developed electrically conducting and incompressible fluid through ducts under the action of transverse magnetic field are considered to be very important because of their practical applications such as in nuclear reactors, MHD generators, MHD flow meters, blood flow measurement, pumps, and accelerators and so on. These classes of flow problems are governed by coupled equations of fluid flow and equations of electrodynamics whose exact solutions are difficult in most practical cases. There may arise complexities if Hartmann boundary layer is very thin. This Hartmann layers may arise when flow is passing through region of very strong magnetic field like confining field of nuclear fusion reactors. In such case of very high Hartmann number flow, it requires high resolution of flow in the very thin Hartmann layers and may involve additional effects which are required to include in the study. Therefore, many researchers employed numerical techniques which give improved numerical solutions.

Hartmann and Lazarus [1] first studied the MHD flow through a duct under the action of transverse magnetic field. They considered flow of mercury as a conducting fluid in pipes of different cross sections experimentally. In their investigations, the influence of transverse magnetic field in such a flow was elucidated. Theoretical investigation for the motion of electrically conducting fluid through pipe was subsequently studied by Shercliff [2, 3] and improved results were obtained. Gold [4] obtained an analytical solution of the MFM flow in a circular tube with zero wall conductivity. Hunt and Stewartson [5] studied the laminar motion of a conducting liquid in a rectangular duct under a uniform transverse magnetic field; they considered perfectly conducting walls parallel to the field and non-conducting walls perpendicular to the field. Sterl [6] used finite difference code to investigate MHD flow in rectangular duct at high Hartmann numbers with wall conductance ratio and changing magnetic field. Gupta and Singh [7] investigated analytically unsteady MHD flow in a circular pipe analytically for the case of insulated wall under the action of a uniform magnetic field parallel to a diameter of the cross section. Singh and Lal [8, 9] obtained numerical solutions of steady state MHD flow through channels of triangular cross sections by using finite difference method (FDM) together with the Kantorovich technique. Morley and Roberts [10] explored the flow of an electrically conducting fluid in an open channel in the presence of a strong magnetic field of oblique incidence to both the channel walls and the force of gravity. Liu and Zhu [11] studied the problem of the steady state fully developed MHD flow of a conducting fluid through a rectangular duct with arbitrary wall

conductivity in the presence of a transverse external magnetic field with various inclined angles by using dual reciprocity boundary element method (DRBEM). Smolentsev et al. [12] developed a new numerical code for analysis of a fully developed MHD flow in a duct of a liquid metal blanket using various insulation techniques. Alkhawaja and Selmi [13] studied the MHD flow in a square duct under the action of strong transverse magnetic field. Ibrahim [14] studied numerically magnetohydrodynamics (MHD) flow equations in a rectangular duct in the presence of transverse external oblique magnetic field for values of Hartmann number $H \leq 1000$. He employed Chebyshev collocation method to solve partial differential equations. Bozkaya and Tezer-Sezgin [15] investigated steady magnetohydrodynamics (MHD) duct flow in the presence of an external oblique magnetic field using boundary element method (BEM) with the most general form of wall conductivities and for large values of the Hartmann number (H). Hsieh et al. [16] proposed a development of the finite difference method, called the tailored finite point method for solving steady MHD duct flow problems with a high Hartmann number. Hosseinzadeh et al. [17] obtained the numerical solution of the coupled equations in velocity and induced magnetic field for the steady magnetohydrodynamic (MHD) flow through a pipe of rectangular and circular sections having arbitrary conducting walls. Sarma et al. [18] investigated numerically steady MHD flow of liquid metal through a square duct under the action of strong transverse magnetic field. They have considered the walls of the duct were electrically insulated as well as isothermal. Young [19] studied the effects of constant applied magnetic field as a function of its angle with the channel walls using finite elements method for insulating channel walls and for two insulating and two conducting walls forming a short-circuited magnetohydrodynamics generator. Tao and Ni [20] obtained two analytical solutions for MHD flows in a rectangular duct with unsymmetrical walls with side walls insulated and unsymmetrical Hartmann walls of arbitrary conductivity, and unsymmetrical side walls of arbitrary conductivity and Hartmann walls perfectly conductive.

