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Effect of Uniform Magnetic Field on Rayleigh Bènard Marangoni Convection in a Relatively Hotter or Cooler Layer of Liquid

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Abstract: *The effect of uniform vertical magnetic field on the onset of steady Rayleigh-Bènard-Marangoni convection in a relatively hotter or cooler layer of electrically conducting liquid is studied theoretically by means of modified linear stability analysis. The top surface of the layer is non-deformable free where surface tension gradients arise on account of variation of temperature, and the bottom surface is rigid and thermally conducting. The eigenvalue equation is obtained by using the Fourier series method. Numerical results are obtained and presented. The results of this analysis indicate that the critical eigenvalues in the presence of magnetic field are greater in a relatively hotter layer of liquid than a cooler one under identical conditions otherwise. A detail description of marginal stability curves showing the influence of the magnetic field is presented and discussed. The asymptotic behavior of the magnetic field for large values of Chandrasekhar number is also obtained.*

Keywords: *Buoyancy, Convection, Linear stability, Surface tension, Chandrasekhar number.*

1. Introduction

The mechanism of the onset of surface tension induced convection in a thin horizontal fluid layer heated from below with free upper surface was reported experimentally by Block [5] and explained theoretically by Pearson [14]. They established that the patterned hexagonal cells observed by Bénard [3, 4] and explained by Rayleigh [15] in terms of buoyancy, were in fact due to temperature dependent surface tension. Convection driven by surface tension gradients is now commonly known as Bénard-Marangoni convection in contrast to the buoyancy driven Rayleigh-Bénard convection. Quantitative disagreement between experiment and theory has indicated that gravity was present in Bénard's experiments as well as in other experiments involving convection in a fluid layer with free surface in a laboratory on the earth, therefore, Nield [11] considered the combined effects of both the surface tension and buoyancy on the onset of convection in a liquid layer heated from below with free upper surface, called Rayleigh-Bénard-Marangoni convection, and found that the two effects causing instability are tightly coupled. For a detail study of convection one may be referred to the work of Chandrasekhar [6], Normand et al. [13], Koschmieder [9], and Schatz and Neitzel [16].

Since the process of controlling convection in a fluid has become important in material processing and because of its applications extending from producing large crystals of uniform properties to manufacturing new materials with unique properties. Recently, Gupta and Shandil [7] studied the surface tension driven convection in a relatively hotter or cooler layer of liquid, and Gupta et al. [8] considered the combined effects of both surface tension and buoyancy on the onset of convection in a relatively hotter or cooler layer of liquid. They established that irrespective of the driving force causing instability namely, surface tension or buoyancy or both, the hotter layer with its heat diffusivity apparently increased as a consequence of actual decrease in its specific heat at constant volume, must exhibit convection at a higher temperature difference, hence more stable than a cooler layer under identical conditions otherwise. The stabilizing nature of the magnetic field on buoyancy driven or/and surface tension driven convection in a liquid layer heated from below has been well established, notably by Chandrasekhar [6], Nield [12], Maekawa and Tanasawa [10] and by Wilson [18] and reference therein. Nevertheless, the effect of the magnetic field on the onset of Rayleigh-Bénard-Marangoni convection in a relatively hotter or cooler layer of liquid heated from below has not been given any attention in the literature despite its importance in controlling and understanding qualitatively the mechanism of convective instability problems encountered in geophysics, oceanography, atmospheric sciences and chemical engineering of paints and detergents.

The aim of the present study is to investigate the effect of a uniform vertical magnetic filed on the onset of Rayleigh-Bénard-Marangoni convection in a relatively hotter or cooler layer of liquid heated from below, thus extending the work of Gupta et al. [8] to include the effect of magnetic field. The Fourier series method is used to obtain the eigenvalue equation analytically. The numerical results are obtained for a wide range of parameters relevant to the problem in the present context. The result of this analysis indicate that the critical eigenvalues in the presence of magnetic field are greater in a relatively hotter layer of liquid than a cooler one under identical conditions otherwise, and also establish the inhibiting effect of the magnetic field. A detail description of marginal stability curves showing the influence of the magnetic field is presented. We find that when the magnetic field becomes sufficiently large, the two mechanisms causing instability become less tight as compared to the case when there is no magnetic field present. The asymptotic behavior of the magnetic field for large values of Chandrasekhar number is also obtained.

