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Description of the Ground and Super Bands in Xenon Nuclei Using the Rotational Limit of the Interacting Vector Boson Model

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Abstract: *The interacting vector boson model (IVBM) has been applied to describe the ground state band with positive parity in even* $^{120\text{-}130}$Xe *nuclei. The model is extended to investigate the high spin states in the super band. The analysis of the excitation energies of the two bands reveals the presence of backbending phenomena at the positions between the* $I^{\pi} = 10^{+}$ *and* 12^{+} *levels in* $^{120\text{-}126}$*Xe or between* $I^{\pi} = 8^{+}$ *and* 10^{+} *levels in* ^{128,130}*Xe due to band crossing between the ground band and the super band with a* $\nu h_{11/2}$ *configuration. Chi squared fitting search program has been employed to extract the model parameters in order to obtain a minimum root mean square (rms) deviation between the calculated and the experimental excitation energies . Good agreement is obtained between experimental and IVBM calculations for the two bands in all six Xe isotopes.*

1. INTRODUCTION

In recent years the phenomena of shape transition and moment of inertia anomaly [1-3] have created considerable special interest in nuclear structure physics. Nuclei around mass region A~130 are among the nuclei which are rich in these phenomena and are suitable for comprehensive experimental and theoretical studies. The even –even nuclei in this region seem to be soft with regard to the γ-deformation with an almost maximum effective triaxiality of $\gamma = \pi/6$ [4,5]. Since they are neither vibrational nor rotational. The excitation energies in this mass region were investigated extensively in terms of various models ,such as , the interacting boson model (IBM) [6-8],the fermion dynamical symmetry model (FDSM) [9,10] ,the nucleon pair shell model [11] and the pair truncated shell model (PTSM) [12,13].

In the algebraic models like the IBM [6] and the interacting vector boson model (IVBM) [14] the use of the dynamical symmetries defined by a certain reduction chain of the group of dynamical symmetry yields exact solutions for the eigenvalues and eigenfunctions of the model Hamiltonian , which is constructed from the invariant operators of the subgroups in the chain.

In the region of high spins where large change in nuclear structure can be expected as induced by the presence of large quantities of angular momentum, these models must be modified. In a nucleus, the lowest state is referred as bandhead. Many states of different intrinsic structure can in principle become bandheads, the band built on the ground state of the nucleus is the ground state band (gb). All other excited high spin bands are called the super bands (sb). Near the crossing point between two bands, several physical phenomena occur. As example the backbending phenomena [15,16] is understood as a consequence of crossing between two bands, one being the gb and the other having a pair of aligned high j intruder particles [17-20]. Its magnitude is related to the crossing angle between the two crossing bands [21].

The symplectic extension to Sp(12,R) of the unitary dynamical symmetry U(6) of the IVBM was used to describe the well deformed even –even nuclei, the mixed mode dynamics [22], and the energy staggering in rare earth nuclei [23].

In this paper, the algebraic IVBM [14] is used to describe the gb and sb of the even –even Xenon nuclei. The most general spectrum generating algebra of the model is the algebra of the Sp(12,R) dynamical symmetry. It has a rather rich sub-algebraic structure. In the rotational limit [24] of the model the reduction of Sp(12,R) to the So (3) angular momentum group is carried out through the compact unitary U(6) subgroup, which defines the number of bosons preserving version of the model.

Section 2 provides information on the IVBM and its rotational limit, this section also gives expressions for excitation energies to be described in this paper. The backbending phenomenon is the subject of section 3. The results of calculations and comparison with experimental data for $^{120-130}Xe$ isotopic chain are presented in section 4.Finally a conclusion is given in section 5.

2. FRAMEWORK OF THE INTERACTING VECTOR BOSON MODEL (IVBM)

In this section the IVBM [14] is used to describe the excitation energies of collective states having fixed values of angular momentum L. The fundamental dynamical symmetry of this model is the group Sp(12,R) [25,26], which is the group of linear canonical transformation in a 12 dimensional phase. In the rotational limit [24], the group Sp (12,R) is reduced to the So(3) angular momentum group through the compact unitary U(6) subgroup which defines the boson number N.