In this present paper, the fully developed steady laminar MHD flow in an insulated square duct under the action of strong transverse and inclined magnetic field is investigated. The numerical solutions for velocity and induced magnetic field are obtained by employing finite difference method. The effects of inclination parameter (γ) and Hartmann number (H) on velocity and induced magnetic field are analyzed and presented graphically.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider steady two dimensional motion of an electrically conducting fluid through a square duct under the action of inclined transverse magnetic field. The length of the cross-section of the duct is a . The duct is bounded by the fixed planes $x = 0, x = a, y = 0$ and $y = a$ and the flow is driven by constant pressure gradient $\partial p / \partial z$. It is assumed that all sides of the duct are electrically insulated. The flow is assumed to be along in Z -axis, the applied uniform magnetic field B_0 acts in a direction lying in the XY plane but makes an angle θ with the Y -axis, which induces a magnetic field $B_z(x, y)$ in the flow direction. In this study following assumptions are made:

- i. The flow is steady, laminar and the fluid is viscous, incompressible.
- ii. The fluid is finitely conducting and the flow is fully developed.
- iii. The duct is considered to be infinite so that all the fluid properties except pressure gradient, are independent of the variable z .
- iv. Displacement currents are negligible and there is no net of current in the z -direction.

Under these assumptions, the velocity \vec{V} and magnetic field \vec{B} will be of the form

$$\vec{V} = \{0, 0, V_z(x, y)\}$$

$$\vec{B} = \{B_{0x}, B_{0y}, B_z(x, y)\}$$

$$= \{B_0 \sin \theta, B_0 \cos \theta, B_z(x, y)\}$$

$$= \left\{ \sqrt{(1 - \gamma^2)} B_0, \gamma B_0, B_z(x, y) \right\}$$

Where, $\gamma = \cos\theta$, is the inclination parameter.

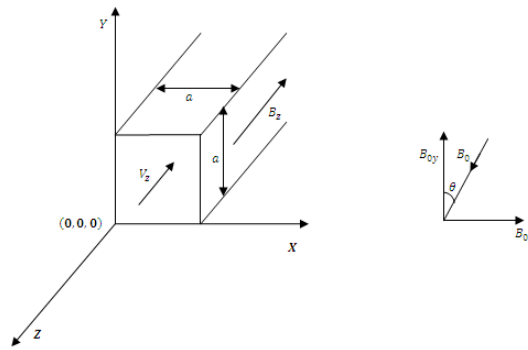


Fig1. Geometry of the problem

3. GOVERNING EQUATIONS

The governing equations of MHD flow are:

$$\nabla \cdot \vec{V} = 0 \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \tag{2}$$

$$\rho \left\{ \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \right\} = -\nabla p + \vec{j} \times \vec{B} + \mu \nabla^2 \vec{V} \tag{3}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B}) + \lambda \nabla^2 \vec{B} \tag{4}$$

$$\vec{j} = \sigma (\vec{E} + \vec{V} \times \vec{B}) \tag{5}$$

$$\nabla \times \vec{B} = \mu_e \vec{j} \tag{6}$$

Where \vec{j} is the current density due to the magnetic field, \vec{E} is the electric field intensity, ρ is density of the fluid, σ is the electrical conductivity, $\mu = \rho\nu$, is the co-efficient of viscosity, ν is the kinematic viscosity, μ_e is magnetic permeability and λ is the magnetic diffusivity.

Using (1) and (2) in equation (4), we get

$$\frac{\partial \vec{B}}{\partial t} + (\vec{V} \cdot \nabla) \vec{B} = (\vec{B} \cdot \nabla) \vec{V} + \lambda \nabla^2 \vec{B} \tag{7}$$

In steady case, and using velocity and magnetic field distribution stated as above, equations (3) and (7) become

$$\mu \left(\frac{\partial^2 V_z}{\partial x^2} + \frac{\partial^2 V_z}{\partial y^2} \right) + \sqrt{(1-\gamma^2)} \frac{B_0}{\mu_e} \frac{\partial B_z}{\partial x} + \gamma \frac{B_0}{\mu_e} \frac{\partial B_z}{\partial y} - \frac{\partial p}{\partial z} = 0 \tag{8}$$

$$\lambda \left(\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} \right) + \sqrt{(1-\gamma^2)} B_0 \frac{\partial V_z}{\partial x} + \gamma B_0 \frac{\partial V_z}{\partial y} = 0 \tag{9}$$