2. Mathematical Formulation of the Problem

The physical configuration of the problem consists of an infinite horizontal layer of viscous fluid of uniform thickness *d* in the presence of a uniform vertical magnetic field *H* and which is heated from below. The lower rigid boundary of the layer is kept at a constant temperature T_0 and the upper free surface is open to the atmosphere at temperature T_1 $(< T_0)$ subject to general heat radiative condition. We choose a Cartesian co-ordinate system of axis with the *x* and *y*-axis in the plane of the lower surface of the fluid layer and *z*-axis along the vertically upward direction so that the fluid layer is confined between the planes at $z = 0$ and $z = d$ as shown in Fig (1). The surface tension on the upper free surface of fluid layer is regarded as a function of temperature only which is given by the simple linear law

$$\tau = \tau_1 - \sigma (T - T_1) \tag{1}$$

where the constant τ_1 is the unperturbed value of τ at the unperturbed surface temperature $T = T_1$ and $-\sigma = (\partial\tau / \partial T)_{T=T_1}$ represents the rate of change of surface tension with temperature evaluated at temperature T_1, and surface tension being a monotonically decreasing function of temperature, σ is positive.

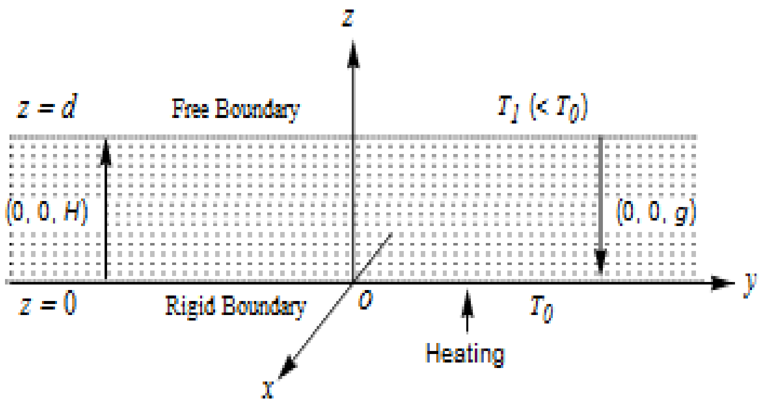

Fig1. *Schematic representation of the physical configuration of the problem*

Under the Boussinesq approximation, the modified linearized perturbation equations for an electrically conducting liquid in the presence of a uniform magnetic field in the present context (Banerjee et al. [2]) are given as

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\nabla^2 w = g\alpha\nabla_1^2\theta + \frac{\mu H}{4\pi\rho}\frac{\partial}{\partial z}\left(\nabla^2 h_z\right), \tag{2}$$

$$(1-\alpha_2 T_0)\left(\frac{\partial \theta}{\partial t}-\beta w\right)= \kappa\nabla^2\theta \quad , \tag{3}$$

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z} \tag{4}$$

where *w* is the perturbation velocity, θ the perturbation temperature, h_z is the *z*-component of the perturbation from the basic vertical magnetic field *H* and ρ is the density of the fluid. The adverse uniform temperature gradient $\beta=(T_0-T_1)/d>0$, the gravitational acceleration *g*, the coefficient of volume expansion α, the kinematic viscosity *v*, the thermal diffusivity κ, the magnetic permeability μ and the magnetic resistivity $\eta = 4\pi\mu\in$ (where $\in$ is the electrical conductivity) are each assumed constant, ∇^2 and ∇_1^2 represent respectively

$$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} \text{ and } \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},$$

and *t* represents time. Further, the coefficient α_2 (due to variation in specific heat at constant volume on account of variations in temperature) lies in the range 0 to 10^{-4} and range of the dimensionless parameter $\alpha_2 T_0$ covering the usual laboratory conditions is $0\le\alpha_2T_0<1$ for liquids with which we are mostly concerned. In this range, any given value of $\alpha_2T_0(\neq 0)$ corresponds to the layer of liquid which is relatively hotter compared to that associated with its value less than (including $\alpha_2T_0=0$) the given one. A detailed account of this has been given in the research monograph by Banerjee and Gupta [1].

