International Journal of Advanced Research in Physical Science (IJARPS)
Volume 2, Issue 1, January 2015, PP 53-58
ISSN 2349-7874 (Print) & ISSN 2349-7882 (Online)
www.arcjournals.org

Odd- Even Staggering Within Rotational Vibrational Model for Ground- Octupole Bands in Some Actinide Nuclei

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The ΔI=1staggering effect in energies of the states between the even spin positive parity ground band and the odd spin negative parity octupole band in actinide nuclei $^{224,226}Ra$, $^{224,226,228,232}Th$ and $^{236,238}U$ have been examined in framework of rotational vibrational model. Optimum values of the model parameters have been determined from the comparison of the experimental and calculated values for the excitation energies. for each nucleus we calculated the deviation of ΔI=1 gamma transition energies from smooth reference representing the finite difference approximation of the fourth order derivative of transition energies at a given spin. Our selected nuclei exhibit a beat pattern.

1. Introduction

One of the most interesting effects occurring in rapidly deformed rotating nuclei is the anomalous odd- even staggering (OES) between the energies of positive parity ground band and the negative parity octupole band. Collective models based on point symmetry group considerations were used for the description of the energy levels of the ground and octupole bands and reproduce the OES between these levels [1, 2]. An extensive experimental data were obtained on octupole deformed bands of negative parity [3, 4]. Theoretical studies of such octupole bands have been presented in framework of the cranked random phase approximation (RPA) [5,6], the collective model [7], the interacting boson model (IBM) [8,9], the variable moment of inertia (VMI) model [10], the alpha-particle cluster model [11], the reflection asymmetric shell model [13] and symplectic extension of the interacting vector boson model (IVBM) [14]. The purpose of the present work is to use the rotational –vibrational model to investigate the collective negative parity states and to study the behavior of odd- even ΔI=1 staggering in energies. In section 2 a brief review of the rotational-vibrational model is presented. Section 3 describes the odd- even staggering patterns between the energies of states from ground and octupole bands. The results for Ra– Th– U nuclei are described in section 4. The main conclusion is summarized in section 5.

2. Brief Review of Rotational – Vibrational Model

The symmetric rotor model is generally rather good for the well deformed even –even nuclei. This is because the pairing of particles in an even –even nucleus gives the intrinsic system a stability that is not readily perturbed by the rotation. The rotational energy for the ground band may be written as function of spin I as:

$$E_g(I) = A_g I(I+1) \tag{1}$$

where $A_g = \hbar^2/2\theta_o$ is the inertial parameter. The moment of inertia θ_o for the rotational band can be related to the transition energy

$$E_\gamma(I) = E(I) - E(I-2) \tag{2}$$

by the relation:

$$\frac{2\theta}{\hbar^2} = \frac{4I-2}{E_\gamma(I)} \tag{3}$$

In vibrational model the members of these bands have excitation energies

$$E(I) = B_g I \tag{4}$$

where B_g is the vibrational constant

So that the members of rotational – vibrational model for ground band have the general form

$E_g(I) = A_g I(I+1) + B_g I$ (5)

Octupole correlations have attracted much interest over the years, especially in deformed actinide, because they are often associated with some of the lowest collective modes being observed. Rotational octupole bands of states with odd spins and negative parity appear at excitation energies E_o < 1 MeV, which are interpreted as structure based on one– phonon octupole vibration. This leads to a simplified expression for the energies of the same types of ground band

$E_o(I) = A_o I(I+1) + B_o I + E_o$ (6)

where the phenomenological parameters A_o , B_o and E_o are the octupole energy parameters.

3. The Odd-Even ΔI=1 Staggering

The ΔI=1 staggering patterns between the energies of states from the ground and octupole bands have been investigated [1]. In order to test the proposed model, we applied it on the ground and octupole bands, the staggering displacement function defined as [15].

$$\Delta^4 E_\gamma(I) = \frac{1}{16} [E_\gamma(I+2) - 4E_\gamma(I+1) + 6E_\gamma(I) - 4E_\gamma(I-1) + E_\gamma(I-2)] \quad (7)$$

where $E_\gamma(I)$ denote the dipole transition energy

$$E_\gamma(I) = E(I) – E(I-1) \quad (8)$$

Gives the odd- even energy difference. Function (7) is a finite difference of the fourth order derivative of $E_\gamma(I)$, and is very sensitive even to small deviations of the energies. This formula includes five consecutive transition energies and is denoted by five point formula. We say that ΔI=1 staggering is observed if the quantity $\Delta^4 E_\gamma(I)$ exhibits alternating signs with increasing the spin I.