The equations (8) and (9) are to be solved subject to the following boundary conditions:

$$\left. \begin{aligned} V_z = 0, B_z = 0 & \quad \text{at } y = 0 \\ V_z = 0, B_z = 0 & \quad \text{at } y = a \\ V_z = 0, B_z = 0 & \quad \text{at } x = 0 \\ V_z = 0, B_z = 0 & \quad \text{at } x = a \end{aligned} \right\} \tag{10}$$

We use dimensionless quantities as

$$x^* = \frac{x}{a}, y^* = \frac{y}{a}, B^* = \frac{B_z}{B_0}, V^* = \frac{V_z}{V_0} \tag{11}$$

Where, $B_0 = -a^2 \mu_e \left(\frac{\sigma}{\rho\nu} \right)^{1/2} \frac{\partial P}{\partial z}$

$$V_0 = -\frac{a^2}{\rho\nu} \frac{\partial P}{\partial z}$$

Using dimensionless quantities (11) in equations (8) and (9) and dropping asterisks, we obtain

$$\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) + \sqrt{(1 - \gamma^2)}H \frac{\partial B}{\partial x} + \gamma H \frac{\partial B}{\partial y} + 1 = 0 \tag{12}$$

$$\left(\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2}\right) + \sqrt{(1 - \gamma^2)}H \frac{\partial V}{\partial x} + \gamma H \frac{\partial V}{\partial y} = 0 \tag{13}$$

Where, $H = B_0 a \left(\frac{\sigma}{\nu \rho}\right)^{1/2}$, is the Hartmann number

The corresponding boundary conditions (10) become

$$\left. \begin{aligned} V = 0, \quad B = 0 \quad \text{at } y = 0 \\ V = 0, \quad B = 0 \quad \text{at } y = 1 \\ V = 0, \quad B = 0 \quad \text{at } x = 0 \\ V = 0, \quad B = 0 \quad \text{at } x = 1 \end{aligned} \right\} \tag{14}$$

4. NUMERICAL SOLUTION

The non dimensional governing equations (12) and (13) along with boundary conditions (14) are discretized using finite difference scheme. We have divided the computational domain into a uniform grid system. The second derivative and first derivative terms are discretized using the central differences of second order accuracy respectively. For example the finite difference form of $\frac{\partial^2 V}{\partial x^2}$ and $\frac{\partial V}{\partial x}$ are discretized as $\frac{\partial^2 V}{\partial x^2} = \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{\Delta x^2} + O(\Delta x^2)$ and $\frac{\partial V}{\partial x} = \frac{V_{i+1,j} - V_{i-1,j}}{2\Delta x} + O(\Delta x^2)$, respectively. Therefore the resulting difference equations (12) and (13) are as follows:

$$\begin{aligned} V_{i,j} = C_3(V_{i+1,j} + V_{i-1,j}) + C_4(V_{i,j+1} + V_{i,j-1}) + C_5(B_{i+1,j} - B_{i-1,j}) \\ + C_6(B_{i,j+1} - B_{i,j-1}) + C_7 \end{aligned} \tag{15}$$

$$\begin{aligned} B_{i,j} = C_3(B_{i+1,j} + B_{i-1,j}) + C_4(B_{i,j+1} + B_{i,j-1}) + C_5(V_{i+1,j} - V_{i-1,j}) \\ + C_6(V_{i,j+1} - V_{i,j-1}) \end{aligned} \tag{16}$$

Where, index i refers to x and j refers to y , and $\Delta x = h$, $\Delta y = k$ and

$$C_1 = H\sqrt{(1 - \gamma^2)}, \quad C_2 = \gamma H, \quad C_3 = \frac{k^2}{2(h^2+k^2)}, \quad C_4 = \frac{h^2}{2(h^2+k^2)}, \quad C_5 = \frac{C_1 h k^2}{4(h^2+k^2)}, \quad C_6 = \frac{C_2 h^2 k}{4(h^2+k^2)}$$

and $C_7 = \frac{h^2 k^2}{2(h^2+k^2)}$ are constants.