Equations (2) – (4), must be solved subject to appropriate boundary conditions. For the rigid and perfectly thermally conducting (the perturbation temperature is thus zero) lower boundary at *z* = 0, the boundary conditions are

$$w=0,\quad \frac{\partial w}{\partial z}=0,\quad \theta=0 \tag{5a, b, c}$$

Since the tangential viscous stress experienced by the liquid at the upper free surface is balanced by the traction due to variation with temperature of surface tension, and the general heat radiation condition (Sparrow et al. [17]) at *z* = *d*, the boundary conditions are

$$w=0,\quad \rho\nu\frac{\partial^2 w}{\partial z^2}=\sigma\nabla_1^2\theta,\quad -k\frac{\partial\theta}{\partial z}=q\theta\ , \tag{6a, b, c}$$

where *k* is the heat conductivity and *q* is the rate of change with temperature of the time rate of heat loss per unit area from the upper surface.

We now analyze an arbitrary disturbance in terms of normal modes assuming that the perturbations w, θ and h_z are of the form

$$[w(x,y,z,t),\theta(x,y,z,t),h_z(x,y,z,t)]=[W(z),\Theta(z),K(z)]\,exp\,\{i(a_x x+a_y y)+st\}$$

where a_x and a_y are components of the horizontal wave number $a\left(=\sqrt{a_x^2+a_y^2}\right)$ of the disturbance, and *s* is the time growth rate (a complex number in general). Using the above expressions for *w*, θ and h_z in equations (2) – (4) and then making the resulting equations dimensionless by choosing $d, d^2/\nu, d/\nu, \beta d(\nu/k)$ and *H* as units of length, time, velocity, temperature and magnetic field scales respectively; on putting

$$D^*=d\frac{d}{dz},\quad a^*=ad,\quad W^*=\frac{wd}{\nu},\quad \Theta^*=\frac{\theta\kappa}{\beta d\nu},$$

$$p^*=\frac{sd^2}{\nu},\quad P_r^*=\frac{\nu}{\kappa},\quad P_m^*=\frac{\nu}{\eta},\quad K^*=\frac{K\eta}{H\nu}$$

and omitting asterisks(*) for convenience, we obtain

$$\left(D^2-a^2\right)\left(D^2-a^2-p\right)W+Q\,D\left(D^2-a^2\right)K=Ra^2\Theta\ ,\tag{7}$$

$$\left(D^2-a^2-(1-\alpha_2T_0)P_r\,p\right)\Theta=-(1-\alpha_2T_0)W\ ,\tag{8}$$

$$\left(D^2-a^2-P_m\,p\right)K=-DW\ ,\tag{9}$$

where $P_r\,,P_m\,,\ R=g\alpha\beta\,d^4/\kappa\nu$ and $Q=\mu H^2d^4/4\pi\rho\,\nu\eta$ are respectively the thermal Prandtl number, magnetic Prandtl number, Rayleigh number and Chandrasekhar number.

We restrict our analysis to the case when the principle of exchange of stabilities is valid for the present problem so that instability first sets in as stationary convection. In that case, we can put p=0 in equations (7) – (9) and when K is eliminated from the resulting equations, we obtain

$$\left[\left(D^2-a^2\right)^2-QD^2\right]W=Ra^2\Theta\ ,\tag{10}$$

$$\left(D^2-a^2\right)\Theta=-(1-\alpha_2T_0)W\ ,\tag{11}$$

In terms of new variables, the non-dimensional form of boundary conditions (5a, b, c) – (6a, b, c) can be written as

$$W(0)=0,\quad DW(0)=0\ \text{and}\ \Theta(0)=0\ ,\tag{12a, b, c}$$

evaluated on the lower boundary z = 0, and

$$W(1)=0,\quad D^2W(1)+a^2M\Theta(1)=0\,,D\Theta(1)+L\Theta(1)=0\ ,\tag{13a, b, c}$$

evaluated on the upper boundary z = 1. Here $M=\sigma\beta\,d^2/\rho\kappa\,\nu$ is the Marangoni number and $L=qd/k$ is the Biot number. We note that the boundary conditions on magnetic field do not affect the stability condition.

Equations (10) – (11) together with boundary conditions (12a, b, c) – (13a, b, c) constitute an eigenvalue problem of order six with M as an eigenvalue for given values of the remaining parameters.

3. SOLUTION OF THE PROBLEM

The Fourier series method as presented by Nield [11] is convenient for the problem under consideration. The method consists of expanding each of the two variables W(z) and $\Theta(z)$ in two equivalent forms:

- As the sum of a polynomial and a Fourier series which can be differentiated the required number of times, and
- As a single Fourier series.

