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Imaginary Background of Universe Expansion (A Fairy Tale)

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Abstract: *The expansion of universe in the light of modified Heraclitean dynamics has been discussed.*

Keywords: *expansion of universe, Heraclitean dynamics, complex ground mass*

1. Introduction

Let us see how Heraclitean dynamics fits with the expansion of the universe.

2. The Nature of Expansion

The gravitation between real masses is explained by Newton's law [1]:

$$F = G\frac{m_1 . m_2}{r^2} > 0 \; for \; m \in \mathbb{R}. \tag{1a}$$

The expansion of universe could be explained by the same law as a consequence of anti-gravitation between imaginary masses:

$$F = G\frac{m_1 . m_2}{r^2} < 0 \; for \; m \in i\mathbb{R}. \tag{1b}$$

Thus, the mass of ordinary matter m should be complex, i.e.: real m_{real} enabling gravitation as well as imaginary $m_{imaginary}$ enabling expansion:

$$m = m_{real} + m_{imaginary}. \tag{2}$$

3. Heraclitean Dynamics

Heraclitean dynamics is defined as [2]:

$$F = \frac{dp}{dt} + \frac{d\left(\frac{k}{p}\right)}{dt}. \tag{3}$$

And expressed as

$$m_{relativistic}^2 c^2 a^2 = e^{\frac{m_{ground}^2 c^2 - k(1 - lnk) + m_{relativistic}^2 c^2 (a^2 - 1)}{k}}. \tag{4}$$

Where c is the speed of light and k is the dynamics constant. The relativistic mass $m_{relativistic}$ and energy $m_{relativistic}c^2$ are upside limited by the maximum speed $a_{maximum} = \frac{v_{maximum}}{c}$ characteristic for the given ground mass m_{ground} as follows:

$$a_{maximum} = \sqrt{1 + \frac{k}{e^{\frac{m_{ground}^2 c^2}{k} + lnk} - k}}. \tag{5a}$$

So, the next ground mass m_{ground} belongs to the given maximum speed $a_{maximum}$ (See Appendix 1):

$$m_{ground} = \sqrt{\frac{k}{c^2}\, ln\left(\frac{1}{a_{maximum}^2 - 1} + 1\right)}. \tag{5b}$$

The above Eq. (5b) offers three types of ground masses m_{ground} with regard to the maximum speed $a_{maximum}$:

a) The real ground mass at the superluminal maximum speed $\left(a_{maximum} > 1 \rightarrow m_{ground} \in \mathbb{R}\right)$

b) The infinite ground mass at the luminal maximum speed $\left(a_{maximum} = 1 \rightarrow m_{ground} = \infty\right)$

And

c) The complex ground mass at the subluminal maximum speed $\left(a_{maximum} < 1 \rightarrow m_{ground} \in \mathbb{C}\right)$

As already proposed in Section 2 the latter may allow the universe to expand (1b).

4. The Complex Ground Mass

Using the relation $\ln(-1) = i\pi$ [3] and applying the square root of complex number formula [4] the equation (5b) can be modified for the needs of complex ground mass $m_{ground(\mathbb{C})}$ at the subluminal maximum speed expressed in the units of the speed of light $a_{maximum(\mathbb{C})} < 1$ (See Appendix 2):

$$m_{ground(\mathbb{C})} = \frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2}+ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)}{2}} + i\frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2}-ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)}{2}}. \quad (6a)$$

Thus, the complex ground mass $m_{ground(\mathbb{C})}$ consists of a real part $m_{ground(\mathbb{RC})}$ as well as of an imaginary part $m_{ground(i\mathbb{RC})}$:

$$m_{ground(\mathbb{C})} = m_{ground(\mathbb{RC})} + i m_{ground(i\mathbb{RC})}. \quad (6b)$$

Then the real part of the complex ground mass is given as

$$m_{ground(\mathbb{RC})} = \frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2}+ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)}{2}}. \quad (7a)$$

And is written explicitly for the maximum speed $a_{maximum(\mathbb{C})}$related to the real part of complex ground mass $m_{ground(\mathbb{RC})}$(See Appendix 3):

$$a_{maximum(\mathbb{C})} = \sqrt{1 - \frac{1}{e^{\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2-\pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}}}}}. \quad (7b)$$