The rotational limit [24] of the IVBM is defined by the chain as follows

$$Sp(12,R) \supset \underset{N}{U(6)} \supset \underset{(\lambda,\mu)}{SU(3)} \otimes \underset{(N,T)K}{U(2)} \supset \underset{L}{So(3)} \otimes U(1) \qquad (1)$$

where the labels below the subgroup are the quantum numbers corresponding to their irreducible representation (irreps). Their values are obtained by means of standard reduction rules [24]. Since the reduction from U(6) to So (3) is carried out by the mutually complementary group Su(3) and U(2), their quantum numbers are related in the following way

$T = \lambda/2$, $N = 2\mu + \lambda$

So that, we can write the basis as

$$|N, T, K, L, T_0\rangle \qquad (2)$$

The Hamiltonian operator is expressed in terms of the first and second order invariant operators of the different subgroups in the chain (1)

$$\hat{H} = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 {T_0}^2 \qquad (3)$$

The Hamiltonian operator is obviously diagonal in the basis (2). The eigen values for states of a given L value are therefore reads

$$E(N, T, L, T_0) = aN + bN^2 + \alpha_3 T(T+1) + \beta_3 L(L+1) + \alpha_1 {T_0}^2 \qquad (4)$$

For even value of the number of bosons N(N =0 ,2,4,…) , the possible values for the pseudospin are

$$T = \frac{N}{2}, \frac{N}{2} - 1, \ldots\ldots, 1, 0$$

When N and T are fixed, (2T+1) equivalent representation of the group Su(3) arise (T0= -T , - T+1 , …. + T).

For ground and super states, N = 2L with the sequence of states with different number of bosons N = 0 , 4 , 8 ,.. and pseudospin T =0 (T0=0).

3. THE BACKBENDING PLOTS

The band diagram is a diagram in which the excitation energies E(L) are plotted for the ground state band (gb) and superband (sb) as function of spin L .

In this diagram, the slope of the excitation energy is the angular velocity, which can be defined for each band

$$\omega(L) = \frac{dE(L)}{dL} \qquad (5)$$

The kinematic moment of inertia J(1) is then defined by

$$J^{(1)} = \frac{L}{\omega} \qquad (6)$$

The backbending plot is a conventional diagram in which twice the kinematic moment of inertia (2 J(1)) is plotted against the square of rotational frequency $\hbar^2\omega^2$. In the vibrational limit the angular velocity ω becomes spin independent and upbending can be seen. On the other hand in the rotational limit the kinematic moment of inertia J(1) becomes spin independent and a plateau can be seen in the backbending plot. In such a case the dynamic moment of inertia J(2) becomes equal to the inverse of the curvature of the excitation energy

$$J^{(2)} = \left(\frac{d\omega}{dL}\right)^{-1} = \left(\frac{d^2E}{dL^2}\right)^{-1} \tag{7}$$

Conventionally the kinematic moment of inertia J(1) and the square of the rotational frequency $\hbar^2\omega^2$are extracted from the definitions

$$\frac{J^{(1)}}{\hbar^2} = \frac{2L-1}{E_\gamma(L)} \tag{8}$$

$$\hbar^2\omega^2 = \frac{L^2-L+1}{(2L-1)^2} E_\gamma^2(L) \left(\cong \frac{1}{4} E_\gamma^2(L), \text{for } L > 6\right) \tag{9}$$

with

$$E_\gamma(L) = E(L) - E(L-2) \tag{10}$$

In the definition, a derivative in energy is involved, and therefore it reflects changes in band energies as spin varies.

4. NUMERICAL CALCULATIONS AND DISCUSSIONS

Xenon isotopes are in a typical transitional region, in which the nuclear structure varies from that of the spherical shape to deformed one. The even-even 120-130Xe isotopic chain has been considered.

The model parameters a , b and β3 for the ground state band (gb) and superband (sb) for each nucleus are evaluated by using a computer simulated search program in order to fit the calculated IVBM energy levels up to Iπ= 24+ with the experimental ones using the standard χ^2minimization procedure. The quality of the energy fit is

$$\chi^2 = \frac{1}{N}\sum_{L_i} \frac{\left(E^{exp}(L_i) - E^{cal}(L_i)\right)^2}{(\text{exp. errors})^2}$$

which is the standard energy root mean square (rms) deviation with N being equal to the number of energy levels enter the fitting. All experimental data on energy levels are taken from Ref.[27]. The adopted best model parameters are given in Table(1) for (gb) and (sb).