Thus, we let

$$W(z)=\frac{1}{6}\left[D^2W(1)-D^2W(0)\right]z^3+\frac{1}{2}D^2W(0)z^2+$$
$$+\left[W(1)-W(0)-\frac{1}{3}D^2W(0)-\frac{1}{6}D^2W(1)\right]z+W(0)+\sum_{n=1}^{\infty}A_n\sin\,n\pi z\tag{14a}$$

$$W(z)=\sum_{n=1}^{\infty}\left[\begin{array}{l}A_n-\dfrac{2}{n^3\pi^3}\left\{D^2W(0)-(-1)^nD^2W(1)\right\}\\[2mm]+\dfrac{2}{n\pi}\left\{W(0)-(-1)^nW(1)\right\}\end{array}\right]\sin\,n\pi z\tag{14b}$$

and
$$\Theta(z)=\left[\Theta(1)-\Theta(0)\right]z+\Theta(0)+\sum_{n=1}^{\infty}B_n\sin\,n\pi z\ ,\tag{15a}$$

$$\Theta(z)=\sum_{n=1}^{\infty}\left[B_n+\frac{2}{n\pi}\left\{\Theta(0)-(-1)^n\Theta(1)\right\}\right]\sin\,n\pi z\ .\tag{15b}$$

The forms (14a) and (15a) are suitable for satisfying the boundary conditions. The complete Fourier expansions for W and Θ given by (14b) and (15b) are substituted in the differential equations and the coefficients of $\sin n\pi z$ are equated. The resulting system of equations can be solved for A_n and B_n, which are then substituted in the boundary conditions. Finally elimination of $D^2W(0)$ and $D^2W(1)$ gives the eigenvalue equation which may be written as

$$\begin{vmatrix} \sum_{n=1}^{\infty}\frac{F_n}{H_n} & M\sum_{n=1}^{\infty}\frac{(-1)^n F_n}{H_n}+R\sum_{n=1}^{\infty}\frac{(-1)^n E_n}{H_n} \\ \sum_{n=1}^{\infty}\frac{(-1)^n En}{H_n} & M\sum_{n=1}^{\infty}\frac{E_n}{H_n}-\frac{1}{a^2(1-\alpha_2 T_0)}\left(\sum_{n=1}^{\infty}\frac{G_n}{H_n}+\frac{L+1}{2}\right)+R\sum_{n=1}^{\infty}\frac{1}{H_n}-\frac{Q}{(1-\alpha_2 T_0)}\sum_{n=1}^{\infty}\frac{E_n}{H_n} \end{vmatrix}=0 \tag{16}$$

Where

$$\left.\begin{aligned} E_n &= n^2\pi^2 \\ F_n &= n^2\pi^2(n^2\pi^2+a^2) \\ G_n &= a^2(n^2\pi^2+a^2)^2 \\ H_n &= (n^2\pi^2+a^2)^3-Ra^2(1-\alpha_2 T_0)+Qn^2\pi^2(n^2\pi^2+a^2) \end{aligned}\right\} \tag{17}$$

We can obtain M from the eigenvalue equation (16) in terms of $a,\ R,\ L,\ \alpha_2 T_0$ and Q as the ratio for two determinants given by

$$M=\frac{\begin{vmatrix} \sum_{n=1}^{\infty}\frac{F_n}{H_n} & -R\sum_{n=1}^{\infty}\frac{(-1)^n E_n}{H_n} \\ \sum_{n=1}^{\infty}\frac{(-1)^n E_n}{H_n} & -R\sum_{n=1}^{\infty}\frac{1}{H_n}+\frac{1}{a^2(1-\alpha_2 T_0)}\left(\sum_{n=1}^{\infty}\frac{G_n}{H_n}+\frac{L+1}{2}\right)+\frac{Q}{(1-\alpha_2 T_0)}\sum_{n=1}^{\infty}\frac{E_n}{H_n} \end{vmatrix}}{\begin{vmatrix} \sum_{n=1}^{\infty}\frac{F_n}{H_n} & \sum_{n=1}^{\infty}\frac{(-1)^n F_n}{H_n} \\ \sum_{n=1}^{\infty}\frac{(-1)^n E_n}{H_n} & \sum_{n=1}^{\infty}\frac{E_n}{H_n} \end{vmatrix}} \tag{18}$$