On the other hand, the imaginary part of the complex ground mass is given as

$$m_{ground(i\mathbb{RC})} = \frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2}-ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)}{2}}. \quad (8a)$$

And is written explicitly for the maximum speed $a_{maximum(\mathbb{C})}$ related to the imaginary part of complex ground mass $m_{ground(i\mathbb{RC})}$(See Appendix 4):

$$a_{maximum(\mathbb{C})} = \sqrt{1 - \frac{1}{e^{\frac{(\pi)^2-\left(m^2_{ground(i\mathbb{RC})}\frac{2c^2}{k}\right)^2}{2m^2_{ground(i\mathbb{RC})}\frac{2c^2}{k}}}}}. \quad (8b)$$

5. The Imaginary Ground Mass Attributed to Real Ground Mass

Taking into account the complex mass relations just mentioned in Section 4, the imaginary ground mass can be attributed to each real ground mass at subluminal speed as follows (See Appendix 5):

$$m_{ground(i\mathbb{RC})} = \frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2-\pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}}\right)^2+\pi^2}-\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2-\pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}}}{2}}. \quad (9)$$

For ordinary matter, the dynamics constant $k = hc$ is proposed [5], where h is Planck constant and c is the speed of light. So, we can write again:

$$m_{ground(i\mathbb{RC})} = \sqrt{\frac{h}{c}}\sqrt{\frac{\sqrt{\left(\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c}{h}\right)^2-\pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c}{h}}\right)^2+\pi^2}-\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c}{h}\right)^2-\pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c}{h}}}{2}}. \quad (10)$$

6. The Consequences of Proposed Model

To the real ground mass of observable universe yielding $2.777\,266\,999\, x\, 10^{-20} kg$ [6], becoming a part of complex world $m^{universe}_{ground(\mathbb{R})} \rightarrow m^{universe}_{ground(\mathbb{RC})}$the next complex ground mass can be ascribed (10):

$$m^{universe}_{ground(\mathbb{C})} = 2.772\,266\,999.10^{-20}kg + i\ 0.012\,523\,339\,.\,10^{-20}kg. \quad (11)$$

As well as the next maximum speed of universe expansion belongs to it in the complex world (8b):

$$a^{universe}_{maximum(\mathbb{C})} = 1 - 10^{-151}. \quad (12)$$

Speed of universe expansion a is related to the relativistic energy of expansion and the latter can be expressed with help of modified exponent x [7] which in our case having $a = a^{universe}_{maximum(\mathbb{C})} = 1 - 10^{-151}$(12) yields:

$$x = \frac{ln2+2lnc+ln\left(\frac{1}{\sqrt{1-a^2}}-1\right)}{lnc} = 11. \quad (13)$$

The modified x is related to the occurrence of n-dimensional space [7] written as:

$$p_x(n) = \frac{p_x(1)}{n^x} \qquad where \qquad p_x(1) = \frac{1}{\sum_{n=1}^{n=\infty}\frac{1}{n^x}}. \quad (14)$$

In our case having $p_x(1) = p_{11}(1) \approx 1$ the next minimum occurrence of 3-dimensional space is given at maximum speed of expansion $a = a^{universe}_{maximum(\mathbb{C})}$:

$$p_{minimum}(3) = p_{11}(3) = \frac{1}{3^{11}} = 6.10^{-6}. \quad (15)$$

And the next maximum occurrence of 1-dimensional space is proposed at the same speed:

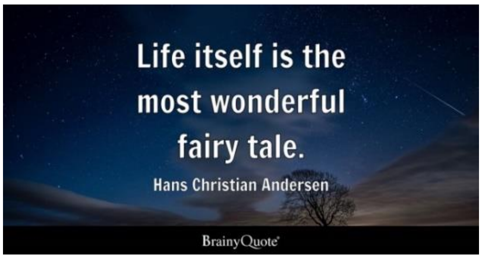
$$p_{maximum}(1) = 1 - p_{11}(2) = 1 - \frac{1}{2^{11}} = 1 - 0{,}0005 = 0{,}9995. \quad (16)$$

7. Conclusion

Modified Heraclitean dynamics could support the expansion of universe

DEDICATION

To my granddaughter Noemi and the quote [8]:



ADDENDUM

We should be grateful that we belong to such a heavy mass of the universe $m_{ground(\mathbb{C})}^{universe} = 2.772\,266\,999.10^{-20}kg + i\,0.012\,523\,339\,.10^{-20}kg$. If it were as light as $m_{ground(\mathbb{RC})} = \sqrt{\frac{\pi}{2}}\sqrt{\frac{h}{c}} = 1.863\,277\,766.10^{-21}kg$, the maximum speed available would be zero (See Appendix 6 and 7) and the universe, with all its imaginary potential $m_{ground(i\mathbb{RC})}\, i\sqrt{\frac{\pi}{2}}\sqrt{\frac{h}{c}} = i\,1.863\,277\,766.10^{-21}kg$, could not expand. It could only leave the fairy world forever at superluminal speed $a_{maximum(\mathbb{R})}^{universe} = 1.123\,581\,020\,c$ (See Appendix 8).

Hamilton in May 2023

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APPENDIX 1

We have available $a_{maximum} = f\left(m_{ground}\right)$. But we would like to have $m_{ground} = f(a_{maximum})$ available. The next steps should be done:

$$a_{maximum} = \sqrt{1 + \frac{k}{e^{\frac{m_{ground}^2 c^2}{k} + lnk} - k}} \quad and \quad a_{maximum}^2 = 1 + \frac{k}{e^{\frac{m_{ground}^2 c^2}{k} + lnk} - k} \qquad (1a)$$

$$a_{maximum}^2 - 1 = \frac{k}{e^{\frac{m_{ground}^2 c^2}{k} + lnk} - k} \quad and \quad e^{\frac{m_{ground}^2 c^2}{k} + lnk} - k = \frac{k}{a_{maximum}^2 - 1}. \qquad (1b)$$

$$e^{\frac{m_{ground}^2 c^2}{k} + lnk} = k + \frac{k}{a_{maximum}^2 - 1} = k\left(1 + \frac{1}{a_{maximum}^2 - 1}\right). \qquad (1c)$$

$$\frac{m_{ground}^2 c^2}{k} + lnk = lnk + ln\left(1 + \frac{1}{a_{maximum}^2 - 1}\right) \quad and \quad \frac{m_{ground}^2 c^2}{k} = ln\left(1 + \frac{1}{a_{maximum}^2 - 1}\right). \qquad (1d)$$

$$m_{ground}^2 = \frac{k}{c^2} ln\left(1+\frac{1}{a_{maximum}^2-1}\right) \quad and \quad m_{ground} = \sqrt{\frac{k}{c^2} ln\left(1+\frac{1}{a_{maximum}^2-1}\right)}. \quad (1e)$$

APPENDIX 2

The complex form of the ground mass $m_{ground(\mathbb{C})}$ is given applying equality $ln(-1) = i\pi$ [3] as well as with the help of square root of complex number formula [4]:

$$m_{ground} = \sqrt{\frac{k}{c^2} ln\left(1+\frac{1}{a_{maximum}^2-1}\right)} \quad and \quad m_{ground} = \sqrt{\frac{k}{c^2}}\sqrt{ln\left(-1\left(-\frac{1}{a_{maximum}^2-1}\right)\right)}. \quad (2a)$$

$$m_{ground}^2 = \frac{k}{c^2} ln\left(-1\left(-\frac{1}{a_{maximum}^2-1}\right)\right) \quad and \quad m_{ground}^2 = \frac{k}{c^2}\left(ln(-1) + ln\left(-\frac{1}{a_{maximum}^2-1}\right)\right). \quad (2b)$$

$$m_{ground}^2 = \frac{k}{c^2}\left(ln(-1) + ln\left(\frac{1}{1-a_{maximum}^2}\right)\right) \quad and \quad m_{ground}^2 = \frac{k}{c^2}\left(i\pi + ln\left(\frac{1}{1-a_{maximum}^2}\right)\right). \quad (2c)$$

$$m_{ground}^2 = \frac{k}{c^2}\left(ln\left(\frac{1}{1-a_{maximum}^2}\right) + i\pi\right) \quad and \quad m_{ground} = \sqrt{\frac{k}{c^2} ln\left(\frac{1}{1-a_{maximum}^2}\right) + i\pi\frac{k}{c^2}}. \quad (2d)$$

