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Mathematical Modeling on MHD Free Convective Couette Flow between Two Vertical Porous Plates with Chemical Reaction and Soret Effect

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Abstract: *An analytical solution of the problem of a two dimensional steady MHD free convective couette and mass transfer flow of a viscous incompressible and electrically conducting fluid between two infinite vertical parallel plates with the viscous and magnetic dissipations of energy has been presented. A uniform magnetic field is assumed to be applied transversely to the direction of the flow taking into account the induced magnetic field. The plates are subjected to constant suction and injection. The governing equations are solved by regular perturbation technique. The expressions for the velocity field, temperature field, species concentration, skin friction, the coefficient of heat transfer (in terms of Nusselt numbers), the coefficient of mass transfer (in terms of Sherwood numbers) at the walls are obtained and their numerical values are demonstrated in graphs. The effects of Hartman number M, the Reynolds number Re, the magnetic Reynolds number Rm, the Soret number S_0, the chemical reaction parameter Ch, the Schmidt number Sc on the flow and heat, mass transfer are discussed.*

2006 Mathematics subject classification: *76D*

Keywords: *MHD, free convective, viscous incompressible, skin friction, Nusselt number, chemical reaction, Soret number, couette flow.*

1. INTRODUCTION

Analytical solutions of the problem of convective flows, which arise in fluids due to interaction of the force of gravity and density differences caused by simultaneous diffusion of thermal energy and chemical species, have been presented by many authors due to applications of such problems in Geophysics and Engineering. Some of them are Bejan and Khair (1985), Helmy (1988), Acharya et. al. (2000). A theoretical analysis of steady MHD free convective and mass transfer flow of an incompressible viscous and electrically conducting fluid through porous medium occupying a semi infinite region of the space bounded by an infinite vertical plate has been presented by Raptis and Kafousias (1982). The influence of variations of temperature dependent physical variables on the steady MHD couette flow with heat transfer was studied by Atta (2006). Recently Ahmed et. al. (2010) have studied the problem of the MHD free convective poisellile flow and mass transfer bounded by two infinite vertical porous plates.

In the above mentioned works the induced magnetic field and thermal diffusion effect were ignored. The assumption of ignorance of induced magnetic field is valid for electrically conducting fluid of low conductivity. In case of fluids with large magnetic Reynolds number (high conductivity) the induced magnetic field is turn changes the flow and heat transfer characteristics and hence ignorance of it is not justified. Moreover the thermal diffusion effects commonly known as Soret effect plays an important role in flow and transfer characteristics. The assumption of not considering Soret effect is true when the concentration level is very low. The thermal diffusion effect is applied for isotope separation in mixture between gases with very light molecular weight (H_2, H_e) and medium molecular weight (N_2, air), where the diffusion thermo effect is found to be a magnitude such that it can't be neglected. Keeping in view of the importance of the diffusion of this thermo effect Sattar and Alam (1994) investigate the effect of free convection and mass transfer flow past on accelerated vertical porous plate taking into account the Soret effect. Kalita and Ahmed (2011) studied the combined effect of thermal diffusion and magnetic field on a free convective poiseuille flow through a porous medium bounded by two infinite vertical porous plates.

In many times it has been observed that the foreign mass reacts with the fluid and in such a situation chemical reaction plays an important role in chemical industry. The problem of MHD natural convective flow with mass transfer, Soret effect and chemical reaction are recently studied by Ahmed and Kalita (2010), Ahmed et. al. (2011). In view of the importance of the combined effects of the magnetic field, thermal diffusion and chemical reaction it is proposed to study a problem of two-dimensional MHD free convective couette flow bounded by two infinite vertical porous plates with chemical reaction taking into account of Soret effect in presence of induced magnetic field.

The present work is an extension work done by Ahmed and Kalita (2009) to consider the thermal diffusion effect and chemical reaction.

2. MATHEMATICAL FORMULATION

The basic equations governing the motion of an incompressible viscous electrically conducting fluid in presence of a magnetic field in steady case (neglecting the displacement currents and free charges) are

The Gauss's law of magnetism

$$\vec{\nabla}.\vec{B}=0 \tag{2.1}$$

The equation of continuity

$$\vec{\nabla}.\vec{q}=0 \tag{2.2}$$

The momentum equation

$$\rho\left[\vec{q}.\vec{\nabla}\ \vec{q}\right]=-\vec{\nabla}p+\vec{J}\times\vec{B}+\mu\nabla^2\vec{q} \tag{2.3}$$

The energy equation

$$\rho C_p\ \vec{q}.\vec{\nabla}\ \overline{T}=\lambda\nabla^2\overline{T}+\Phi+\frac{J^2}{\sigma}+Q\ \overline{T}_w-\overline{T} \tag{2.4}$$

The magnetic induction equation

$$\eta\nabla^2\vec{B}+\vec{\nabla}\times\ \vec{q}\times\vec{B}\ =0 \tag{2.5}$$

The Ohm's law

$$\vec{J}=\sigma\ \vec{E}+\vec{q}\times\vec{B} \tag{2.6}$$

where $\vec{q}$ is the velocity vector, p is the pressure, μ is the coefficient of viscosity, $\overline{T}$ is the temperature, $\overline{T}_W$ is the temperature at the either plate, $\vec{B}$ is the magnetic induction vector, $\vec{J}$ is the electric current density, $\vec{E}$ is the electric field (here assumed to zero), σ is the electrical conductivity, λ is the thermal conductivity, C_p is the specific heat at constant pressure, ρ is the density of fluid, Φ is the viscous dissipation per unit volume, η is the magnetic diffusivity, $\vec{J}\times\vec{B}$ is the Lorentz force per unit volume, $\frac{J^2}{\sigma}$ is the Julian heat per unit volume, Q is the heat source parameter and other symbols have their usual meanings.

We consider a steady two-dimensional couette flow of an incompressible viscous electrically conducting fluid as mentioned earlier under the influence of transverse magnetic field through a channel bounded by two infinite vertical porous plates separated by a distance h.

We make the following assumptions:

(1) All fluid properties except the density in the buoyancy force term are constant.
(2) The plates are subjected to constant suction / injection.
(3) The Eckert number Ec is small.
(4) The plates are non-conducting.

We introduce a coordinate system $\bar{x}, \bar{y}, \bar{z}$ with x-axis vertically upwards along the stationary plate and y-axis perpendicular to it directed to the fluid region and Z-axis along the width of the stationary plate. Let $\vec{q} = u, v, 0$ be the fluid velocity at the point $\bar{x}, \bar{y}, \bar{z}$ and $\bar{B} = \bar{b}_{x}, \bar{b}_{y}, 0$ be the magnetic induction vector at the point $\bar{x}, \bar{y}, \bar{z}$ in the fluid. Since the plates are infinite in length therefore all the physical quantities except possibly the pressure p are independent of $\bar{x}$.

With the foregoing assumptions and Boussinesq approximation the equations governing the flow reduces to

<u>Equation of continuity</u>

$$\frac{d\bar{v}}{d\bar{y}} = 0 \Rightarrow \bar{v} = -v_0 (\text{constant suction/injection}) \tag{2.7}$$

<u>Momentum equation</u>

$$-v_0 \frac{d\bar{u}}{d\bar{y}} = g\beta(\bar{T} - \bar{T}_s) + g\beta(\bar{C} - \bar{C}_s) - \frac{\sigma B_0^2 \bar{u}}{\rho} - \frac{\sigma v_0 B_0}{\rho} \bar{b}_x + \upsilon \frac{d^2 \bar{u}}{d\bar{y}^2} \tag{2.8}$$

<u>Energy equation</u>

$$-v_0 \frac{d\bar{T}}{d\bar{y}} = \frac{\lambda}{\rho c_p} \frac{d^2 \bar{T}}{d\bar{y}^2} + \frac{\upsilon}{c_p} \left(\frac{d\bar{u}}{d\bar{y}} \right)^2 + \frac{\sigma}{\rho c_p} \; 2 B_0 v_0 \bar{u} \bar{b}_x \tag{2.9}$$

(Neglecting the higher powers of $\bar{u}$)

<u>Species continuity equation</u>

$$-v_0 \frac{d\bar{C}}{d\bar{y}} = D_M \frac{d^2 \bar{C}}{d\bar{y}^2} + D_T \frac{d^2 \bar{T}}{d\bar{y}^2} + (\bar{C}_S - C)\xi \tag{2.10}$$

<u>Magnetic induction equation</u>

$$0 = \eta \frac{d^2 \bar{b}_x}{d\bar{y}^2} + B_0 \frac{d\bar{u}}{d\bar{y}} + v_0 \frac{d\bar{b}_x}{d\bar{y}} \tag{2.11}$$