4. Numerical Results and Discussion

The numerical calculations may be carried out as follows. For fixed values of Q, $\alpha_2 T_0$ and L the eigenvalue equation (18) gives the relations between M and R with a as a parameter. If we fix the values of R, Q, $\alpha_2 T_0$ and L, the expression (18) determines the Marangoni number M as a function of the wave number a. The minimum value of M is the critical Marangoni number M_c and the value of a at which M attains the minimum is the critical wave number a_M. Alternatively, for fixed values of M, Q, $\alpha_2 T_0$ and L, we may determine the Rayleigh number R as a function of the wave number a. The minimum value of R is the critical Rayleigh number R_c and the value of a at which R attains the minimum is the critical wave number a_R. By either means we can plot the (R, M)-curve for fixed values of Q, $\alpha_2 T_0$ and L corresponding to the marginal stability.

The numerical values of M_c and a_M (R = 0) computed from expression (18) when L = 0 and $L = 10^6 (\to \infty)$ are respectively presented in Table 1 and Table 2 for various given values of $\alpha_2 T_0$ and Q. The values of Q are chosen so as to compare the results obtained by us with the results of Nield [12]. We find that when $\alpha_2 T_0 = 0$ values of M_c and a_M for various values of Q obtained here are in close agreement with corresponding values obtained by Nield [12]. When Q=0 the results obtained here for various values of $\alpha_2 T_0$ agree precisely with corresponding values obtained by Gupta et al. [8].

Table1. *Values of M_c and a_M for various values of Q and $\alpha_2 T_0$ when L = 0*

Q	$\alpha_2 T_0 = 0$		$\alpha_2 T_0 = 0.3$		$\alpha_2 T_0 = 0.5$	
	M_c	a_M	M_c	a_M	M_c	a_M
0	79.61	1.99	113.73	1.99	159.22	1.99
2.5	85.97	2.05	122.82	2.05	171.94	2.05
12.5	110.08	2.22	157.26	2.22	220.16	2.22
25	138.09	2.39	197.28	2.39	276.19	2.39
50	189.87	2.63	271.25	2.63	379.75	2.63
125	328.69	3.08	469.56	3.08	657.38	3.08
250	536.91	3.53	767.01	3.53	1073.81	3.53
500	919.78	4.08	1313.97	4.08	1839.55	4.08
1000	1632.47	4.74	2332.10	4.74	3264.94	4.74
2500	3624.93	5.84	5178.47	5.84	7249.85	5.84
5000	6773.06	6.86	9675.80	6.86	13546.10	6.86
10000	12830.20	8.09	18328.80	8.09	25660.30	8.09

Table2. *Values of M_c and a_M for various values of Q and $\alpha_2 T_0$ when $L = 10^6$*

Q	$\alpha_2 T_0 = 0$		$\alpha_2 T_0 = 0.3$		$\alpha_2 T_0 = 0.5$	
	M_c	a_M	M_c	a_M	M_c	a_M
0	32.07 x10^6	3.01	45.81 x10^6	3.01	64.14 x10^6	3.01
2.5	33.68 x10^6	3.11	48.11 x10^6	3.11	67.36 x10^6	3.11
12.5	39.41 x10^6	3.46	56.31 x10^6	3.46	78.82 x10^6	3.46
25	45.53 x10^6	3.81	65.04 x10^6	3.81	91.06 x10^6	3.81
50	55.74 x10^6	4.36	79.63 x10^6	4.36	111.48 x10^6	4.36
125	78.91 x10^6	5.55	112.73 x10^6	5.55	157.82 x10^6	5.55
250	107.34 x10^6	6.96	153.34 x10^6	6.96	214.68 x10^6	6.96
500	149.52 x10^6	9.12	213.60 x10^6	9.12	299.04 x10^6	9.12
1000	210.70 x10^6	12.50	301.00 x10^6	12.50	421.40 x10^6	12.50
2500	333.03 x10^6	19.66	475.76 x10^6	19.66	666.06 x10^6	19.66
5000	470.97 x10^6	27.79	672.81 x10^6	27.79	941.94 x10^6	27.79
10000	666.06x10^6	39.31	951.51 x10^6	39.31	1332.12 x10^6	39.31