The corresponding discretized boundary conditions are

$$\left. \begin{aligned} V_{i,1} = 0, \quad B_{i,1} = 0 \quad \text{at } j = 1 \\ V_{i,n+1} = 0, \quad B_{i,n+1} = 0 \quad \text{at } j = n + 1 \\ V_{1,j} = 0, \quad B_{1,j} = 0 \quad \text{at } i = 1 \\ V_{m+1,j} = 0, \quad B_{m+1,j} = 0 \quad \text{at } i = m + 1 \end{aligned} \right\} \tag{17}$$

The numerical solution of the equations (15) and (16) for velocity $V_{i,j}$ and induced magnetic field $B_{i,j}$ are obtained by first the selecting the non-dimensional flow parameters γ and H that are involved in dimensionless equations (12) and (13). Knowing the values of $V_{i,1}$, $B_{i,1}$; $V_{i,n+1}$, $B_{i,n+1}$; $V_{1,j}$, $B_{1,j}$; $V_{m+1,j}$, $B_{m+1,j}$ at the boundary grid points on the bottom, top, left and right respectively from the discretized boundary conditions (17), we substitute $i = 2,3,4, \dots, m$ and $j = 2,3,4, \dots, n$ in equation (15) which results in a tri-diagonal linear system of $(m - 1)(n - 1)$ equations in $(m - 1)(n - 1)$ unknowns values for V . Using initial and boundary conditions (17), the system of linear equations can be solved by Gauss elimination method. Thus V is known at all values of x and y . Then knowing the values of V and applying the same procedure and using boundary conditions (17), we can calculate B from equation (16). The procedure is continued to obtain the solutions till the converged solutions for V and B in the grid system are obtained.

5. NUMERICAL RESULT AND DISCUSSION

In the present study a numerical investigations on MHD flow in a square duct is considered. The basic coupled linear equations are solved simultaneously using finite difference method. Finite difference equations are derived by making the mesh size to be uniform and have the same value for both

Numerical Solution for Coupled MHD Flow Equations in a Square Duct in the Presence of Strong Inclined Magnetic Field

directions. Numerical solutions are carried out in the spatial domain $0 \leq x \leq 1$ and $0 \leq y \leq 1$ using Matlab programming (Al-Khawaja and Selmi, [13]; Mathews and Fink, [21]). In this numerical computation a uniform 101×101 mesh is used for low to moderate Hartmaan number ($H = 0$ to 150), while for high Hartmaan numbers ($H = 300, 500$), a uniform 151×151 and 201×201 meshes are used respectively. The solutions are obtained for the velocity and induced magnetic field for Hartmann number H up to 500.

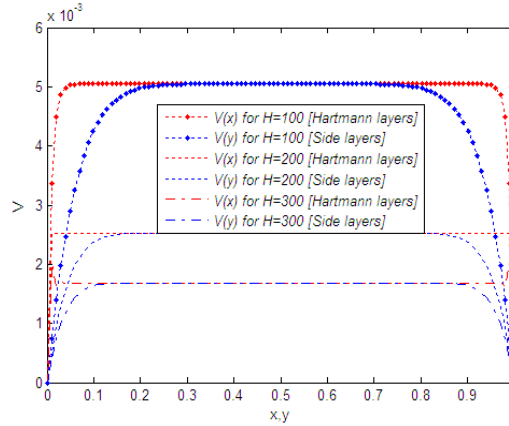


Fig2. Axial velocity profiles along Hartmann and side layers for $\gamma = 0$ at various H

Fig. 2 presents the axial velocity profiles along Hartmann and side layers for $\gamma = 0$ at various Hartmann numbers H , it is observed that velocity profile along Hartmann layers is more flattening than that of side layers for same values of H and velocity profile decreases as H increases. It is observed in fig. 3 that the intensity of axial induced magnetic field decreases for increasing values of Hartmann number H .