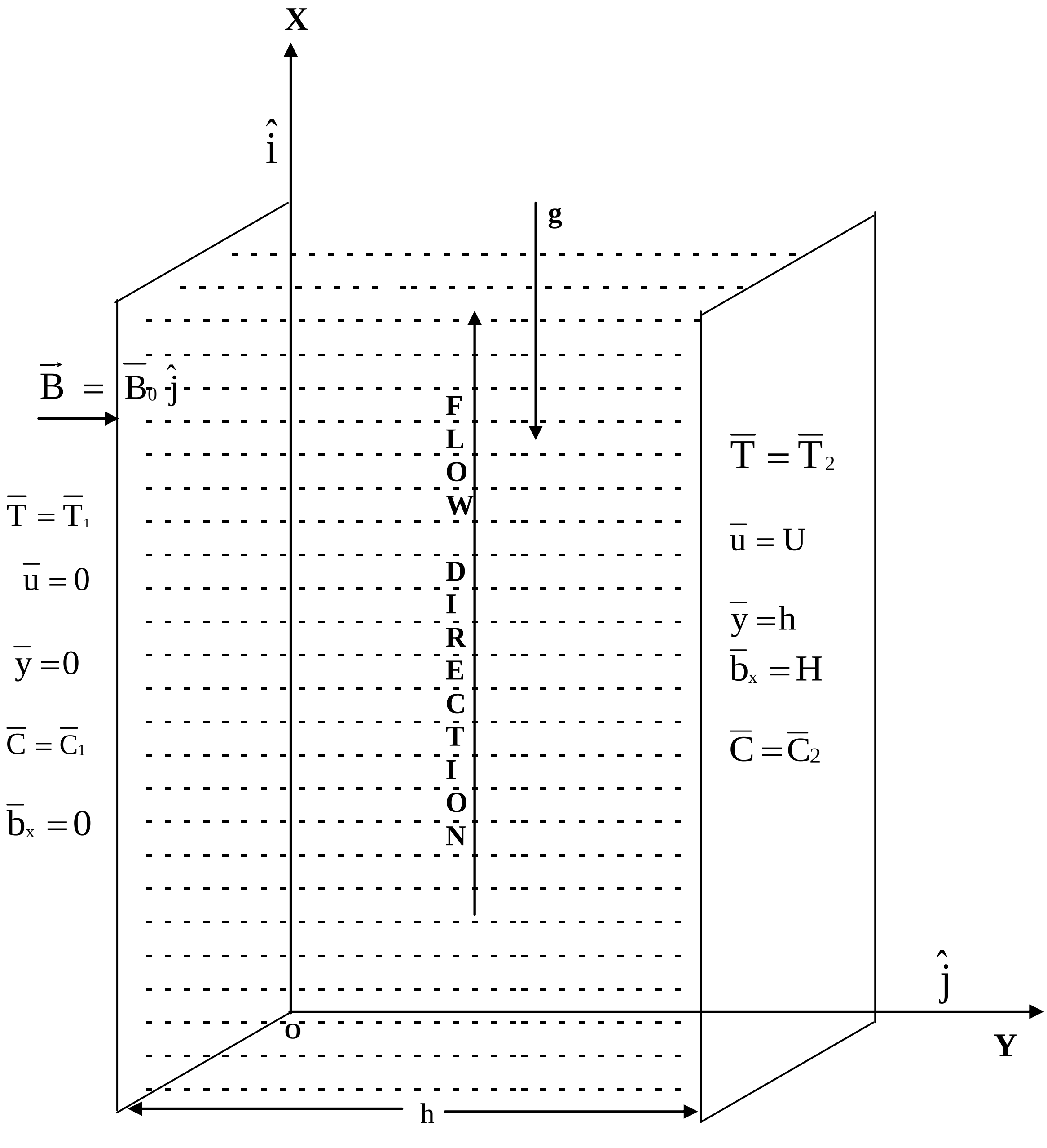

Figure 1. *Flow configuration of the problem*

where υ is the kinematic viscosity, g is the acceleration due to gravity, v_0 is the constant suction / injection velocity, β is the coefficient of volume expansion for heat transfer, $\bar{\beta}$ is the coefficient of volume expansion for mass transfer, D_M is the chemical molecular diffusivity, D_T is the chemical thermal diffusivity, K is the permeability of porous medium, B_0 is the strength of applied magnetic field, $\bar{T}_S$ is the temperature at static condition, $\bar{C}$ is the species concentration, $\bar{C}_S$ is the concentration at static condition and the other symbols have their usual meanings.

The relevant boundary conditions are

$$\left.\begin{array}{l} \bar{y} = 0; \bar{u} = 0, \bar{T} = \bar{T}_1, \bar{b}_x = 0, \bar{C} = \bar{C}_1 \\ \bar{y} = h; \bar{u} = U, \bar{T} = \bar{T}_2, \bar{b}_x = H, \bar{C} = \bar{C}_2 \end{array}\right\} \tag{2.12}$$

We introduce the following non-dimensional quantities

$y=\frac{\bar{y}}{h}$ (distance), $u=\frac{\bar{u}}{U}$ (fluid velocity), $\theta=\frac{\bar{T}-\bar{T}_s}{\bar{T}_1-\bar{T}_s}$ (fluid temperature), $\phi=\frac{\bar{C}-\bar{C}_s}{\bar{C}_1-\bar{C}_s}$ (Species concentration), $G_r=\frac{hg\beta(\bar{T}_1-\bar{T}_s)}{U^2}$ (Grashoff number for heat transfer) $G_m=\frac{hg\bar{\beta}(\bar{C}_1-\bar{C}_s)}{U^2}$ (Grashoff number for mass transfer), $K=\frac{v_0}{U}$ (the ratio of cross-flow suction velocity to the plate velocity), $E_C=\frac{U^2}{c_p(\bar{T}_1-\bar{T}_s)}$ (Eckert number), $P_r=\frac{\mu c_p}{\lambda}$ (Prandtl number), $M=\frac{\sigma B_0{}^2h^2}{\rho\upsilon}$ (Hartmann number), $b_x=\frac{\bar{b}_x}{H}$ (induced magnetic field) , $\xi=\frac{H}{B_0}$ (the ratio of induced magnetic field at the moving plate to the applied magnetic field), $R_e=\frac{v_0h}{\upsilon}$ (Reynolds number) , $R_m=\frac{v_0h}{\eta}$ (magnetic Reynolds number) , $S_C=\frac{\upsilon}{D_M}$ (Schmidt number), $m=\frac{\bar{T}_1-\bar{T}_s}{\bar{T}_0-\bar{T}_s}$ (non dimensional temperature at the plate $\bar{y}=h$), $n=\frac{\bar{C}_1-\bar{C}_s}{\bar{C}_0-\bar{C}_s}$ (non-dimensional concentration at the plate $\bar{y}=h$), $S_0=\frac{D_T\ \bar{T}_1-\bar{T}_S}{\upsilon\ \bar{C}_1-\bar{C}_S}$ (Soret number), $C_h=\frac{S_C\xi}{v_0{}^2}D_M$ (chemical reaction parameter)

The non-dimensional equations with boundary conditions are

$$\frac{d^2u}{dy^2}+R_e\frac{du}{dy}-Mu=-\frac{G_r\theta R_e}{K}-\frac{G_m\phi R_e}{K}+MKb_x\xi \tag{2.13}$$

$$\frac{d^2\theta}{dy^2}+P_rR_e\frac{d\theta}{dy}=-E_CP_r\left(\frac{\partial u}{\partial y}\right)^2-2ME_CP_rKUb_x \tag{2.14}$$

$$\frac{d^2\phi}{dy^2}+R_eS_C\frac{d\phi}{dy}-C_hR_e{}^2S_C\phi=-S_CS_0\frac{d^2\theta}{dy^2} \tag{2.15}$$

$$k\xi\frac{d^2b_x}{dy^2}+K\xi R_m\frac{db_x}{dy}+R_m\frac{du}{dy}=0 \tag{2.16}$$

Subjected to boundary conditions

$$\left.\begin{aligned} y=0;u=0,\theta=1,b_x=0,\phi=1\\ y=1;u=1,\theta=m,b_x=1,\phi=n\end{aligned}\right\} \tag{2.17}$$

3. Solution of the Problem

To solve the equations (2.13), (2.14), (2.15) and (2.16) subject to boundary conditions (2.17) we assume the solutions of the equations to be of the form

$$\left.\begin{aligned} u&=u_0(y)+E_cu_1(y)+O(E_c{}^2)\\ \theta&=\theta_0(y)+E_c\theta_1(y)+O(E_c{}^2)\\ \varphi&=\varphi_0(y)+E_c\varphi_1(y)+O(E_c{}^2)\\ b_x&=b_{x0}(y)+E_cb_{x1}(y)+O(E_c{}^2)\end{aligned}\right\} \tag{3.1}$$

Substituting from the equations (3.1) in (2.13), (2.14), (2.15) and (2.16) and by equating the coefficient of similar powers of Ec and neglecting the higher powers of Ec, the following equations are obtained

$$u_0'' + R_e u_0' - M u_0 = -\frac{G_r R_e \theta_0}{K} - \frac{G_m R_e}{K}\varphi + MK\xi b_{x_0} \tag{3.2}$$

$$u_1'' + R_e u_1' - M u_1 = -\frac{G_r R_e}{K}\theta_1 - \frac{G_m R_e}{K}\varphi_1 + MK\xi b_{x_1} \tag{3.3}$$

$$\theta_0'' + R_e P_r \theta_0' = 0 \tag{3.4}$$

$$\theta_1'' + R_e P_r \theta_1' = -P_r {u_0'}^2 - 2MP_r K\xi u_0 b_{x_0} \tag{3.5}$$

$$\phi_0'' + R_e S_c \phi_0' - C_h R_e^{\,2} S_c \phi_0 = -S_0 S_c \theta_0'' \tag{3.6}$$

$$\phi_1'' + R_e S_c \phi_1' - C_h R_e^{\,2} S_c \phi_1 = -S_0 S_c \theta_1'' \tag{3.7}$$

$$K\xi b_{x_0}'' + K\xi R_m b_{x_0}' + R_m u_0' = 0 \tag{3.8}$$

$$K\xi b_{x_0}'' + K\xi R_m b_{x_0}' + R_m u_1' = 0 \tag{3.9}$$

Where dashes denote differentiation with respect to y.

The boundary conditions (2.11) reduce to

$$\left.\begin{aligned} y = 0; u_0 = 0, u_1 = 0, \theta_0 = 1, \theta_1 = 0, b_{x_0} = 0, b_{x_1} = 0, \phi = 1 \\ y = 1; u_0 = 1, u_1 = 0, \theta_0 = m, \theta_1 = 0, b_{x_0} = 1, b_{x_1} = 0, \phi = n \end{aligned}\right\} \tag{3.10}$$

The solutions of the equations (3.2) to (3.9) subject to boundary conditions (3.10) are obtained and shown bellow

$$\theta_0(y) = A_2 + A_1 e^{-R_e P_r y} \tag{3.11}$$

$$\phi_0(y) = A_6 e^{\lambda_1 y} + A_5 e^{\lambda_2 y} + A_4 e^{-R_e P_r y} \tag{3.12}$$

$$u_0(y) = A_{14} e^{\lambda_3 y} + A_{13} e^{\lambda_4 y} + A_{10} e^{-R_e P_r y} + A_{11} e^{\lambda_1 y} + A_{12} e^{\lambda_2 y} \tag{3.13}$$

$$b_{x_0}(y) = A_{21} + A_{20} e^{-R_m y} + A_{15} e^{\lambda_3 y} + A_{16} e^{\lambda_4 y} + A_{17} e^{-R_e P_r y} + A_{18} e^{\lambda_1 y} + A_{19} e^{\lambda_2 y} \tag{3.14}$$

$$\begin{aligned}\theta_1(y) = {} & A_{79} + A_{78} e^{-R_e P_r y} + A_{52} e^{2\lambda_3 y} + A_{53} e^{2\lambda_4 y} + A_{67} e^{2\lambda_1 y} + A_{54} e^{-2R_e P_r y} + A_{55} e^{2\lambda_1 y} + A_{56} e^{2\lambda_2 y} + \\ & A_{57} e^{(\lambda_3+\lambda_4)y} + A_{58} e^{(\lambda_3 - R_e P_r)y} + A_{59} e^{(\lambda_3+\lambda_1)y} + A_{60} e^{(\lambda_3+\lambda_2)y} + A_{61} e^{(\lambda_4 - R_e P_r)y} + A_{62} e^{(\lambda_1+\lambda_4)y} + \\ & A_{63} e^{(\lambda_2+\lambda_4)y} + A_{64} e^{(\lambda_1 - R_e P_r)y} + A_{65} e^{(\lambda_2 - R_e P_r)y} + A_{66} e^{(\lambda_1+\lambda_2)y} + A_{67} e^{\lambda_3 y} + A_{68} e^{\lambda_4 y} + A_{69} e^{\lambda_1 y} + A_{70} e^{\lambda_2 y} \\ & + A_{71} y e^{-R_e P_r y} + A_{72} e^{(\lambda_4 - R_m)y} + A_{73} e^{(\lambda_3 - R_m)y} + A_{74} e^{(\lambda_2 - R_m)y} + A_{75} e^{(\lambda_1 - R_m)y} + A_{76} e^{-(R_m + R_e P_r)y}\end{aligned} \tag{3.15}$$