4. Numerical Calculations and Discussions

In the present applications to the deformed even- even nuclei, the model parameters A_g, B_g, A_o, B_o and E_o are adjusted by fitting the experimental level excitation energies of the ground and octupole bands to the calculated ones using the rotational vibrational model. The quality of the fit is determined by minimizing the root-mean square (rms) deviation χ defined by

$$\chi(N) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{E^{exp}(I_i) - E^{cal}(I_i)}{E^{exp}(I_i)}\right)^2}$$

where the sum are over the available data points and N is the number of energy levels entering the fitting. The best fit parameters are listed in Table (1), and have been used to calculate the transition energies. The experimental and calculated energy spectra are compared in Figures(1, 2). The experimental energies are taken from the National Nucear Data Center [16]. Fig.(3) Presents the energy staggering displacement function $\Delta^4 E_\gamma(I)$ between the even- spin positive parity of ground band and odd- spin negative parity of octupole band as a function of nuclear spin I, for our eight actinide Ra- Th- U nuclei. A more complicated beat staggering structure is observed. The respective ΔI=1 staggering patterns has been reproduced successfully.

Table1. *The adopted best model parameters for ground and octupole bands in the studied Ra/Th/U nuclei.*

		Ag A_o	Bg B_o	E_o
^{224}Ra	Ground Octupole	4.03532 6.62422	65.48988 15.00812	171.26912
^{226}Ra	Ground Octupole	3.87539 4.10643	56.16166 46.9832	94.5604
^{224}Th	Ground Octupole	5.68456 5.95391	52.91004 40.17108	119.90908
^{226}Th	Ground Octupole	5.44376 5.60832	42.847 31.69164	148.77224
^{228}Th	Ground Octupole	5.72766 5.55372	26.60732 11.24032	314.52832
^{232}Th	Ground	4.3707	39.6394	

	Octupole	4.07224	30.2525704	651.42704
^{236}U	Ground	4.82196	27.9052	
	Octupole	4.52257	26.27484	585.29884
^{238}U	Ground	4.055959	31.2505	
	Octupole	4.21662	26.04784	579.67784