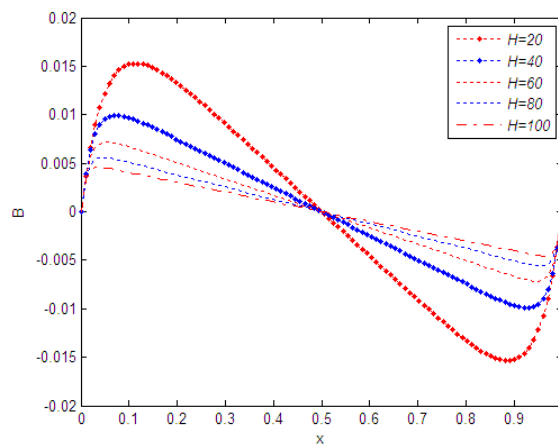


Fig3. Axial induced magnetic field for $\gamma = 0$ at various H

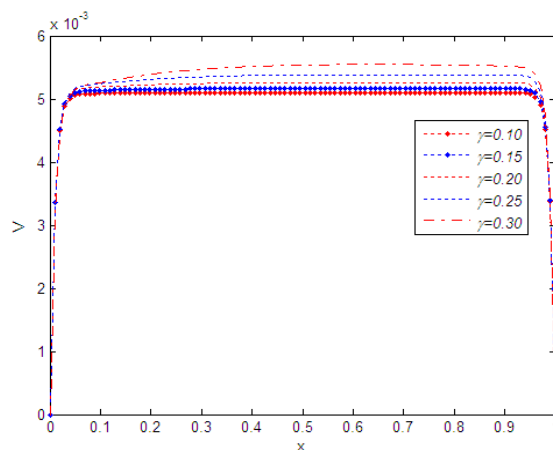


Fig4. Axial velocity profiles for $H = 100$ at various γ

Fig.4 depicts the axial velocity for $H = 100$ at various γ . It is observed that velocity profile increases as γ increases i.e. velocity profile increases for decreasing values of θ .

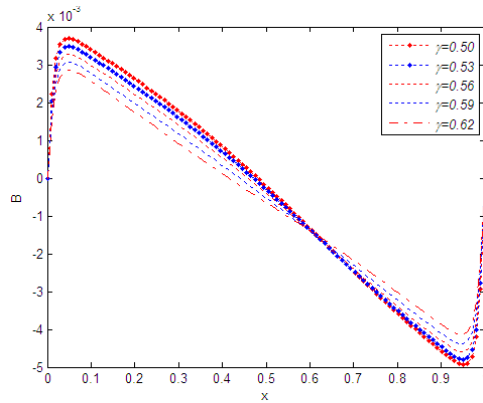


Fig5. Axial induced magnetic field for $H = 100$ at various γ

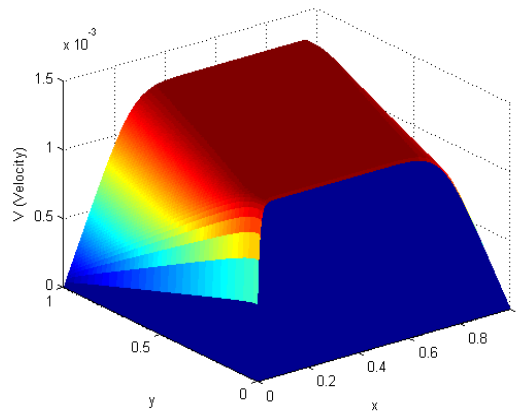


Fig6(a). Velocity profile for $H = 400$ and $\gamma = 0.26$

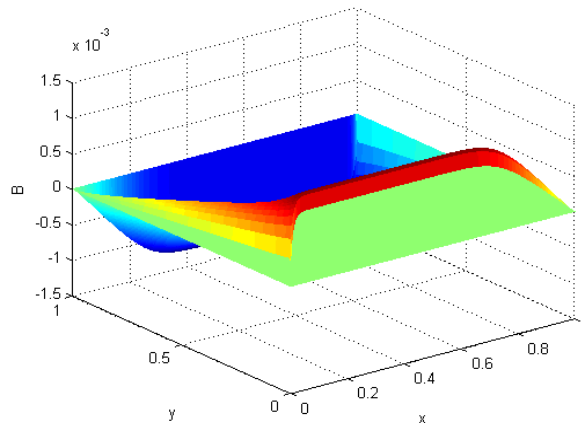


Fig6(b). Induced magnetic field for $H = 400$ and $\gamma = 0.26$.