$$\phi_1 \;\; y \;=A_{140}e^{\lambda_5 y}+A_{139}e^{\lambda_6 y}+A_{136}e^{-R_eP_r y}+A_{109}e^{2\lambda_3 y}+A_{110}e^{2\lambda_4 y}+A_{111}e^{-2R_eP_r y}+A_{112}e^{2\lambda_1 y}+ A_{113}e^{2\lambda_2 y}+A_{114}e^{(\lambda_3+\lambda_4)y}+A_{115}e^{(\lambda_3-R_eP_r)y}+A_{116}e^{(\lambda_3+\lambda_1)y}+A_{117}e^{(\lambda_3+\lambda_2)y}+A_{118}e^{(\lambda_4-R_eP_r)y}+A_{119} e^{(\lambda_1+\lambda_4)y}+A_{120}e^{(\lambda_2+\lambda_4)y}+A_{121}e^{(\lambda_1-R_eP_r)y}+A_{122}e^{(\lambda_2-R_eP_r)y}+A_{123}e^{(\lambda_1+\lambda_2)y}+A_{124}e^{\lambda_3 y}+A_{125}e^{\lambda_4 y}+ A_{126}e^{\lambda_1 y}+A_{127}e^{\lambda_2 y}+A_{128}ye^{-R_eP_r y}+A_{131}e^{(\lambda_4-R_m)y}+A_{132}e^{(\lambda_3-R_m)y}+A_{133}e^{(\lambda_2-R_m)y}+A_{135}e^{(\lambda_1-R_m)y} +A_{135}e^{-(R_m+R_eP_r)y} \tag{3.16}$$

$$u_1 \;\; y \;=A_{177}e^{-R_eP_r y}+A_{175}ye^{-R_eP_r y}+A_{178}e^{-2R_eP_r y}+A_{179}e^{-(R_m+R_eP_r)y}+A_{180}e^{2\lambda_1 y}+A_{181}e^{2\lambda_2 y}+ A_{182}e^{2\lambda_3 y}+A_{183}e^{2\lambda_4 y}+A_{184}e^{\lambda_1 y}+A_{185}e^{\lambda_2 y}+A_{208}e^{\lambda_3 y}+A_{209}e^{\lambda_4 y}+A_{188}e^{\lambda_5 y}+A_{189}e^{\lambda_6 y}+A_{190} e^{(\lambda_1+\lambda_2)y}+A_{191}e^{(\lambda_1+\lambda_3)y}+A_{192}e^{(\lambda_1+\lambda_4)y}+A_{193}e^{(\lambda_2+\lambda_3)y}+A_{194}e^{(\lambda_2+\lambda_4)y}+A_{195}e^{(\lambda_3+\lambda_4)y}+A_{196}e^{(\lambda_1-R_eP_r)y} +A_{197}e^{(\lambda_2-R_eP_r)y}+A_{198}e^{(\lambda_3-R_eP_r)y}+A_{199}e^{(\lambda_4-R_eP_r)y}+A_{200}e^{(\lambda_1-R_m)y}+A_{201}e^{(\lambda_2-R_m)y}+A_{202}e^{(\lambda_3-R_m)y} +A_{203}e^{(\lambda_4-R_m)y} \tag{3.17}$$

$$b_{x_1} \;\; y \;=A_{271}+A_{272}e^{-R_m y}+A_{242}e^{-R_eP_r y}+A_{240}ye^{-R_eP_r y}+A_{243}e^{-2R_eP_r y}+A_{244}e^{-(R_m+R_eP_r)y}+ A_{245}e^{2\lambda_1 y}+A_{246}e^{2\lambda_2 y}+A_{247}e^{2\lambda_3 y}+A_{248}e^{2\lambda_4 y}+A_{249}e^{\lambda_1 y}+A_{250}e^{\lambda_2 y}+A_{251}e^{\lambda_3 y}+A_{252}e^{\lambda_4 y}+ A_{253}e^{\lambda_5 y}+A_{254}e^{\lambda_6 y}+A_{255}e^{(\lambda_1+\lambda_2)y}+A_{256}e^{(\lambda_1+\lambda_3)y}+A_{257}e^{(\lambda_1+\lambda_4)y}+A_{258}e^{(\lambda_2+\lambda_3)y}+A_{259}e^{(\lambda_2+\lambda_4)y} +A_{260}e^{(\lambda_3+\lambda_4)y}+A_{261}e^{(\lambda_1-R_eP_r)y}+A_{262}e^{(\lambda_2-R_eP_r)y}+A_{263}e^{(\lambda_3-R_eP_r)y}+A_{264}e^{(\lambda_4-R_eP_r)y}+A_{265}e^{(\lambda_1-R_m)y} +A_{266}e^{(\lambda_2-R_m)y}+A_{267}e^{(\lambda_3-R_m)y}+A_{268}e^{(\lambda_4-R_m)y} \tag{3.18}$$

Where, $\lambda_1=\dfrac{-R_eS_c+\sqrt{R_e^{\,2}S_c^{\,2}+4R_e^{\,2}S_cC_h}}{2}=\lambda_5$, $\lambda_2=\dfrac{-R_eS_c-\sqrt{R_e^{\,2}S_c^{\,2}+4R_e^{\,2}S_cC_h}}{2}=\lambda_6$,

$$\lambda_3=\frac{-\;\;R_e+R_m\;\;+\sqrt{\;R_e+R_m\;\;^2-4\;\;R_eR_m-4\;}}{2},$$

$$\lambda_4=\frac{-\;\;R_e+R_m\;\;-\sqrt{\;R_e+R_m\;\;^2-4\;\;R_eR_m-4\;}}{2}.$$

The other constants A_1, A_2, …………. A_{272} are not shown for the sake of brevity.

The expressions for non-dimensional velocity, temperature, species concentration and induced magnetic field in the following form:

$$u(y)=u_0(y)+E_cu_1(y)$$

$$\theta(y)=\theta_0(y)+E_c\theta_1(y)$$

$$\phi(y)=\phi_0(y)+E_c\phi_1(y)$$

$$b_x(y)=b_{x_0}(y)+E_cb_{x_1}(y)$$

4. COEFFICIENT OF SKIN-FRICTION

The skin-friction in the non-dimensional form on the plate y = 0 and y = 1 are respectively given by

$$\tau_0=\frac{1}{R_e}\left[\frac{du}{dy}\right]_{y=0}=\frac{1}{R_e}\left[u_0'(0)+E_cu_1'(0)\right] \text{ and}$$

$$\tau_1 = \frac{1}{R_e}\left[\frac{du}{dy}\right]_{y=1} = \frac{1}{R_e}\left[u_0'(1) + E_c u_1'(1)\right]$$

5. RATE OF HEAT-TRANSFER

The (Nusselt number) rate of heat transfer from the plates to the fluid in the non-dimensional form on the plate y = 0 and y = 1 are respectively given by

$$N_{u_0} = \left(\frac{d\theta}{dy}\right)_{y=0} = \theta_0'(0) + E_c\theta_1'(0) \text{ and}$$

$$N_{u_1} = \left(\frac{d\theta}{dy}\right)_{y=1} = \theta_0'(1) + E_c\theta_1'(1)$$

6. RATE OF MASS-TRANSFER

The (Sherwood number) rate of mass transfer between the fluid and the plates y=0 and y=1 are respectively given by

$$Sh_0 = \left(\frac{\partial\phi}{\partial y}\right)_{y=0} = \left[\phi_0'(0) + E_c\phi_1'(0)\right] \text{ and}$$

$$Sh_1 = \left(\frac{\partial\phi}{\partial y}\right)_{y=1} = \left[\phi_0'(1) + E_c\phi_1'(1)\right]$$

7. RESULTS AND DISCUSSION

In order to get physical insight into the problem the numerical values of species concentration distribution, velocity distribution, temperature distribution, skin friction, rate of heat transfer in terms of Nusselt number, rate of mass transfer in terms of Sherwood number have been obtained and they are demonstrated graphically.

Throughout our investigation the Prandtl number Pr is taken to be equal to 0.71 which corresponds to the air at 20^0 C. The values of Grashof number for heat transfer Gr is taken to be 5, the Grashof number for mass transfer Gm is considered to be 2 and the Eckert number E_c is assumed to be 0.05. The values of non-dimensional temperature m at the plate y=1, non-dimensional species concentration n at the plate y=1, permeability parameter K, the ratio of induced magnetic field at the moving plate to the applied magnetic field ξ are fixed at 2, 2, 0.5 and 0.5 respectively. The values of other parameters namely the Hartmann number M, the Reynolds number Re, the Magnetic Reynolds number Rm, the Soret number S_0 , the Chemical reaction parameter Ch and the Schmidt number Sc are chosen arbitrary. The values of Schmidt number Sc are chosen in such a way that they represent the diffusing chemical species of most common interest in air (for example the values of Schmidt number for H_2, He, water vapor, O_2, NH_3, and CO_2 are 0.22, 0.30, 0.60, 0.66, 0.78, 0.94 respectively). Here, Grashof number for heat transfer Gr > 0 corresponds to externally cooled plates.

The concentration profiles against y are presented in figures (2)-(5) for different values of Ch, S_0, Sc and Re. These figures indicate that concentration decreases due to increase values of Ch, S_0 and Sc, whereas concentration is increased due to low viscosity (high Reynolds number). The variation of Sherwood number Sh_0 (at the plate y=0) and Sh_1 (at the plate y=1) against Reynolds number Re are shown in figures (6)-(11) for different values of Ch, S_0 and Sc. It is inferred from these figures that at the plate y=0, Sh_0 decreases due to the effects of chemical reaction and thermal diffusion but it shows reverse effect at the plate y=1 for the same parameters. It is also observed from these figures that high rate of molar diffusivity (small Schmidt number) causes Sh_1 to decrease and Sh_0 to increase. Further, it is also cleared from these figures that at the moving plate rate of mass transfer increases significantly for low viscosity.