Square root of Complex Number Formula cuemath THE MATH EXPERT

$$\sqrt{a+ib} = \pm\left(\sqrt{\frac{|z|+a}{2}} + i\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}\right)$$

where $z = a+ib$ and $b \neq 0$

$$a = \frac{k}{c^2} ln\left(\frac{1}{1-a_{maximum}^2}\right), \quad b = \pi\frac{k}{c^2}, \quad z = \sqrt{a^2+b^2} = \sqrt{\left(\frac{k}{c^2} ln\left(\frac{1}{1-a_{maximum}^2}\right)\right)^2 + \left(\pi\frac{k}{c^2}\right)^2}. \quad (2e)$$

$$m_{ground(\mathbb{C})} = m_{ground(\mathbb{RC})} + i m_{ground(i\mathbb{RC})}. \quad (2f)$$

$$m_{ground(\mathbb{C})} = \sqrt{\frac{\sqrt{\left(\frac{k}{c^2} ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)\right)^2 + \left(\frac{k\pi}{c^2}\right)^2} + \frac{k}{c^2} ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)}{2}}$$
$$+ i\sqrt{\frac{\sqrt{\left(\frac{k}{c^2} ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)\right)^2 + \left(\frac{k\pi}{c^2}\right)^2} - \frac{k}{c^2} ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)}{2}}. \quad (2g)$$

And

$$m_{ground(\mathbb{C})} = \frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)\right)^2 + \pi^2} + ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)}{2}}$$
$$+ i\frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)\right)^2 + \pi^2} - ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)}{2}}. \quad (2h)$$

Or

$$m_{ground(\mathbb{RC})} = \frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2}+ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)}{2}}.$$

And

$$m_{ground(i\mathbb{RC})} = +i\frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2}-ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)}{2}}.$$

APPENDIX 3

We have available $m_{ground(\mathbb{RC})} = f\left(a_{maximum(\mathbb{C})}\right)$. But we would like to have $a_{maximum(\mathbb{C})} = f\left(m_{ground(\mathbb{RC})}\right)$ available. The next steps should be done:

$$m_{ground(\mathbb{RC})} = \frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2}+ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)}{2}}. \quad (3a)$$

$$m_{ground(\mathbb{RC})}\frac{c}{\sqrt{k}}\sqrt{2} = \sqrt{\sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2}+ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)}. \quad (3b)$$

$$m^2_{ground(\mathbb{RC})}\frac{2c^2}{k} = \sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2}+ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right). \quad (3c)$$

$$m^2_{ground(\mathbb{RC})}\frac{2c^2}{k} - ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right) = \sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2}. \quad (3d)$$

$$\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k} - ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2 = \left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2. \quad (3e)$$

$$\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2 - 2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right) + \left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2 = \left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2. \quad (3f)$$

$$\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2 - 2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right) = \pi^2. \quad (3g)$$

$$\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2 - \pi^2 = 2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right). \qquad (3h)$$

$$\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2 - \pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}} = ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right). \qquad (3i)$$

$$e^{\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2 - \pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}}} = \frac{1}{1-a^2_{maximum(\mathbb{C})}}. \qquad (3j)$$

$$\frac{1}{e^{\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2 - \pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}}}} = 1-a^2_{maximum(\mathbb{C})}. \qquad (3k)$$

$$1-\frac{1}{e^{\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2 - \pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}}}} = a^2_{maximum(\mathbb{C})}. \qquad (3l)$$

And

$$a_{maximum(\mathbb{C})} = \sqrt{1-\frac{1}{e^{\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2 - \pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}}}}}. \qquad (3m)$$

APPENDIX 4

We have available $m_{ground(i\mathbb{RC})} = f\left(a_{maximum(\mathbb{C})}\right)$. But we would like to have $a_{maximum(\mathbb{C})} = f\left(m_{ground(i\mathbb{RC})}\right)$ available. The next steps should be done:

$$m_{ground(i\mathbb{RC})} = \frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2 + \pi^2} - ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)}{2}}. \qquad (4a)$$

$$m_{ground(i\mathbb{RC})}\frac{c}{\sqrt{k}}\sqrt{2} = \sqrt{\sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2 + \pi^2} - ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)}. \qquad (4b)$$

$$m^2_{ground(i\mathbb{RC})}\frac{2c^2}{k} = \sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2 + \pi^2} - ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right). \qquad (4c)$$