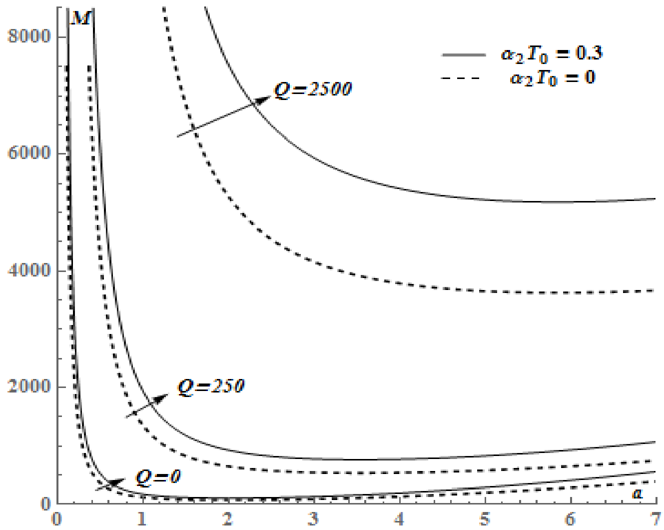

Fig2. *Marginal stability curves at the onset of Bénard-Marangoni convection for various values of Q when $\alpha_2 T_0 = 0$ (dotted curves) and $\alpha_2 T_0 = 0.3$ (thick curves).*

The numerical values of R_c and a_R (M=0) computed from expression (18) when L=0 and $L = 10^6 (\to \infty)$ are respectively presented in Table.3 and Table.4 for various given values of $\alpha_2 T_0$ and Q. When $\alpha_2 T_0 = 0$ values of R_c and a_R for various values of Q obtained here agree precisely with corresponding values as obtained by Nield [12]. When Q=0 the results obtained here for various values of $\alpha_2 T_0$ agree precisely with corresponding values obtained by Gupta et al. [8].

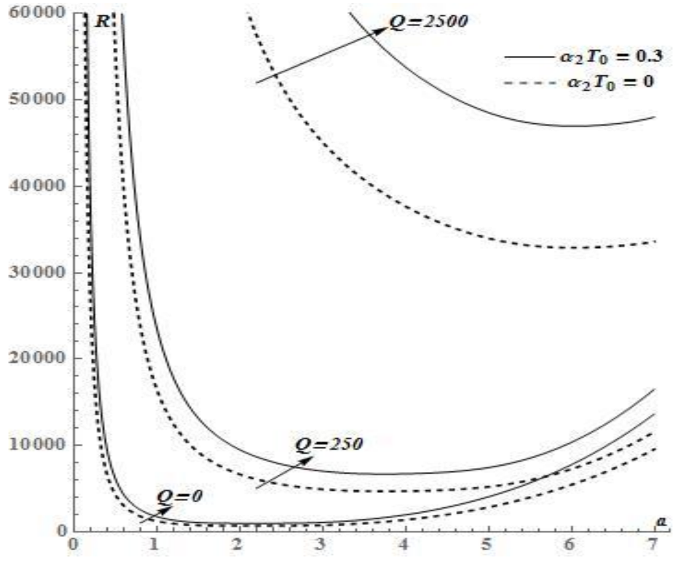

Fig3. *Marginal stability curves at the onset of Rayleigh-Bénard convection for various values of Q when* $\alpha_2 T_0 = 0$ *(dotted curves) and* $\alpha_2 T_0 = 0.3$ *(thick curves).*

For fixed values of $\alpha_2 T_0$ and Q, it may be mentioned here that as L increases values of both M_c and R_c increase. In the limit $L \to \infty$, we find that value of M_c becomes asymptotically proportional to L while that of R_c tends to a finite limit. In either case, the corresponding critical wave number remains finite. The case when L increases from 0 to ∞ corresponds to the situation wherein thermal boundary conditions at the free surface changes from constant heat flux $(D\Theta(1) = 0)$ to constant temperature $(\Theta(1) = 0)$. Therefore, when L is small it is easier for temperature perturbations to be set-up, but when L is large, any temperature variations across the free surface decay rapidly. Thus as L becomes large, the values of M_c tend to infinity since it becomes difficult for the surface tension to be operative, while those of R_c remain finite since the buoyancy force is still operative.