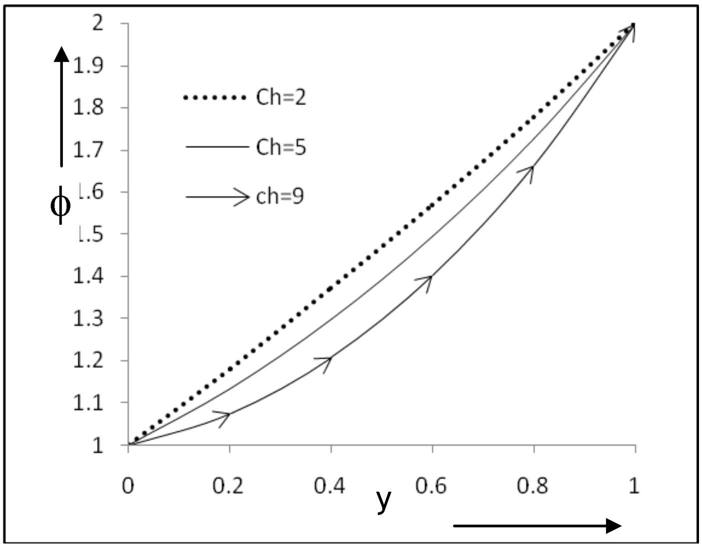

Figure 2. *Species concentration distribution versus y when Re=0.5, M=1, Rm=0.3, Sc=0.6, Sr=0.5*

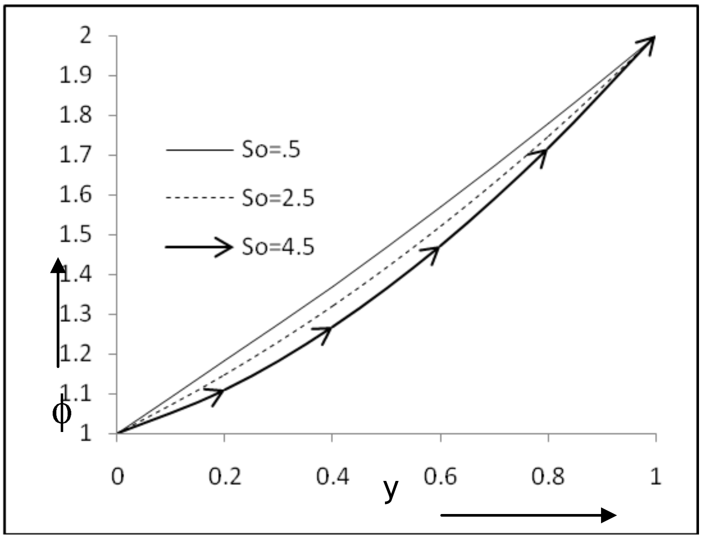

Figure 3. *Species concentration distribution versus y when Re=0.5, Sc=0.6, M=1, Rm=0.3, Ch=2*

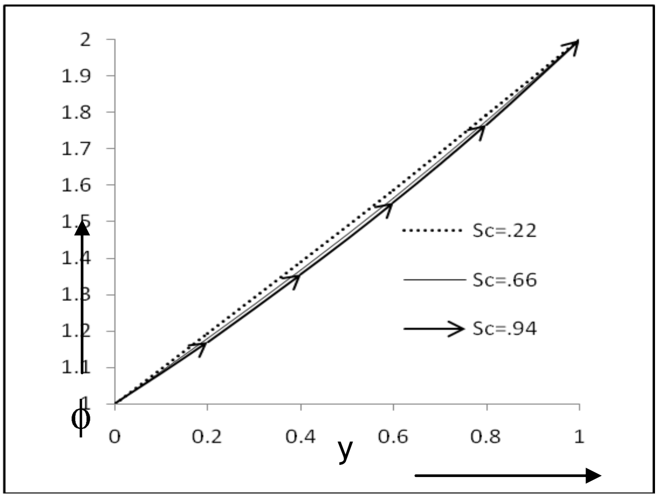

Figure 4. *Species concentration distribution versus y when Re=0.5, So=0.5, M=1, Rm=0.3,Ch=2*

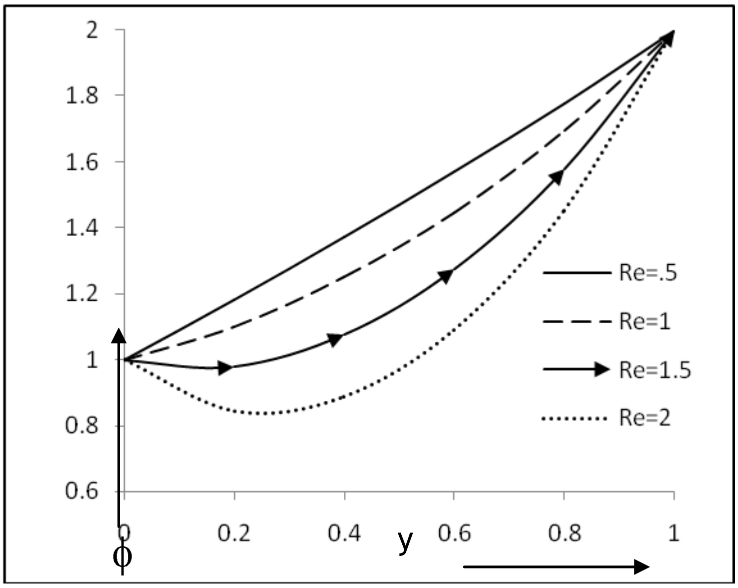

Figure 5. *Species concentration distribution versus y when Sc=0.6, So=0.5, M=1, Rm=0.3,Ch=2*

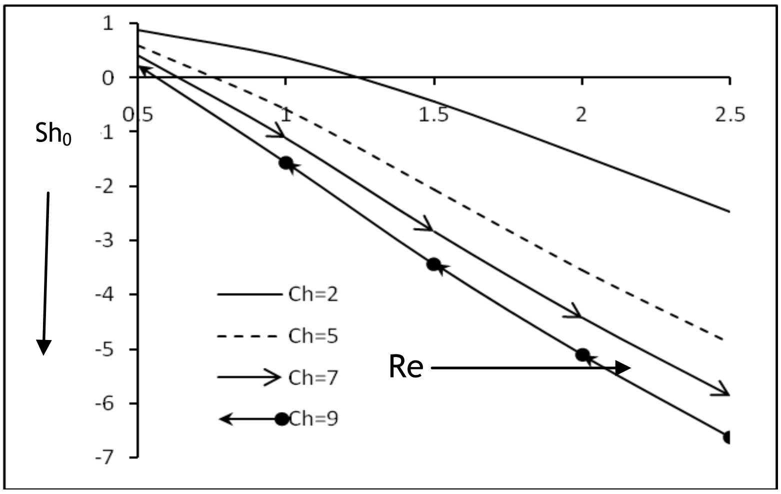

Figure 6. *Sherwood number Sh_0 (at the plate y=0) versus Re when Sc=0.6 M=1, Rm=0.3 and S_0=0.5*

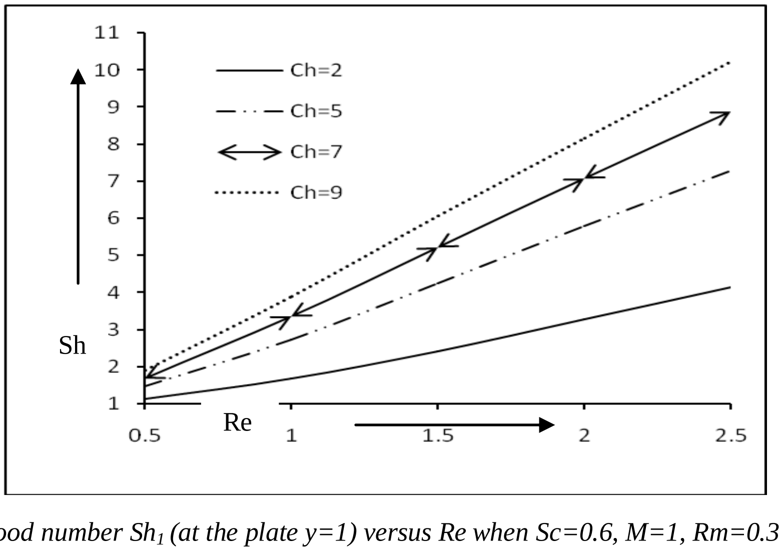

Figure 7. *Sherwood number Sh_1 (at the plate y=1) versus Re when Sc=0.6, M=1, Rm=0.3 and S_0=0.5*

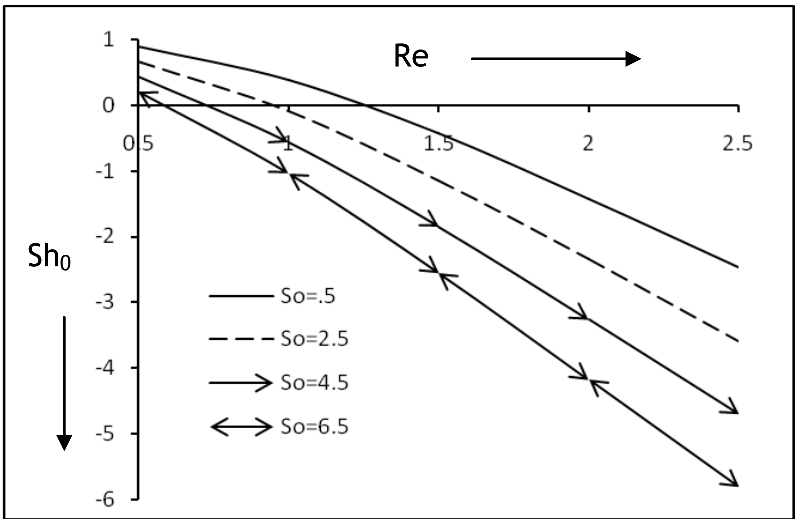

Figure 8. *Sherwood number Sh_0 (at the plate y=0) versus Re when Sc=0.6, M=1, Rm=0.3 and Ch=2*

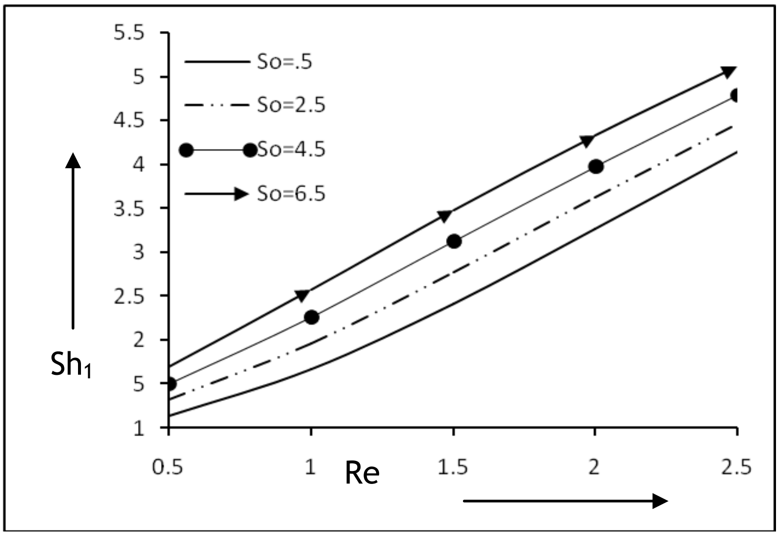

Figure 9. *Sherwood number Sh_1 (at the plate y=1) versus Re when Sc=0.6, M=1, Rm=0.3 and Ch=2*