$$m^2_{ground(i\mathbb{RC})}\frac{2c^2}{k} + ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right) = \sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2 + \pi^2}. \qquad (4d)$$

$$\left(m^2_{ground(i\mathbb{RC})}\frac{2c^2}{k} + ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2 = \left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2 + \pi^2. \qquad (4e)$$

$$\left(m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}\right)^2 + 2m_{ground(\mathbb{RC})}^2 \frac{2c^2}{k} ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right) + \left(ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)\right)^2 = \left(ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)\right)^2 + \pi^2. \quad (4f)$$

$$\left(m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}\right)^2 + 2m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k} ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right) = \pi^2. \quad (4g)$$

$$\left(m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}\right)^2 - \pi^2 = -2m_{ground(\mathbb{RC})}^2 \frac{2c^2}{k} ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right). \quad (4h)$$

$$\frac{\left(m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}\right)^2 - \pi^2}{-2m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}} = ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right). \quad (4i)$$

$$e^{\frac{\left(m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}\right)^2 - \pi^2}{-2m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}}} = \frac{1}{1-a_{maximum(\mathbb{C})}^2}. \quad (4j)$$

$$\frac{1}{e^{\frac{\left(m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}\right)^2 - \pi^2}{-2m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}}}} = 1 - a_{maximum(\mathbb{C})}^2. \quad (4k)$$

$$1 - \frac{1}{e^{\frac{\left(m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}\right)^2 - \pi^2}{-2m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}}}} = a_{maximum(\mathbb{C})}^2. \quad (4l)$$

And

$$a_{maximum(\mathbb{C})} = \sqrt{1 - \frac{1}{e^{\frac{\pi^2 - \left(m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}\right)^2}{2m_{ground(i\mathbb{RC})}^2 \frac{2c^2}{k}}}}}. \quad (4m)$$

APPENDIX 5

Each real ground mass as part of the complex ground mass $m_{ground(\mathbb{RC})}$ at subluminal speed $a < 1$ is related to the maximum speed of the complex ground mass (7b):

$$a_{maximum(\mathbb{C})} = \sqrt{1 - \frac{1}{e^{\frac{\left(m_{ground(\mathbb{RC})}^2 \frac{2c^2}{k}\right)^2 - \pi^2}{2m_{ground(\mathbb{RC})}^2 \frac{2c^2}{k}}}}}. \quad (5a)$$

Rearranging we have:

$$\frac{1}{1-a_{maximum(\mathbb{C})}^2} = e^{\frac{\left(m_{ground(\mathbb{RC})}^2 \frac{2c^2}{k}\right)^2 - \pi^2}{2m_{ground(\mathbb{RC})}^2 \frac{2c^2}{k}}} \quad and \quad \left(ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)\right)^2 = \left(\frac{\left(m_{ground(\mathbb{RC})}^2 \frac{2c^2}{k}\right)^2 - \pi^2}{2m_{ground(\mathbb{RC})}^2 \frac{2c^2}{k}}\right)^2. \quad (5b)$$

Each maximum speed of the complex ground mass $a_{maximum(\mathbb{C})}$ is related to the imaginary part of the complex ground mass (8a):

$$m_{ground(i\mathbb{RC})}=\frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)\right)^2+\pi^2}-ln\left(\frac{1}{1-a^2_{maximum(\mathbb{C})}}\right)}{2}}. \quad (5c)$$

Applying equations (5b) and (5c) the desired relation is given:

$$m_{ground(i\mathbb{RC})}=\frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2-\pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}}\right)^2+\pi^2}-\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}\right)^2-\pi^2}{2m^2_{ground(\mathbb{RC})}\frac{2c^2}{k}}}{2}}. \quad (5d)$$

APPENDIX 6

We have to solve the equation $(5d)$ of type

$$\sqrt{\frac{c}{h}}x=\sqrt{\frac{\sqrt{\left(\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}\right)^2+\pi^2}-\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}}{2}} \quad \text{or} \quad \frac{c}{h}x^2=\frac{\sqrt{\left(\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}\right)^2+\pi^2}-\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}}{2}. \quad (6a)$$