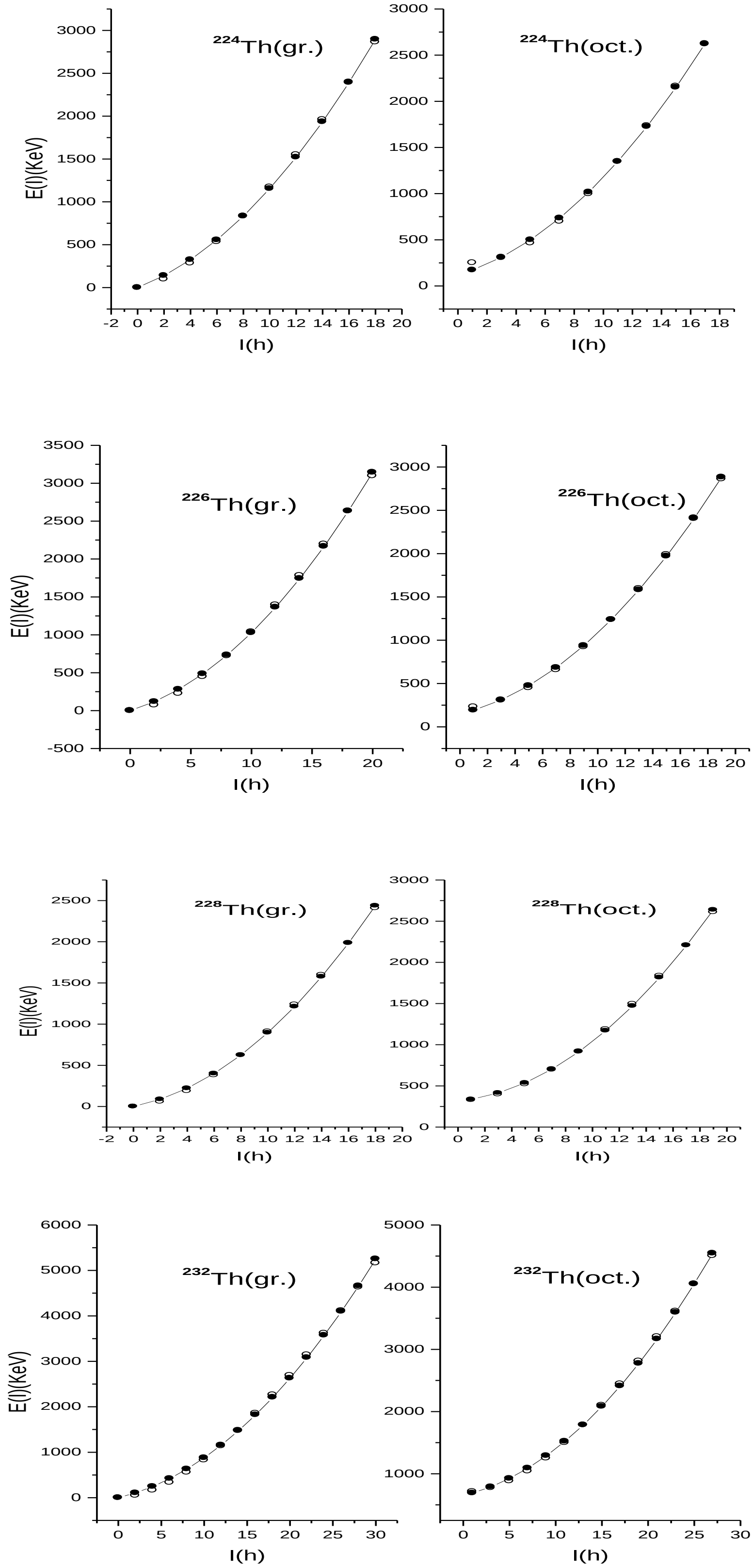

Fig1. *The Calculated energies E(I) (solid curves) of the ground (right panels) and octupole (left panel) for ^{224}Th, ^{226}Th, ^{228}Th and ^{232}Th compared with experimental data (open circles).*