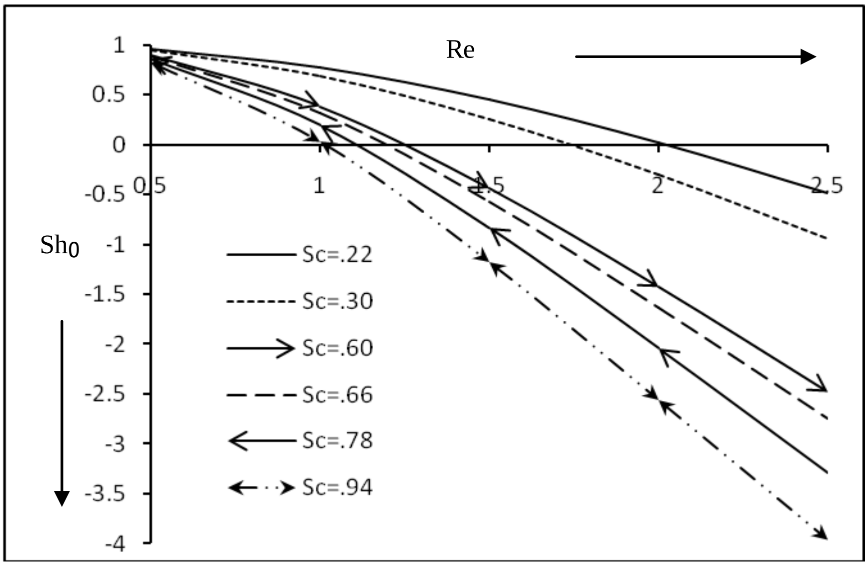

Figure 10. *Sherwood number Sh_0 (at the plate y=0) versus Re when S_0=0.5, M=1, Rm=0.3 and Ch=2*

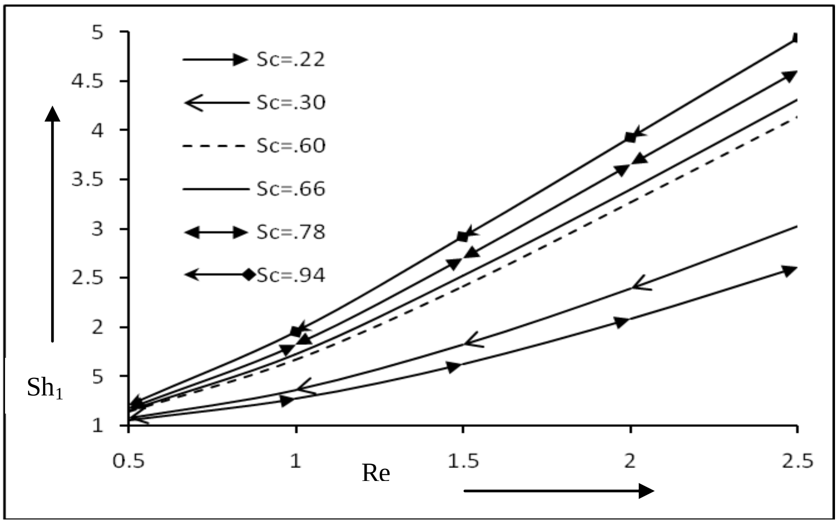

Figure 11. *Sherwood number Sh_1 (at the plate y=1) versus Re when S_0=0.5, M=1, Rm=0.3 and Ch=2*

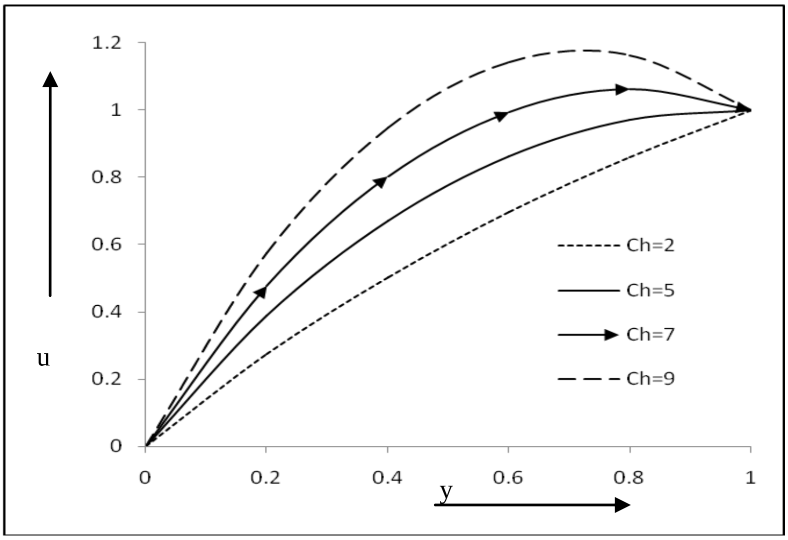

Figure 12. *Velocity distribution versus y when Rm=0.3, Re=0.5, S_0=0.5, Sc=0.6, M=1*

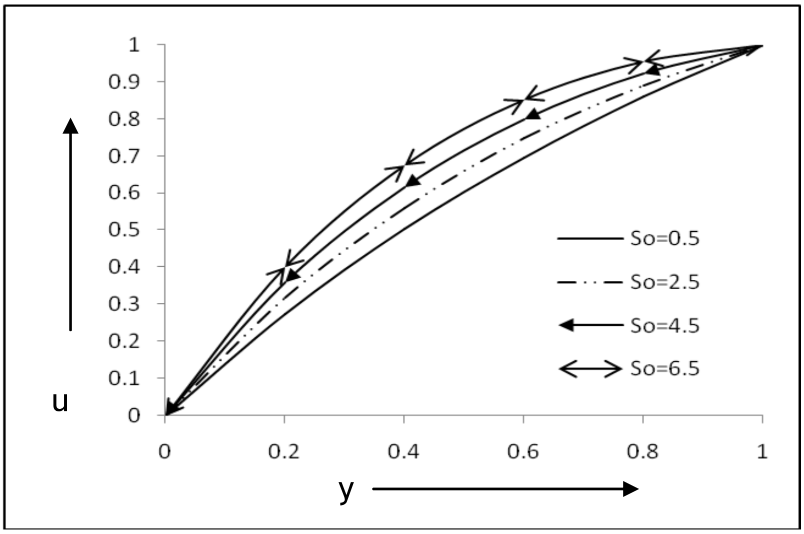

Figure 13. *Velocity distribution versus y when Rm=0.3, Re=0.5, Sc=0.6, Ch=2, M=1*

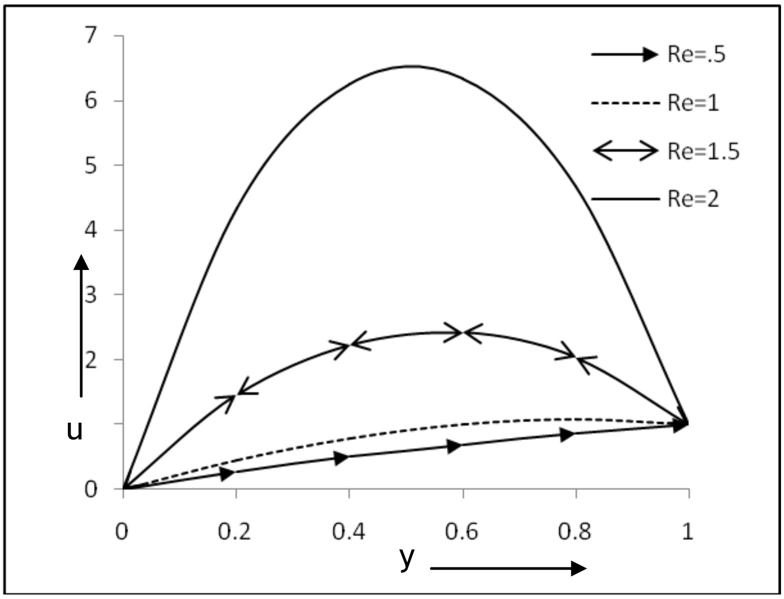

Figure 14. *Velocity distribution versus y when Rm=0.3, S_0=0.5, Sc=0.6, Ch=2, M=1*

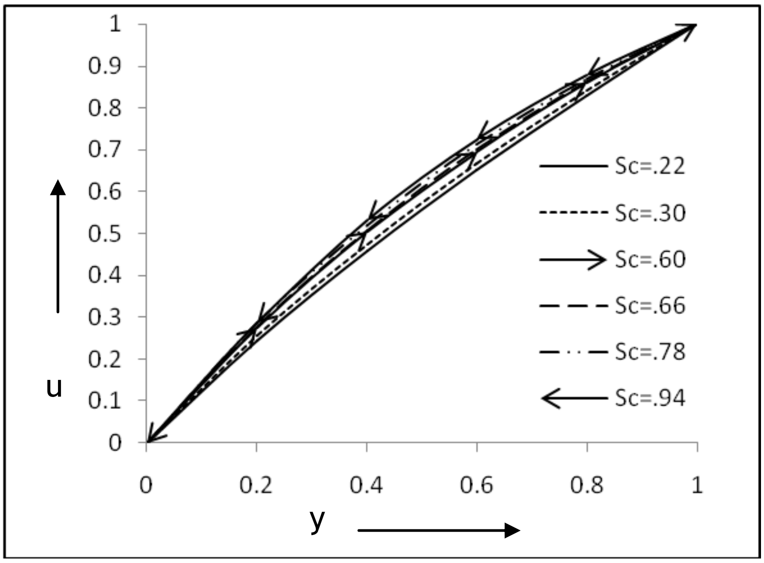

Figure 15. *Velocity distribution versus y when Rm=0.3, S_0=0.5, Re=0.5, Ch=2, M=1*

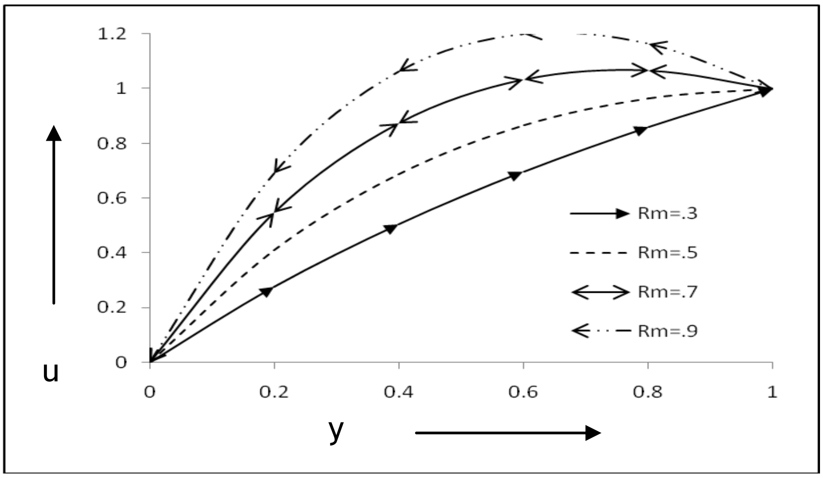

Figure 16. *Velocity distribution versus y when Re=0.5, S_0=0.5, Sc=0.6, Ch=2, M=1*