The neutral stability curves for surface tension driven convection (R=0) and for buoyancy driven convection (M=0) are plotted respectively in Fig.2 and Fig.3 for various values of Q and $\alpha_2 T_0$ when L=0. Points below a curve represent stable state. The upward moving neutral stability curves in Fig 2 or 3, for increasing values of Q when value of $\alpha_2 T_0$ is fixed, illustrate that magnetic field has stabilizing effect on the onset of convection in each case. Further, it is also seen that increasing values of $\alpha_2 T_0$ for a fixed value of Q has stabilizing effect in each case. We find that presence of magnetic field suppresses convection more effectively in a relatively hotter layer of liquid than a cooler one under identical conditions otherwise irrespective of whether the agency causing instability is surface tension or buoyancy.

Table3. *Values of* R_c *and* a_R *for various values of Q and* $\alpha_2 T_0$ *when L = 0*

Q	$\alpha_2 T_0 = 0$		$\alpha_2 T_0 = 0.3$		$\alpha_2 T_0 = 0.5$	
	R_c	a_R	R_c	a_R	R_c	a_R
0	669.00	2.09	955.71	2.09	1338.00	2.09
2.5	722.05	2.14	1031.50	2.14	1441.10	2.14
12.5	924.10	2.33	1320.14	2.33	1848.20	2.33
25	1161.06	2.51	1658.57	2.51	2322.12	2.51
50	1604.30	2.77	2291.86	2.77	3204.60	2.77
125	2815.68	3.26	4022.41	3.26	5631.36	3.26
250	4665.77	3.74	6665.39	3.74	9331.54	3.74
500	8112.65	4.32	11589.48	4.32	16225.30	4.32
1000	14594.50	4.99	20849.29	4.99	29189.00	4.99
2500	32888.40	6.03	46983.57	6.03	65777.00	6.03
5000	62007.40	6.94	88582.00	6.94	124014.80	6.94
10000	118361.00	7.95	169087.14	7.95	236722.00	7.95

Table4. *Values of R_c and a_R for various values of Q and $\alpha_2 T_0$ when $L = 10^6$*

Q	$\alpha_2 T_0 = 0$		$\alpha_2 T_0 = 0.3$		$\alpha_2 T_0 = 0.5$	
	R_c	a_R	R_c	a_R	R_c	a_R
0	1100.65	2.68	1572.36	2.68	2201.30	2.68
2.5	1167.10	2.75	1667.29	2.75	2334.20	2.75
12.5	1415.38	2.97	2021.97	2.97	2830.76	2.97
25	1699.40	3.17	2427.71	3.17	3398.80	3.17
50	2217.39	3.47	3167.70	3.47	4434.78	3.47
125	3585.80	4.00	5122.57	4.00	7171.60	4.00
250	5612.78	4.50	8018.26	4.50	11225.56	4.50
500	9303.60	5.09	13290.86	5.09	18607.20	5.09
1000	16118.02	5.76	23025.74	5.76	32236.04	5.76
2500	35042.01	6.78	50060.01	6.78	70084.02	6.78
5000	64844.00	7.66	92634.29	7.66	129688.00	7.66
10000	122140.00	8.65	174485.71	8.65	244280.00	8.65

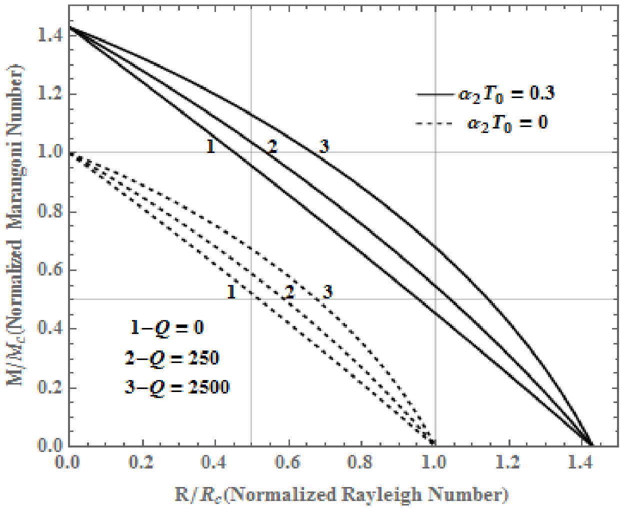

Fig4. *Variation of Marangoni and Rayleigh numbers (normalized to give unit intercepts) at the onset of Rayleigh-Bénard-Marangoni convection for various values of Q when $\alpha_2 T_0 = 0$ (dotted curves) and $\alpha_2 T_0 = 0.3$ (thick curves).*