Table1. *Adopted parameters in (KeV) of the Hamiltonian obtained in the fitting procedure*.

Isotope		a	b	β_3
^{120}Xe	gb	28.51718	-14.25859	72.78012
	sb	57.63437	-28.81718	121.13125
^{122}Xe	gb	28.44000	-14.22000	74.17360
	sb	50.22187	-25.11093	108.78325
^{124}Xe	gb	29.96937	-14.98468	79.00295
	sb	64.05625	-32.02812	133.65550
^{126}Xe	gb	35.72500	-17.86250	88.58700
	sb	64.21250	-32.10625	133.56800
^{128}Xe	gb	44.4997	-22.3495	104.4649
	sb	82.9845	-41.4923	164.8500
^{130}Xe	gb	56.9335	-28.4667	127.8645
	sb	96.8485	-48.4242	186.3606

The comparison between the experimental excitation energies [27] and our IVBM calculations using the values of the model parameter given in Table (1) for the ground state band (gb) and super band (sb) of isotopic chain 120-126Xe is illustrated in Figure (1). The agreement between the theoretical values obtained with only three IVBM parameters and experimental data for all the nuclei under consideration is very good.

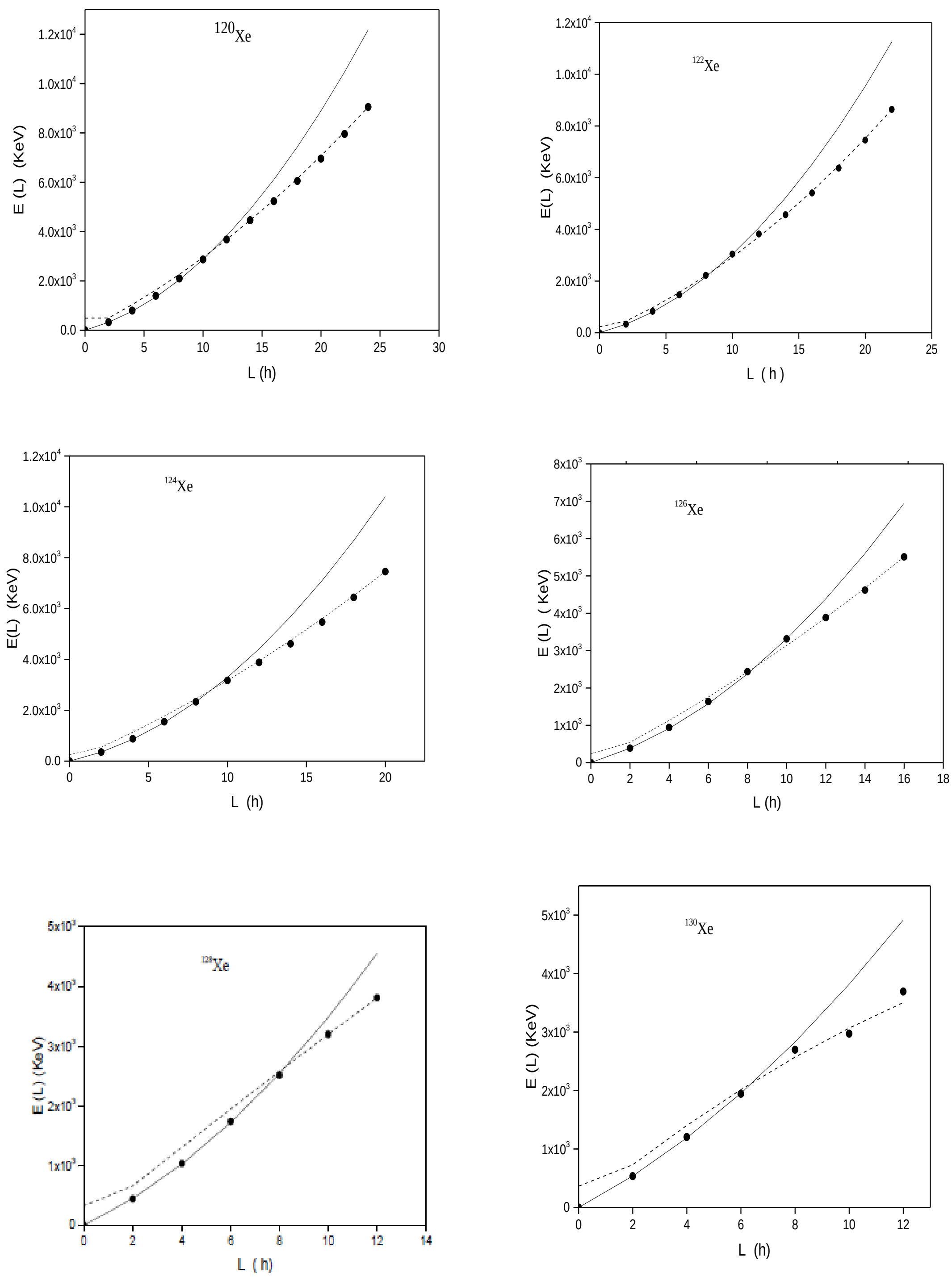

Figure1. *A band diagram for 120-130Xe isotopes showing comparison of IVBM energies with experiment for the ground state band (gb) and super band (sb). The Figure show also the position of the crossing spin point. The experimental data are denoted by solid circles for ground band (gb) and suberband (sb). The IVBM calculation are denoted by solid lines for gb and dotted lines for sb.*

In all considered Xe isotopes one finds that there is a band crossing of the (gb) and (sb).

The position of crossing point is between Lπ = 10+ and 12+ levels for 120-126Xe and Lπ = 8+ and 10+ for the two isotopes 128,130Xe.

In Figure(2) is shown the backbending plot, the variation of twice kinematic moment of inertia 2J(1) with square rotational frequency $\hbar^2\omega^2$ for the ground band (gb) and superband (sb) for our six isotopic chain 120-130Xe. The overall agreement between the calculated and experimental results is very good. As shown from the figure, all isotopes exhibit backbending.