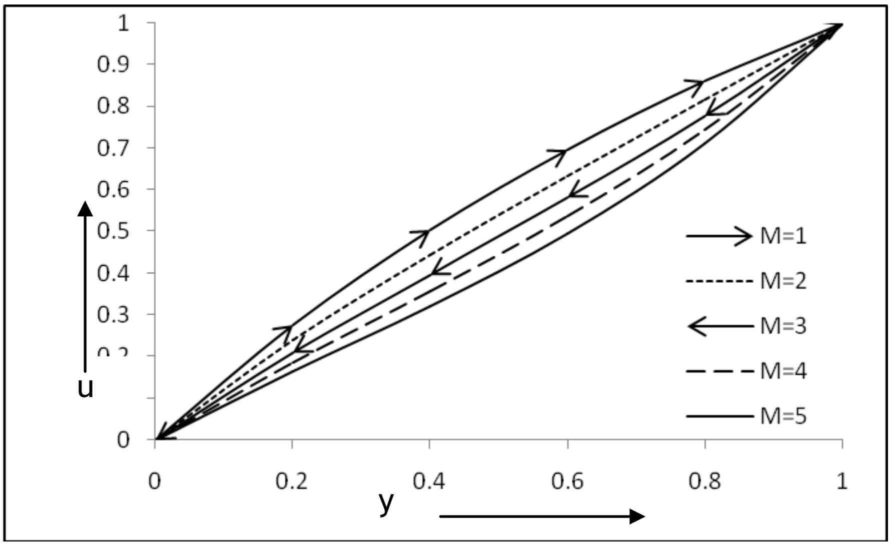

Figure 17. *Velocity distribution versus y when Rm=0.3, Re=0.5, S_0=0.5, Sc=0.6, Ch=2*

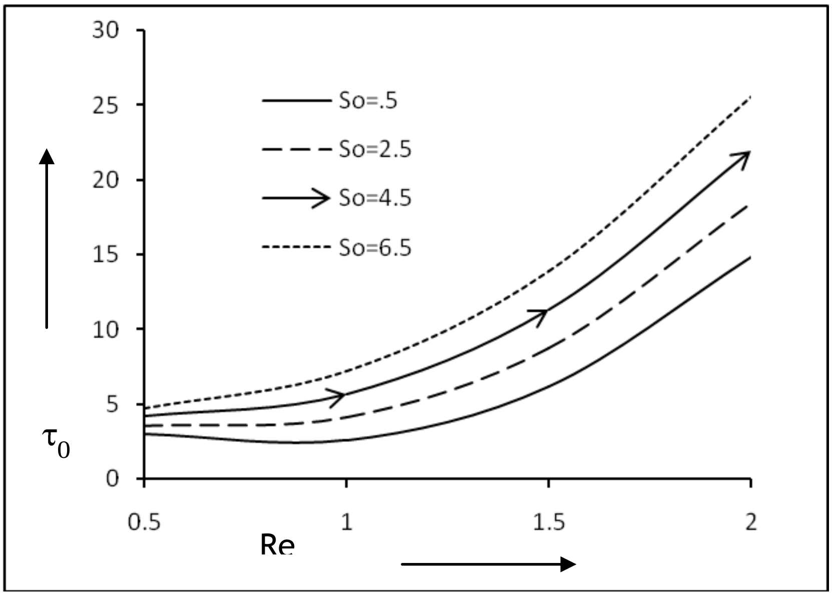

Figure 18. *Skin friction* τ_0 *(at the plate y=0) versus Re when Rm=0.3, M=1, Sc=0.6, Ch=2*

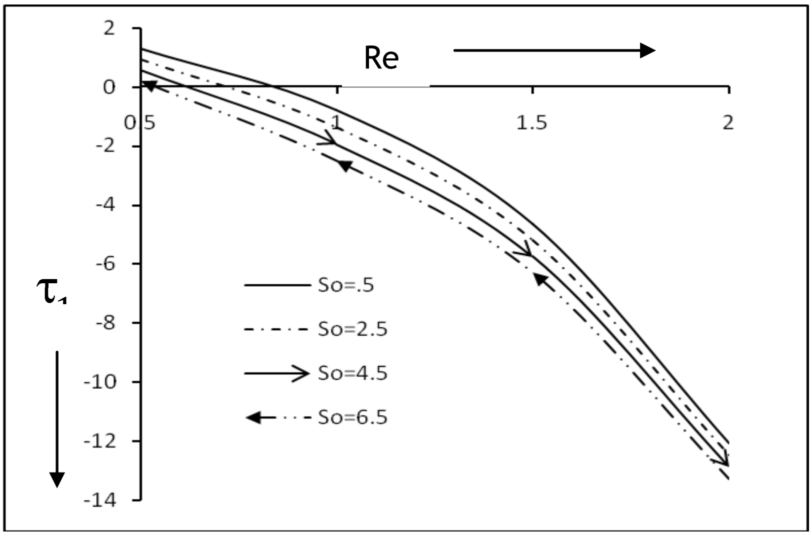

Figure 19. *Skin friction* τ_1 *(at the plate y=1) versus Re when Rm=0.3, M=1, Sc=0.6, Ch=2*

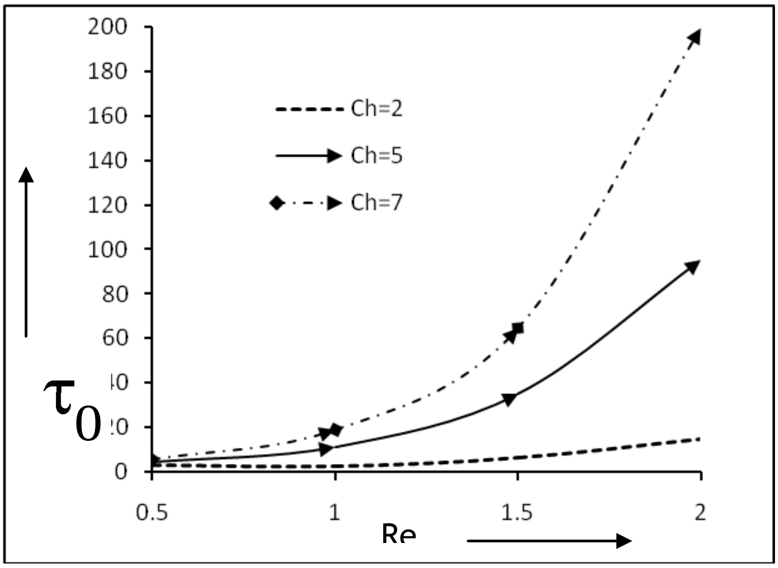

Figure 20. *Skin friction* τ_0 *(at the plate y=0) versus Re when Rm=0.3, M=1, Sc=0.6,* S_0*=2*

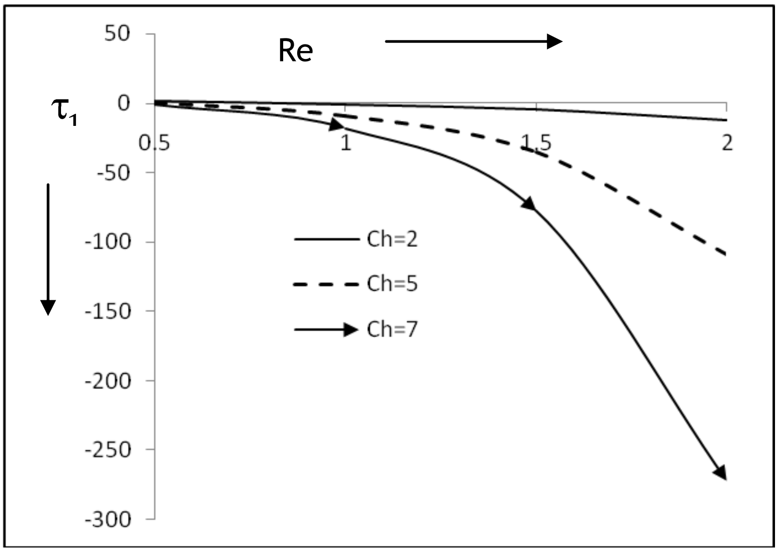

Figure 21. *Skin friction* τ_1 *(at the plate y=1) versus Re when Rm=0.3, M=1, Sc=0.6, Ch=2*

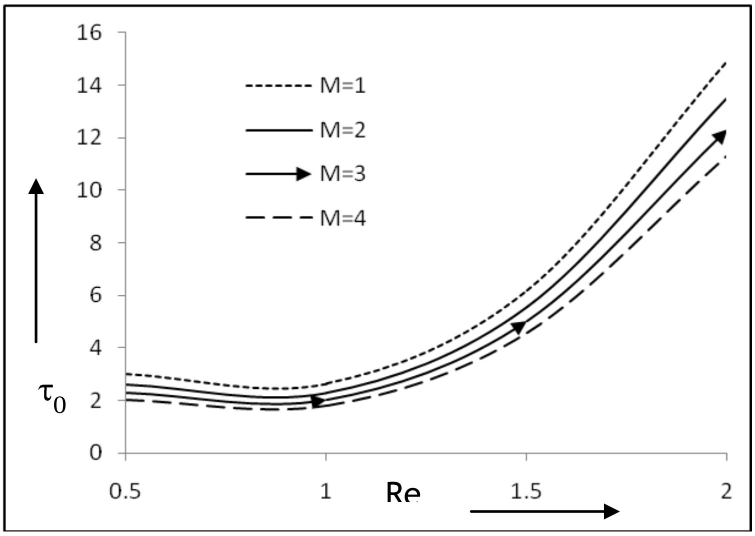

Figure 22. *Skin friction* τ_0 *(at the plate y=0) versus Re when Rm=0.3, Sc=0.6,* S_0*=0.5, Ch=2*