Rearranging we have

$$\frac{2c}{h}x^2=\sqrt{\left(\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}\right)^2+\pi^2}-\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2} \quad \text{and} \quad \frac{2c}{h}x^2+\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}=\sqrt{\left(\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}\right)^2+\pi^2}. \quad (6b)$$

Then by means of squaring

$$\left(\frac{2c}{h}x^2+\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}\right)^2=\left(\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}\right)^2+\pi^2 \quad \text{and}$$

$$\left(\frac{2c}{h}x^2\right)^2+2\frac{2c}{h}x^2\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}+\left(\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}\right)^2=\left(\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}\right)^2+\pi^2. \quad (6c)$$

Further rearranging gives:

$$\left(\frac{2c}{h}x^2\right)^2+2\frac{2c}{h}x^2\frac{\left(\frac{2c}{h}x^2\right)^2-\pi^2}{2\frac{2c}{h}x^2}=\pi^2 \quad \text{and} \quad \left(\frac{2c}{h}x^2\right)^2+\left(\frac{2c}{h}x^2\right)^2-\pi^2=\pi^2 \quad \text{and} \quad \left(\frac{2c}{h}x^2\right)^2+\left(\frac{2c}{h}x^2\right)^2=2\pi^2 \quad \text{and} \quad \left(\frac{2c}{h}x^2\right)^2=\pi^2. \quad (6d)$$

And finally

$$x^2=\frac{\pi h}{2c} \quad and \quad x=\sqrt{\frac{\pi}{2}}\sqrt{\frac{h}{c}}=1.863\,277\,766.10^{-21}kg \quad (6e)$$

APPENDIX 7

For the square of ground mass $m^2_{ground(\mathbb{RC})}=\frac{\pi}{2}\frac{h}{c}$ we can calculate:

$$a_{maximum(\mathbb{C})}=\sqrt{1-\frac{1}{e^{\frac{\left(m^2_{ground(\mathbb{RC})}\frac{2c}{h}\right)^2-(\pi)^2}{2m^2_{ground(\mathbb{RC})}\frac{2c}{h}}}}}=\sqrt{1-\frac{1}{e^{\frac{\left(\frac{\pi h2c}{2c\,h}\right)^2-(\pi)^2}{2\frac{\pi h2c}{2c\,h}}}}}=\sqrt{1-\frac{1}{e^{\frac{(\pi)^2-(\pi)^2}{2\pi}}}}=\sqrt{1-\frac{1}{e^0}}=\sqrt{1-\frac{1}{1}}=0. \quad (7)$$

APPENDIX 8

We have to do with the equality of ground mass in the real world $m_{ground(\mathbb{R})}$ and in the complex world $m_{ground(\mathbb{RC})}$ taking into account (5b) and (7a):

$$m_{ground(\mathbb{R})} = \sqrt{\frac{k}{c^2}\,ln\left(\frac{1}{a_{maximum}^2 - 1} + 1\right)} \;\; and \;\; m_{ground(\mathbb{RC})} = \frac{\sqrt{k}}{c}\sqrt{\frac{\sqrt{\left(ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)\right)^2 + \pi^2} + ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)}{2}}. \quad (8a)$$

Thus

$$m_{ground(\mathbb{R})} = m_{ground(\mathbb{RC})}. \quad (8b)$$

With the help of experience gained in previous calculations, we find out the relation between the maximum speed in the real world $a_{maximum(\mathbb{R})}$ and the maximum speed in the complex world $a_{maximum(\mathbb{C})}$:

$$a_{maximum(\mathbb{R})} = \sqrt{\frac{1}{e^{\frac{\sqrt{\left(ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)\right)^2+\pi^2}+ln\left(\frac{1}{1-a_{maximum(\mathbb{C})}^2}\right)}{2}} - 1} + 1}. \quad (8c)$$

And for the zero subluminal maximum speed $a_{maximum(\mathbb{C})} = 0$ the next superluminal maximum speed is given:

$$a_{maximum(\mathbb{R})} = \sqrt{\frac{1}{e^{\frac{\sqrt{\left(ln\left(\frac{1}{1-0}\right)\right)^2+\pi^2}+ln\left(\frac{1}{1-0}\right)}{2}} - 1} + 1} = \sqrt{\frac{1}{e^{\frac{\pi}{2}} - 1} + 1} = 1{,}123\,581\,020\,\ldots \quad (8d)$$

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