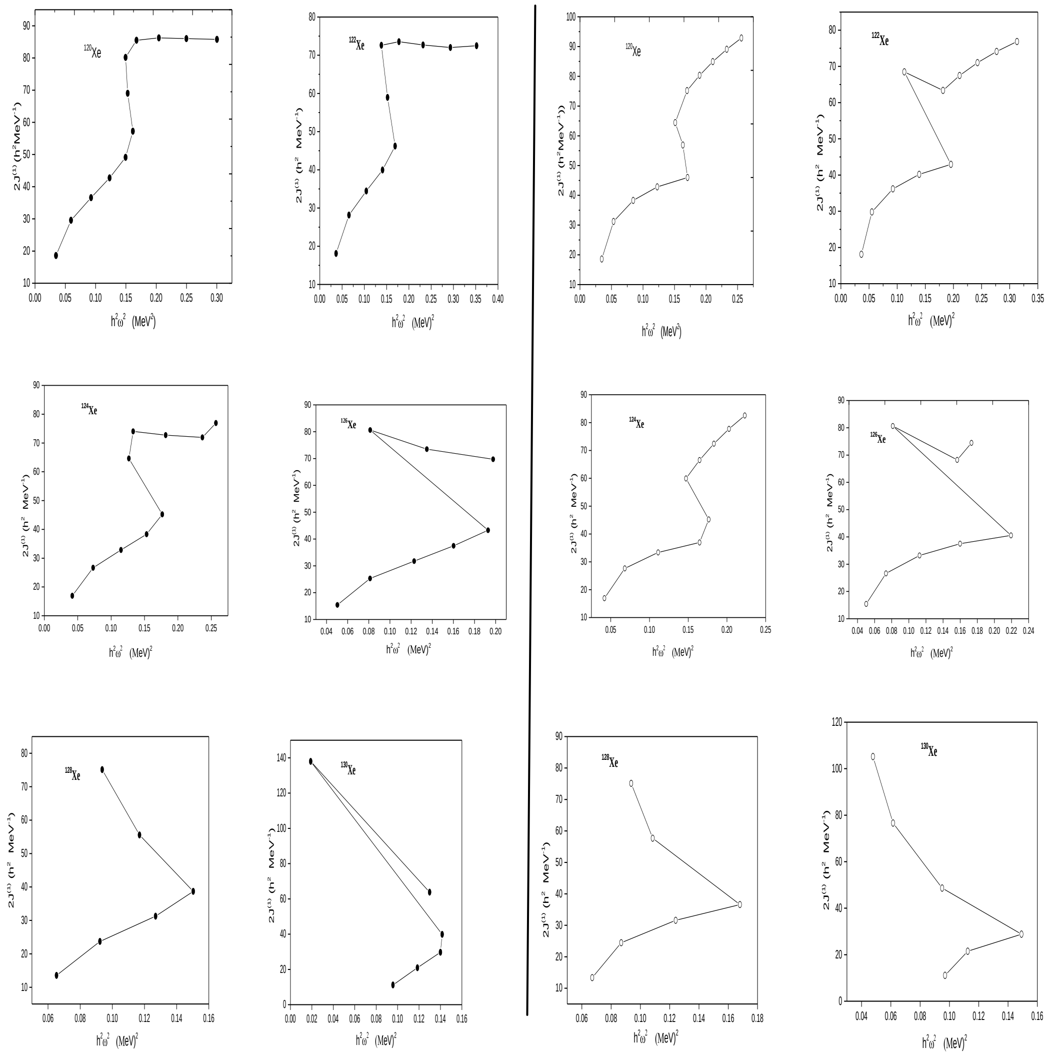

Figure2. *The backbending plot: twice kinematic moment of inertia 2J(1) versus square of rotational frequency ħ2ω2 for the yrast band of 120-126Xe isotopes. The experimental data are denoted by solid circles for ground band (gb) and open circles for super band (sb). The IVBM calculation are denoted by solid lines. The experimental energy levels are taken from Ref [27].*

Table(2) lists our IVBM calculations for excitation energies and backbending plots.

Table2. *Calculated excitation energies (KeV) and twice moment of inertia 2J (ħ2MeV-1) and the square of rotational frequency ħ2ω2 (MeV2) for the yrast bands of 120-130Xe.*

	$E^{exp}(L)$ (KeV)	L^{π} $(\hbar)$	E(L) (KeV)	$E_{\gamma}(L \rightarrow L-2)$ (keV)	$\hbar^2\omega^2$ (MeV^2)	2J ($\hbar^2MeV^{-1}$)
^{120}Xe	0.0	0^+	0.0			
	322.61	2^+	322.612	322.612	0.0346	18.5981
	796.16	4^+	771.190	448.578	0.0533	31.2097
	1397.3	6^+	1345.734	574.544	0.0845	38.2912
	2099.2	8^+	2046.244	700.510	0.1226	42.8259
	2872.7	10^+	2872.720	826.476	0.1707	45.9783
	3676.5	12^+	3681.000	808.280	0.1633	56.9109
	4458.9	14^+	4458.650	777.650	0.1511	69.4399
	5232.3	16^+	5283.200	824.550	0.1699	75.1925
	6051.0	18^+	6154.650	871.450	0.1898	80.3258
	6955.4	20^+	7073.000	918.350	0.2108	84.9349
	7955.1	22^+	8038.250	965.250	0.2329	89.0960
	9051.1	24^+	9050.400	1012.150	0.2561	92.8715