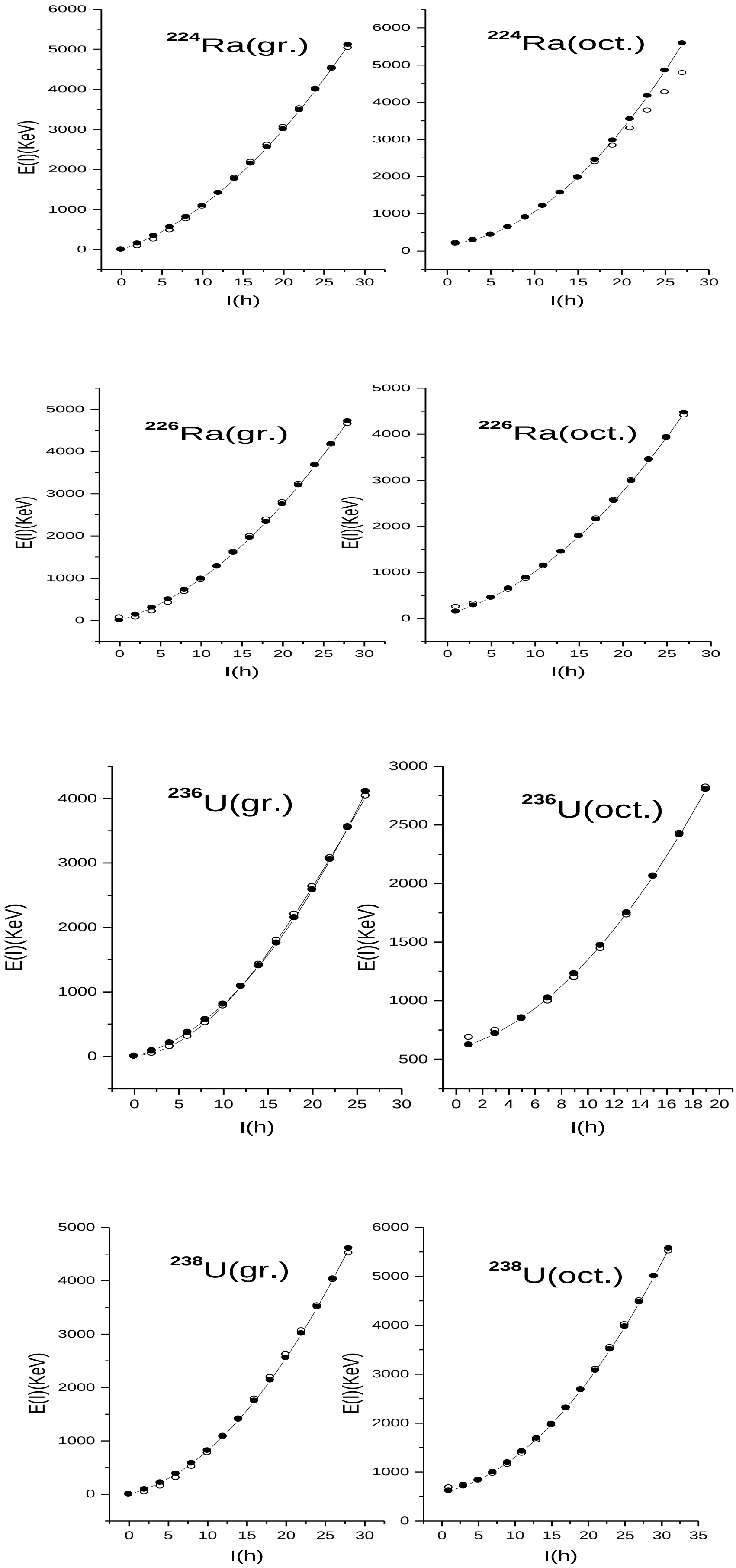

Fig2. *The same as in Fig. (1) but for ^{224}Ra, ^{226}Ra, ^{236}U and ^{238}U*

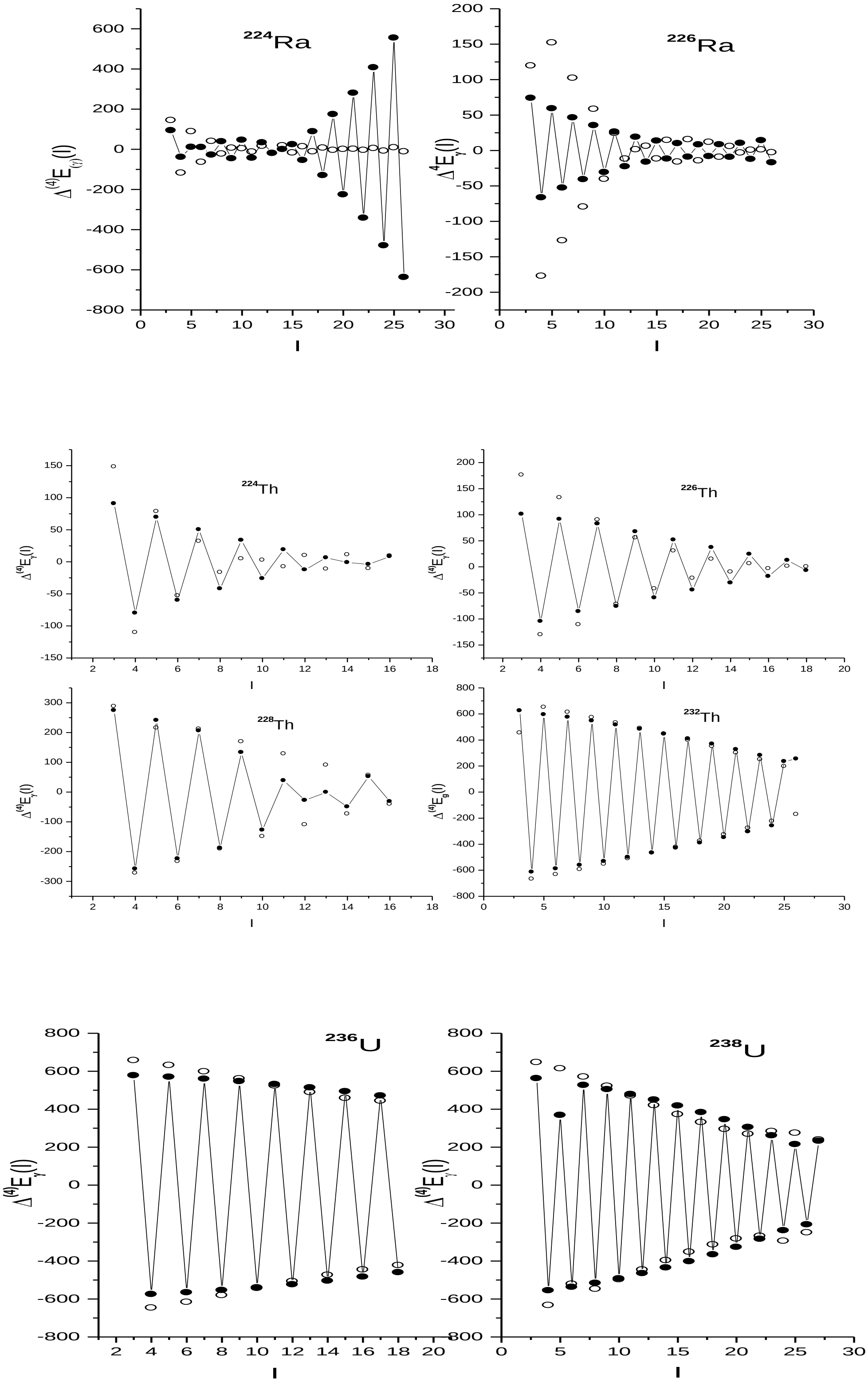

Fig3. *The odd- even $\Delta I=1$ staggering quantity $\Delta^4 E_\gamma(I)$ as a function of spin for the selected Ra/Th/U nuclei.*

5. Conclusion

The ground and octupole bands have been observed by using the rotational- vibrational model. This model is capable to produce the complicated beat staggering patterns of alternating parity bands in light actinide nuclei. The odd- even ΔI=1 staggering is explained as a result of different Carioles induced alignments of the rotational and octupole vibrational angular momentum for odd- and even- spin members of the octupole band. The present results confirm that the rotational-vibrational model is a practical tool in studying ground rotational and octupole vibrational bands.

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