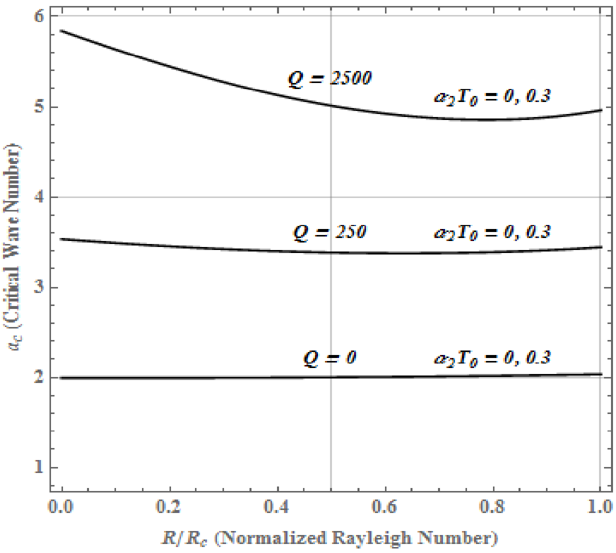

Fig5. *Wave number corresponding to marginal stability, plotted against normalized Rayleigh number at the onset of Rayleigh-Bénard-Marangoni convection for various values of Q when $\alpha_2 T_0 = 0.3$.*

The asymptotic behavior of M_c as $Q \to \infty$ depends critically on both $\alpha_2 T_0$ and L, whereas the asymptotic behavior of a_M depends only on L. When L=0, we find that

$$M_c \approx \frac{Q}{1-\alpha_2 T_0} \text{ and } a_M \approx 0.81\, Q^{\frac{1}{4}}$$

which are in close agreement with the corrected values $M_c \approx Q$ and $a_M \approx 0.79\, Q^{\frac{1}{4}}$ as obtained by Wilson [18] (when $\alpha_2 T_0 = 0$) after correction of the inaccurate asymptotic results of both Nield [12] and Maekawa and Tanasawa [10].

When *L* is large, we find that

$$\frac{M_c}{L} \approx \frac{6.66 \times Q^{\frac{1}{2}}}{1-\alpha_2 T_0} \text{ and } a_M \approx 0.393\, Q^{\frac{1}{3}}$$

which are in close agreement with the results of Nield [12] (when $\alpha_2 T_0 = 0$).

On the other hand, the asymptotic behaviour of R_c as $Q \to \infty$ depends only on $\alpha_2 T_0$, whereas the asymptotic behaviour of both R_c and a_R is independent of *L*. For any value of *L*, we find that

$$R_c \approx \frac{\pi^2}{1-\alpha_2 T_0} Q \text{ and } a_R \approx \left(\frac{\pi^4}{2} Q\right)^{\frac{1}{6}}$$

which are in close agreement with the asymptotic results of both Chandrasekhar [6] and Nield [12] when $\alpha_2 T_0 = 0$.

The (*R*, *M*)-loci corresponding to neutral stability curves for the combined surface tension and buoyancy effects, normalized for critical values of R_c and M_c are plotted in Fig.4 for various values of *Q* and $\alpha_2 T_0$ for the limiting case when *L*=0 (insulating boundary). The values plotted are the critical values at the corresponding wave numbers. Fig.4 illustrates that the inhibiting effect of magnetic field on the onset of convection remains unchanged for given values of $\alpha_2 T_0$. As *Q* increases the locus goes away from the line

$$\frac{R}{R_c} + \frac{M}{M_c} = \frac{1}{1-\alpha_2 T_0}$$

which (as explained in Gupta et al.[8]) corresponds to perfect coupling showing that coupling between the two agarics causing instability becomes weaker. Fig 5 illustrates the variation of the corresponding critical wave numbers at the marginal stability for various values of *Q*. We note that the value of critical wave number remains unchanged for different values of $\alpha_2 T_0$.

5. CONCLUSION

The linear stability analysis of the Rayleigh-Bénard-Marangoni convection in a relatively hotter or cooler layer of liquid in the presence of magnetic field has been studied theoretically and we conclude that a relatively hotter layer of liquid is more stable than a cooler one under identical conditions otherwise irrespective of whether the two mechanisms causing instability act individually or simultaneously.

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