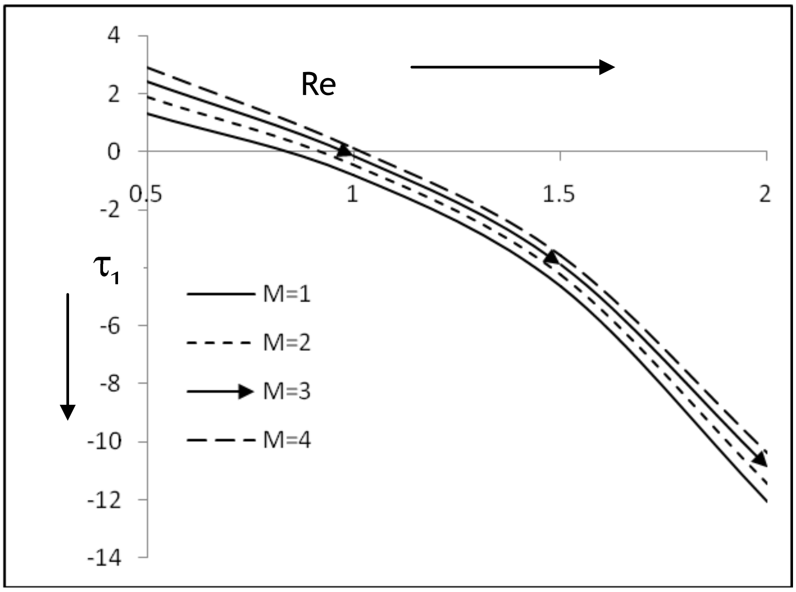

Figure 23. *Skin friction* τ_1 *(at the plate y=1) versus Re when Rm=0.3, M=1, Sc=0.6, Ch=2*

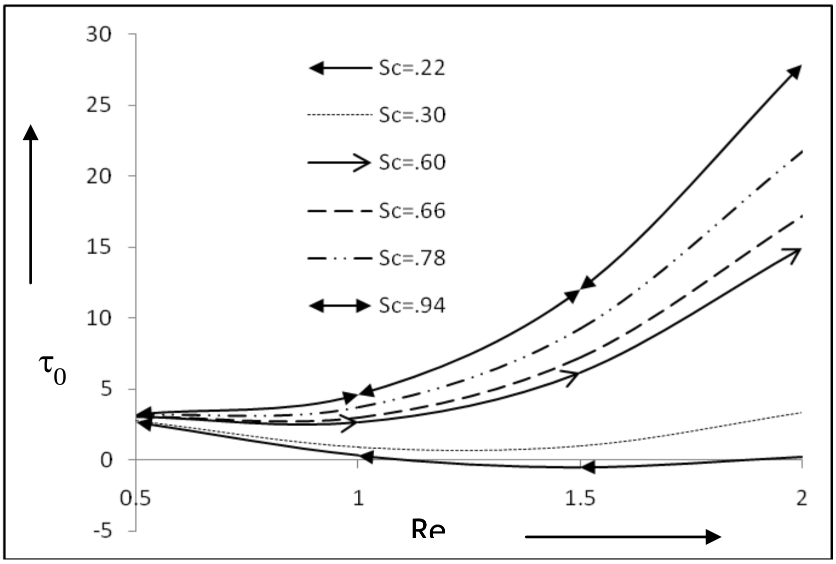

Figure 24. *Skin friction* τ_0 *(at the plate y=0) versus Re when Rm=0.3, M=1,* S_0*=0.5, Ch=2*

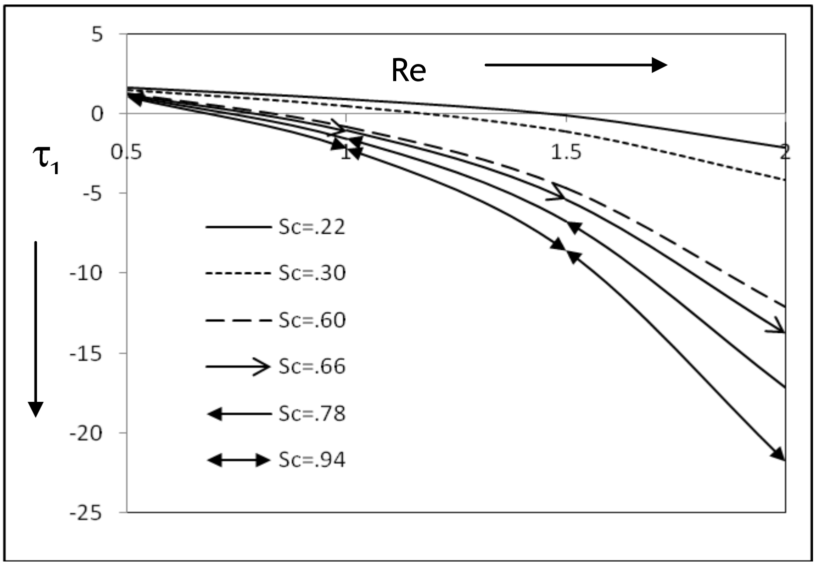

Figure 25. *Skin friction* τ_1 *(at the plate y=1) versus Re when Rm=0.3, M=1,* S_0*=0.6, Ch=2*

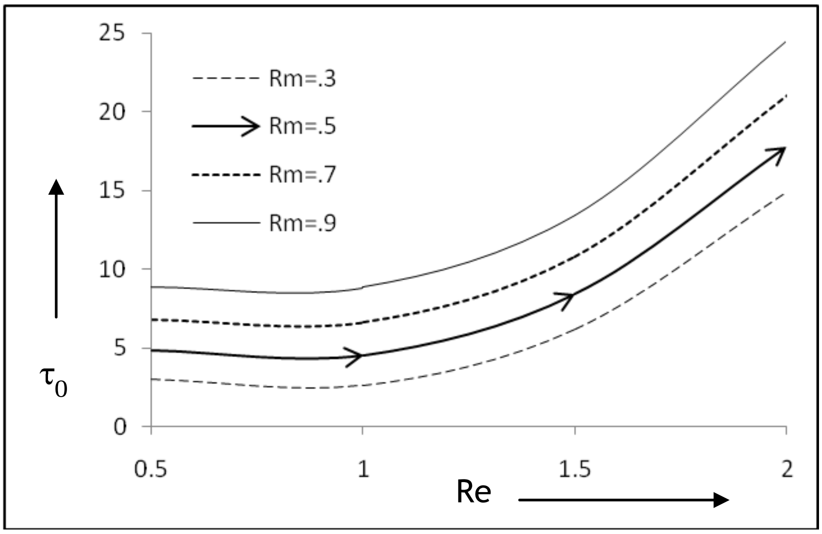

Figure 26. *Skin friction τ_0 (at the plate y=0) versus Re when Sc=0.6, M=1, S_0=0.5, Ch=2*

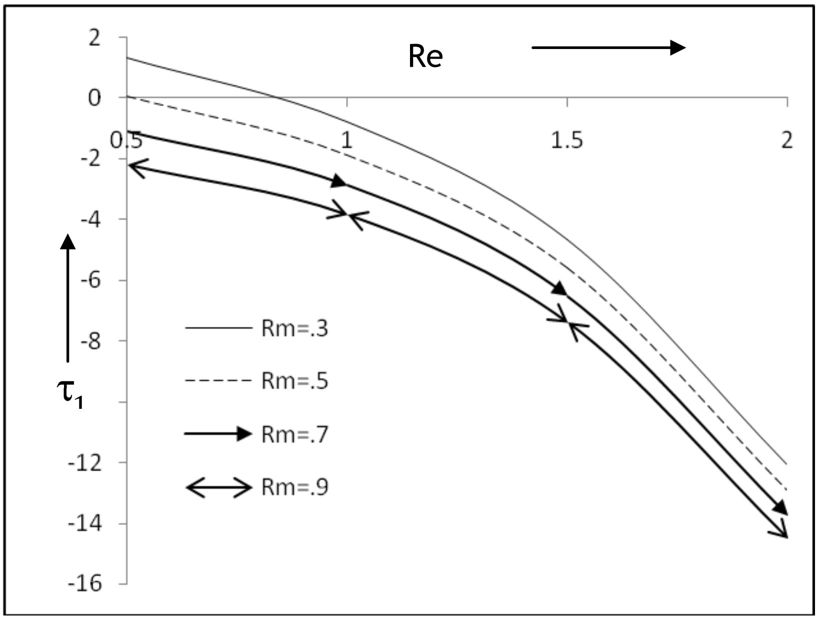

Figure 27. *Skin friction τ_1 (at the plate y=1) versus Re when S_0=0.5, M=1, Sc=0.6, Ch=2*

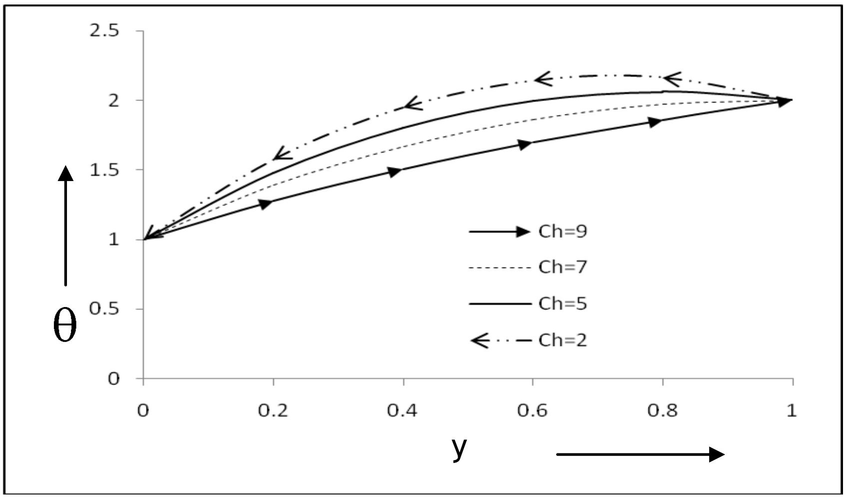

Figure 28. *Temperature distribution versus y when Re=0.5, Rm=0.3, M=1, Sc=0.6, Sr=0.5*

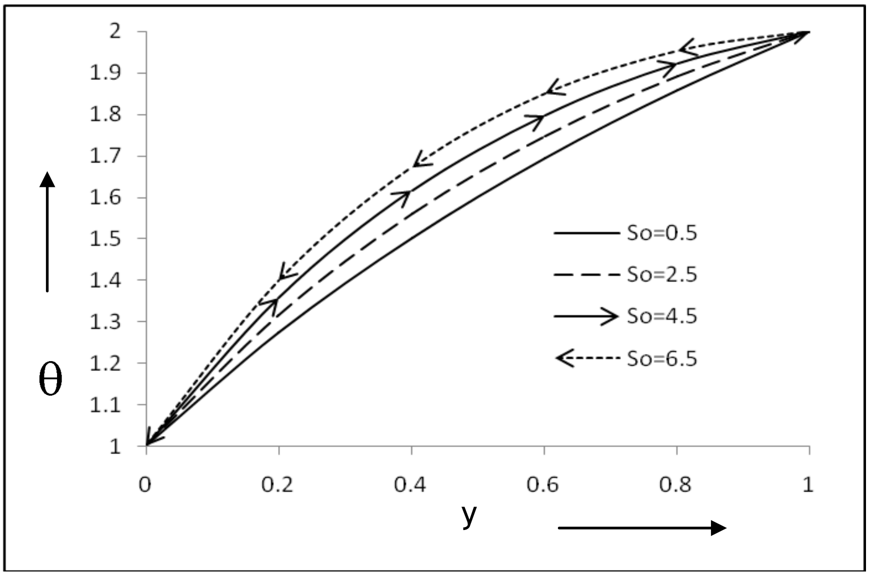

Figure 29. *Temperature distribution versus y when Re=0.5, Rm=0.3, M=1, Sc=0.6, Ch=2*