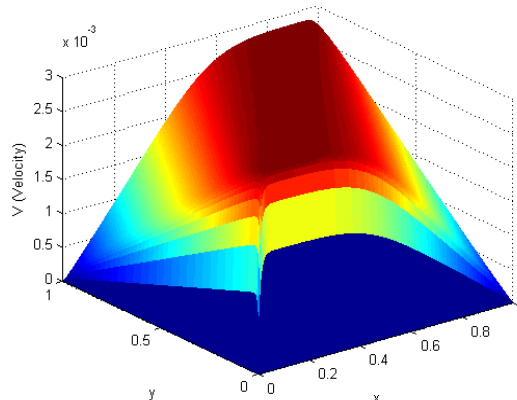


Fig7(a). Velocity profile for $H = 200$ and $\gamma = 0.50$

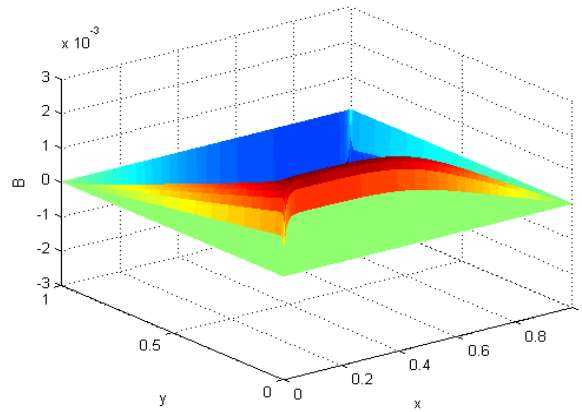


Fig7(b). Induced magnetic field for $H = 200$ and $\gamma = 0.50$

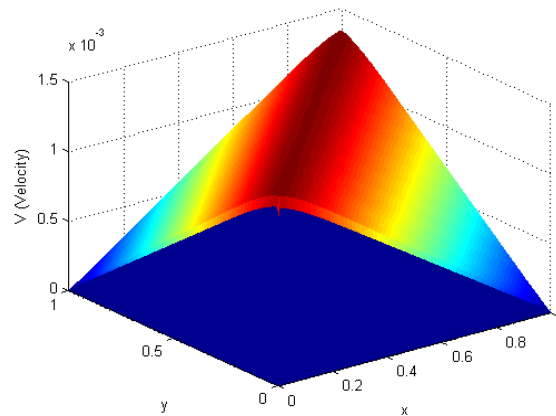


Fig8(a). Velocity profile for $H = 500$ and $\gamma = 0.71$

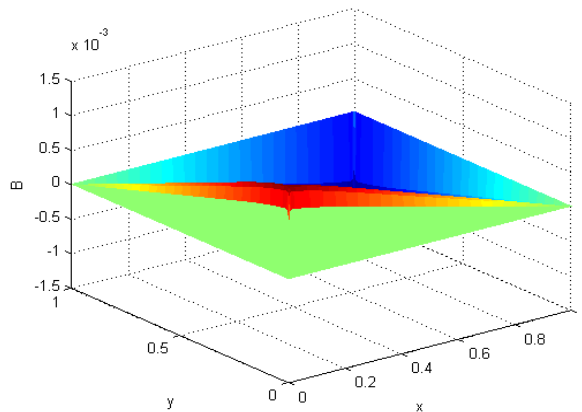


Fig8(b). Induced magnetic field for $H = 500$ and $\gamma = 0.71$

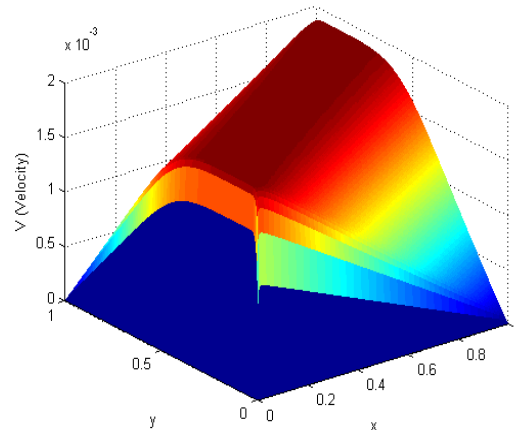


Fig9(a). Velocity profile for $H = 300$ and $\gamma = 0.87$

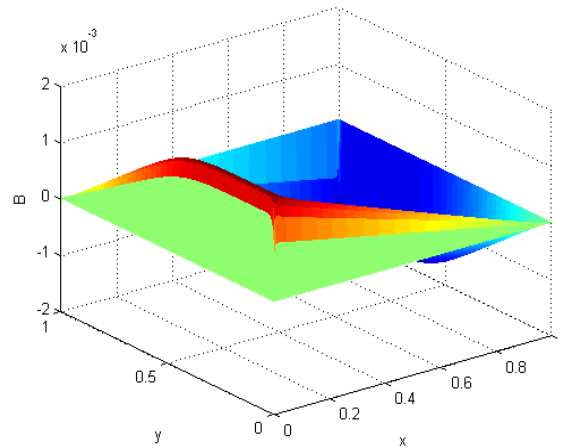


Fig9(b). Induced magnetic field for $H = 300$ and $\gamma = 0.87$

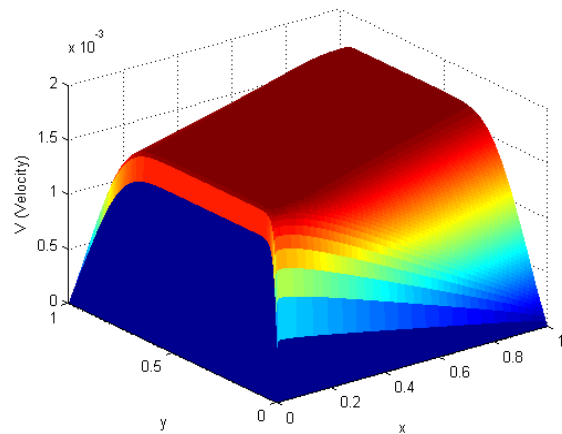


Fig10(a). Velocity profile for $H = 300$ and $\gamma = 0.97$

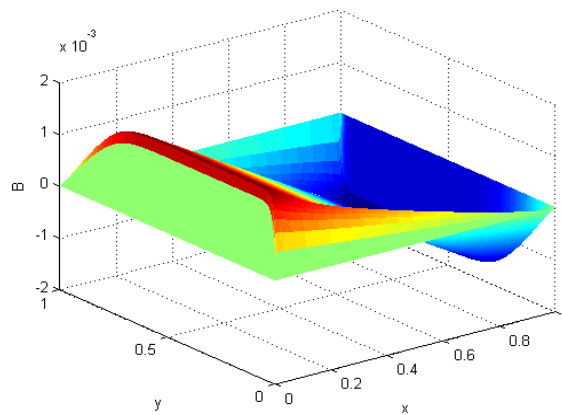


Fig10(b). Induced magnetic field for $H = 300$ and $\gamma = 0.97$

Fig.5 represents the variation of axial induced magnetic field for $H = 100$ for different values of inclination parameter γ , it is seen that induced magnetic field decreases as γ increases.

In Figs. 6-10, we have presented velocity and induced magnetic field with $\gamma = 0.26, \gamma = 0.50, \gamma = 0.71, \gamma = 0.86$ and $\gamma = 0.97$ for $H=400, H=200, H=500$ and $H=300$ respectively. It can be seen from Fig. 8(a), when $\gamma = 0.71$ (i.e., $\theta = 45^\circ$), velocity is symmetric with respect to lines $y = \pm x$. But in Fig.8(b), it is seen that induced magnetic field is symmetric with respect to line $y = -x$ with $\gamma = 0.71$. It is also well observed that for high Hartmann number, the boundary layer formation closes to the walls for both velocity and induced magnetic field. The boundary layers are concentrated near the corners in the direction of the applied inclined magnetic field for both velocity and induced magnetic field. These are the well-known characteristic of magnetohydrodynamic flow through a channel of duct and are in good agreement with the results of research works carried out by Bozkaya and Tezer-Sezgin [15], Ibrahim [14]; Hosseinzadeh et al. [17] and Young [19] in the literature.

6. CONCLUSION

In this work, we have solved coupled MHD flow equations through an insulated square duct under the action of inclined magnetic field using finite difference method. The problem was solved by Bozkaya and Tezer-Sezgin [15], Ibrahim [14], Hosseinzadeh et al. [17] and Young [19] and they employed different numerical techniques such as Boundary Element Method (BEM) and Chebyshev Collocation Method. But here, we have employed finite difference method (FDM) to solve MHD flow equations by Matlab programming. The advantage of this method over the method is that one can solve such type of coupled equation without decoupling the equations and noticed that this numerical technique is so efficient and powerful for solving such problem. There is no complex methodology and applications of this method are very easy and it gives us implicit form of the approximate solution of the problems.