^{122}Xe	0.0	0^+	0.0 331.281	331.281	0.0365	18.1114
	331.28	2^+	800.912	469.630	0.0551	29.8106
	828.53	4^+	1408.891	607.979	0.0924	36.1854
	1467.1	6^+	2155.219	746.328	0.1392	40.1968
	2217.7	8^+	3039.896	884.676	0.1956	42.9535
	3039.9	10^+	3711.612	671.716	0.1128	68.4813
	3820.1	12^+	4563.720	852.108	0.1815	63.3733
	4563.9	14^+	5482.544	918.824	0.2110	67.4775
	5407.0	16^+	6468.084	985.540	0.2428	71.0270
	6370.1	18^+	7520.340	1052.256	0.2768	74.1264
	7453.1	20^+	8639.312	1118.312	0.3130	76.8561
	8639.7	22^+				
^{124}Xe	0.0	0^+	0.0			
	354.14	2^+	354.140	354.140	0.0418	16.9424
	879.03	4^+	860.794	506.653	0.0681	27.6322
	1548.8	6^+	1519.961	659.167	0.1112	33.3754
	2331.6	8^+	2331.642	811.681	0.1647	36.9603
	3172.1	10^+	3171.980	840.337	0.1765	45.2199
	3883.9	12^+	3939.408	767.428	0.1472	59.9404
	4613.4	14^+	4751.180	811.772	0.1647	66.5211
	5466.0	16^+	5607.296	856.116	0.1832	72.4200
	6439.1	18^+	6507.756	900.460	0.2027	77.7380
	7453.1	20^+	7452.560	944.804	0.2231	82.5560
^{126}Xe	0.0	0^+	0.0			
	388.63	2^+	388.622	388.622	0.0502	15.4391
	942.0	4^+	914.340	525.716	0.0733	26.6302
	1635.0	6^+	1577.154	662.814	0.1125	33.1918
	2435.7	8^+	2377.064	799.910	0.1599	37.5042
	3314.1	10^+	3314.070	937.006	0.2194	40.5547
	3884.6	12^+	3884.508	570.438	0.0813	80.6397
	4619.4	14^+	4675.930	791.422	0.1565	68.2316
	5508.6	16^+	5508.496	832.566	0.1732	74.4685
^{128}Xe	0.0	0^+	0.0			
	442.76	2^+	448.790	448.790	0.0671	13.3692
	1032.91	4^+	1021.305	572.515	0.0869	24.4535
	1737.01	6^+	1717.545	696.24	0.1241	31.5982
	2512.47	8^+	2537.508	819.963	0.1680	36.5870
	3196.29	10^+	3196.29	658.782	0.1084	57.6822
	3808.69	12^+	3808.692	612.402	0.0937	75.1140
^{130}Xe	0.0	0^+	0.0			
	536.04	2^+	539.4532	539.4532	0.0970	11.1223
	1204.54	4^+	1190.8864	651.4332	0.1125	21.4910
	1944.06	6^+	1954.2996	763.4132	0.1492	28.8179
	2696.83	8^+	2829.6928	616.6268	0.0950	48.6517
	2972.25	10^+	3817.066	496.0016	0.0615	76.6125
	3693.09	12^+	4916.4192	437.3096	0.0478	105.1886

Table (3) shows the angular velocity ωg(ωs) of ground (super) band and the ratio ωg/ωs at the crossing point Lc for the four Xe isotopes by using the formula

Table3. *Angular velocity ω and the position of the crossing spins Lc of ground band (gb) and super band (sb) for 120-130Xe isotopes*

Isotope	L_c	ω_g	ω_s	ω_g/ω_s
^{120}Xe	10	0.444	0.353	1.257
^{122}Xe	10	0.476	0.376	1.265
^{124}Xe	10	0.520	0.372	1.397
^{126}Xe	10	0.502	0.364	1.379
^{128}Xe	8	0.440	0.312	1.410
^{130}Xe	6	0.409	0.291	1.405

$$\omega(L) = \frac{dE(L)}{dL} = \frac{1}{4}[E(L+2) - E(L-2)]$$

We use the dimensionless quantity ωg/ωs as a measure of mismatch of the two slopes. We observe that the ratio increases with increasing mass number except for the isotope 126Xe. The crossing point between the ground band and superband is at Lc=10ħ for all isotopes. Figure (2) show that backbending becomes indeed more and sharper with increasing mass number. A sharp backbending occurs if the ground and super bands have very different angular velocities, i.e. the spin alignment is large.

5. Conclusion

In this paper, we have studied the exactly solvable rotational limit of IVBM. The theory is based on the reduction of the irreducible representation Sp(12,R) into angular momentum group So(3) through the compact unitary U(6) subgroup which defines a boson number preserving version of the model. As a consequence, we use the model Hamiltonian of the limit and its eigenstates to obtain the excitation energies of ground state band and superband in even-even 120-130Xe nuclei. Simulated search program has been written to determine the optimized model parameters using best fit method in order to obtain a minimum root-mean square deviation between the calculated IVBM and the experimental excitation energies. The backbending plots are investigated, analyzed and discussed in the considered six Xe isotopes.

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