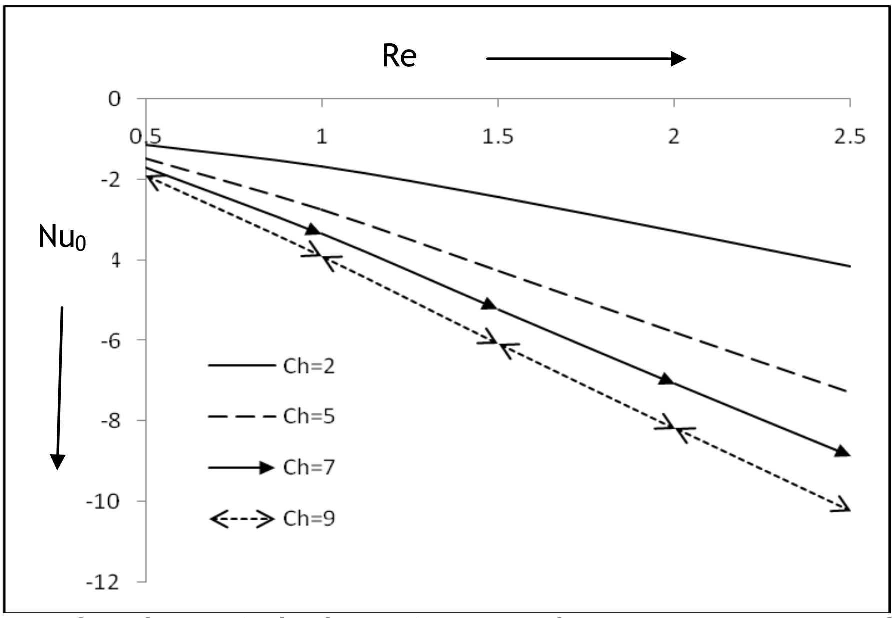

Figure 30. *Nusselt number Nu_0 (at the plate y=0) versus Re when Sc=0.6, Rm=0.3, M=1 and S_0=0.5*

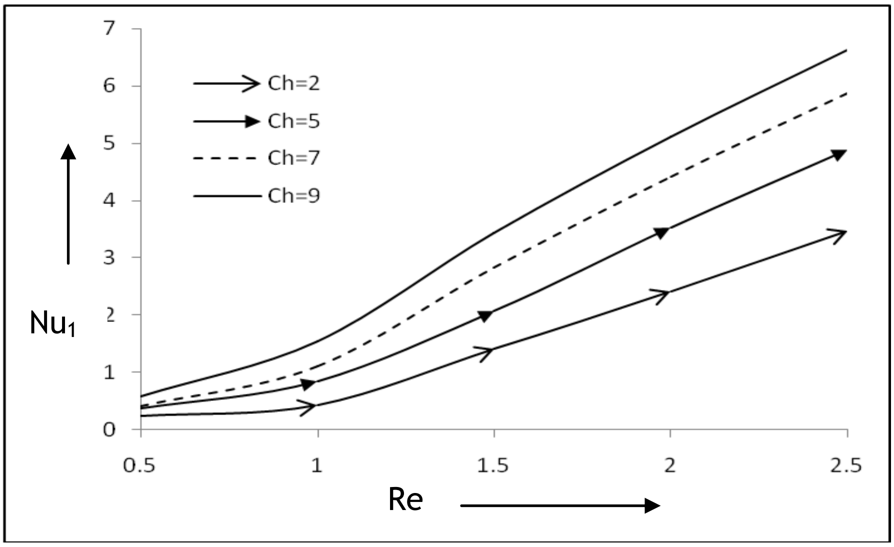

Figure 31. *Nusselt number Nu_1 (at the plate y=1) versus Re when Sc=0.6, Rm=0.3, M=1 and S_0=0.5*

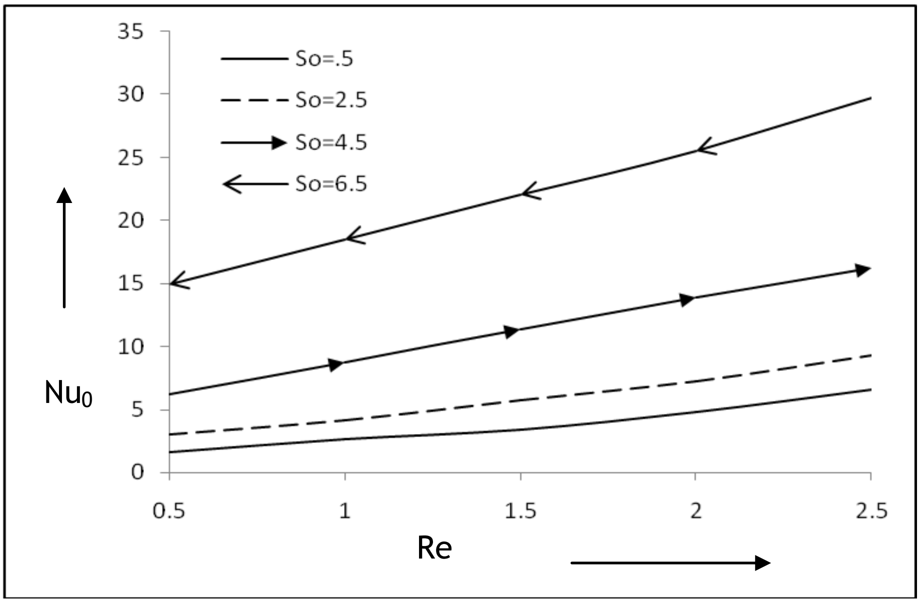

Figure 32. *Nusselt number Nu_0 (at the plate y=0) versus Re when Sc=0.6, Rm=0.3, M=1 and Ch=2*

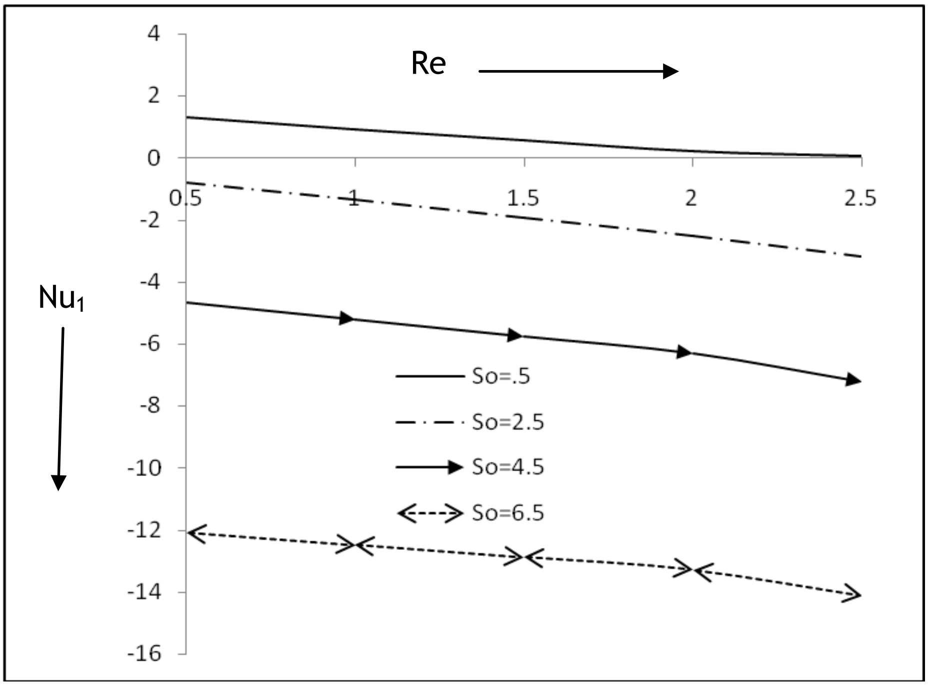

Figure 33. *Nusselt number Nu_1 (at the plate y=1) versus Re when Sc=0.6, Rm=0.3, M=1 and Ch=2*

Figures (12)-(17) depict the change of behavior of velocity profile u against y under the effects of chemical reaction parameter, Soret number, Reynolds number, Schmidt number, magnetic Reynolds number and Hartmann numbers. From these figures we observe that fluid motion is accelerated under the effects of chemical reaction, thermal diffusion and conductivity of the fluid. But due to the application of magnetic field and high rate of molar diffusivity, fluid velocity is retarded. We also notice from these figures that low viscosity causes the fluid velocity to increase.

The variation of skin friction at the plate y=0 and y=1 against Reynolds number for different values of S_0, Ch, M, Sc and Rm are demonstrated in figures (18)-(27). It is seen from these figures that skin friction at the plate y=0 is increased for S_0, Ch, Sc, and Rm but it shows reverse trend at the plate y=1. In other words we can say that drag force at stationary plate increases due to effects of thermal diffusion, chemical reaction and conductivity of the fluid but it is decreased due to molecular diffusivity. We have also noticed from these figures that due to application of magnetic field viscous drag decreases at stationary plate while it is increased at moving plate.

Figures (28) and (29) depict the change of behavior of temperature against y under the effects of chemical reaction and thermal diffusion. It is inferred from these figures that an increase in chemical reaction or a decrease in thermal diffusion leads the temperature to increase. Figures (30)-(33) exhibit how the Nusselt number Nu_0 at the plate y=0 and Nu_1 at the plate y=1 is affected by the parameters Re, Ch and S_0. These figures clearly establish the fact that Nu_0 falls due to the effect of chemical reaction whereas it rises at the plate y=1. It is also clear from these figures that under the effect of thermal diffusion Nu_0 increases at the stationary plate but it is decreased at the moving plate. Further we may conclude from these figures that under the effect of chemical reaction for low viscosity, rate of heat transfer decreases significantly at the stationary plate but it is raised at the moving plate.

8. Conclusions

1. Concentration decreases for chemical reaction and thermal diffusion whereas it is increased due to high rate of molar diffusivity and low viscosity.
2. Rate of mass transfer falls at the stationary plate due to the effect of chemical reaction and thermal diffusion but it shows reverse effect at the moving plate.
3. High rate of molar diffusivity causes rate of mass transfer decreases at the moving plate and increases at the stationary plate.
4. Fluid motion is accelerated under the effects of chemical reaction, thermal diffusion and conductivity of the fluid. But due to the application of magnetic field and high rate of molar diffusivity, fluid velocity is retarded.
5. Due to application of magnetic field viscous drag decreases at stationary plate while it is increased at moving plate.
6. Drag force at stationary plate increases due to effects of thermal diffusion, chemical reaction and conductivity of the fluid but it is decreased due to molecular diffusivity.
7. Fluid temperature falls due to thermal diffusion but it rises for chemical reaction.

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