REFERENCES

- [1] Hartmaan J. and Lazarus F., Experimental investigations on the flow of mercury in a homogeneous magnetic field, *K. Dan.Vidensk. Selsk. Mat. Fys. Medd.*, 15(6), pp. 1-45, (1937).
- [2] Shercliff J.A., Steady motion of conducting fluids in pipes under the action of transverse magnetic field, *Mathematical Proc. of the Cambridge Philosophical Society*, 49(1), pp. 136-144, (1953).
- [3] Shercliff J.A., "Magnetohydrodynamics pipe flow Part II, high Hartmaan number, *Journal of Fluid Mechanics*, 13(4), pp. 513-518, (1962).
- [4] Gold R., Magnetohydrodynamics pipe flow. *J. Fluid Mech. Part 1*, 13, pp. 505-512, (1962).
- [5] Hunt J.C.R. and Stewartson K., Magnetohydrodynamics flow in rectangular duct II, *J. Fluid Mech.*, 23(3), pp. 563-581, (1965).
- [6] Sterl A., Numerical simulation of liquid-metal MHD flows in rectangular ducts, *Journal of Fluid Mechanics*, 216, pp. 161-191, (1990).
- [7] Gupta S.C. and Singh B., Unsteady magnetohydrodynamics flow in a circular pipe under a transverse magnetic field, *Physics of Fluids*, 13(2), pp. 346-352, (1970).
- [8] Singh B. and Lal J., MHD axial flow in a triangular pipe under transverse magnetic field parallel to a side of the triangle, *Indian J. Tech.*, 17, pp. 184-189, (1979).
- [9] Singh B. and Lal J., Finite element method in magnetohydrodynamics channel flow problem. *Int. J. Numer. Methods Eng.*, 18, pp. 1104-1111, (1982).
- [10] Morley N.B. and Roberts P.H., Solutions of uniform, open-channel, liquid metal flow in a strong, oblique magnetic field, *Phys. Fluids*, 8(4), pp. 923-935, (1996).
- [11] Liu H. and Zhu S., The dual reciprocity boundary element method for Magnetohydrodynamics channel flows, *ANZIAM J.*, 44, pp. 305-322, (2002).
- [12] Smolentsev S., Morley N. and Abdou M., Code development for analysis of MHD pressure drop reduction in a liquid metal blanket using insulation technique based on a fully developed flow model, *Fusion Engineering and design*, 73, pp. 83-93, (2005).
- [13] Alkhawaja M.J. and Selmi M., Finite difference solutions of MFM square duct with heat transfer using Matlab Program, *Matlab modeling programming and simulations*, Sciyo, pp. 365-388, (2010).
- [14] Ibrahim C., Solution of magnetohydrodynamics flow in a rectangular duct by Chebyshev collocation Method, *International Journal for Numerical Method in Fluids*, 66, pp. 1325-1340, (2011).
- [15] Bozkaya C. and Tezer-Sezgin M., Fundamental solution of magnetohydrodynamics flow equations, *Journal of Computational and Applied Mathematics*, 203(1), pp. 125-144, (2007).
- [16] Hsieh P.-W. , Shih Y. and Yang S.-Y., A Tailored finite point method for solving steady MHD duct flow problems with boundary layers, *Commun. Comput. Phys.*, 10(1), pp. 161-182 (2011).
- [17] Hosseinzadeh H., Dehghan M. and Mirzaei D., The boundary elements method for magnetohydrodynamics (MHD) channel flows at high Hartmann numbers, *AMS classification: 65N38*, (2012).
- [18] Sarma D., Hazarika G.C. and Deka P.N., Numerical study of liquid metal MHD flow through a square duct under the action of strong transverse magnetic field, *International Journal of Computer Applications(0975-8887)*, 71(8), pp. 29-32, (2013).

- [19] Young F.J., MHD Channel Flow with Skewed Applied Magnetic Induction Field, *Science Research*, 2(4), pp. 62-77, (2014).
- [20] Tao Z. and Ni M.J., Analytical solutions for MHD flow at a rectangular duct with unsymmetrical walls of arbitrary conductivity, *Science China-Physics, Mechanics and Astronomy*, pp. 1-18, (2014).
- [21] J.H. Mathews and K.D. Fink, *Numerical Methods using Matlab*", PHI Learning Private Limited, New Delhi, (2009).

AUTHORS' BIOGRAPHY



Muhim Chutia, assistant professor of Mathematics, Mariani College, Jorhat, Assam, India. He is doing Ph.D. under the supervision of Prof. P.N. Deka, Department of Mathematics, Dibrugarh University and likely to be submitted his thesis by Dec., 2015. He has published a number of research papers in various reputed journals. His subjects of research specialization are in the field of Fluid Mechanics and CFD.



P.N. Deka, is professor of Mathematics, at Dibrugarh University, Assam, India. His area of research interests are: Nonlinear Plasma Dynamics, MHD, Heat and Mass Transfer and CFD. He has published a number of research papers in various reputed journals and guiding M.Phil and Ph.D students in the field of Nonlinear Plasma Dynamics, MHD, Heat and Mass Transfer and CFD. He has 25 years of teaching experience at post graduate level.